Ramsay Maunder ASSOCIATES *Finite Element Specialists and Engineering Consultants*

Limit Analysis Using Finite Element Techniques

Seminar for the Advanced Structural Engineering Module
College of Engineering, Mathematics & Physical Sciences, University of Exeter (6th December 2011)

Aims

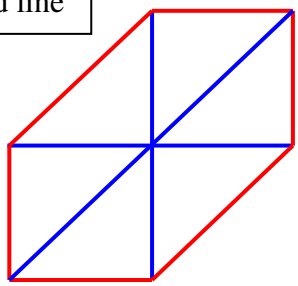
- **RMA** – who are we?
- **Limit analysis using finite element methods**, or Making the most of your structure.
- A few words before showing you what can be done with finite elements.
- **Structural Assessment/Design** – two sides to our lives.
- **Limit analysis** – addresses the question: how much load can the structure support before it collapses?
- **Collapse**, e.g. Piper's Row car park – 4th floor, 1997, a flat slab design

- **Limit analyses provides upper (unsafe) and lower (safe) bounds to this question, when the structure behaves in a ductile way**.

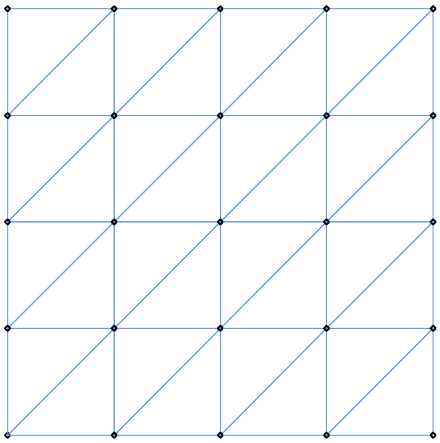
- **Upper bounds from yield line analysis** – potential yield lines delineated by the edges of triangular elements in a finite element mesh

    - **A basic fan mechanism** defined by a local patch of elements

red – hogging yield line

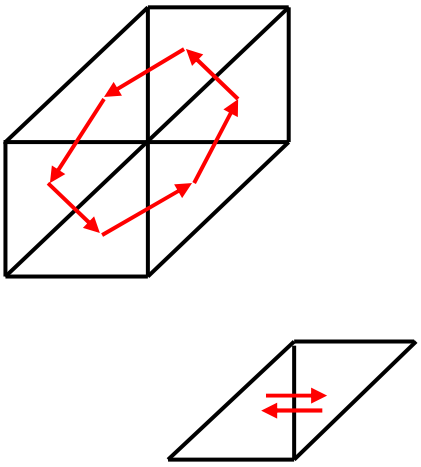


blue – sagging yield line

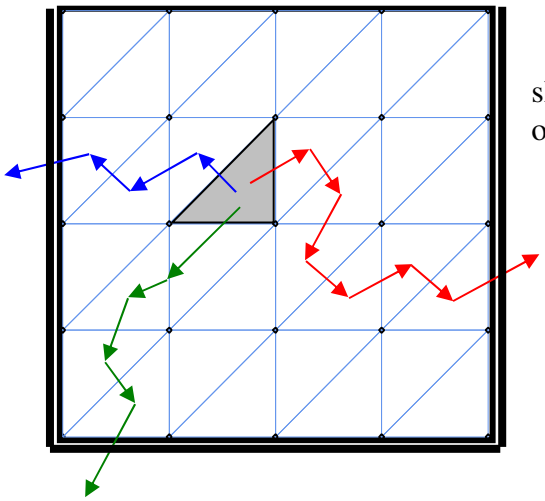


- Each basic fan mechanism provides an upper bound - we seek the **LEAST UPPER BOUND from all possible combinations**!
  - An optimisation problem for the computer.

- **Lower bounds from equilibrium analyses**: alternative load paths
  - **Equilibrium elements** transfer load between them to supports



Circuits of hyperstatic forces/moments



slab supported on three sides

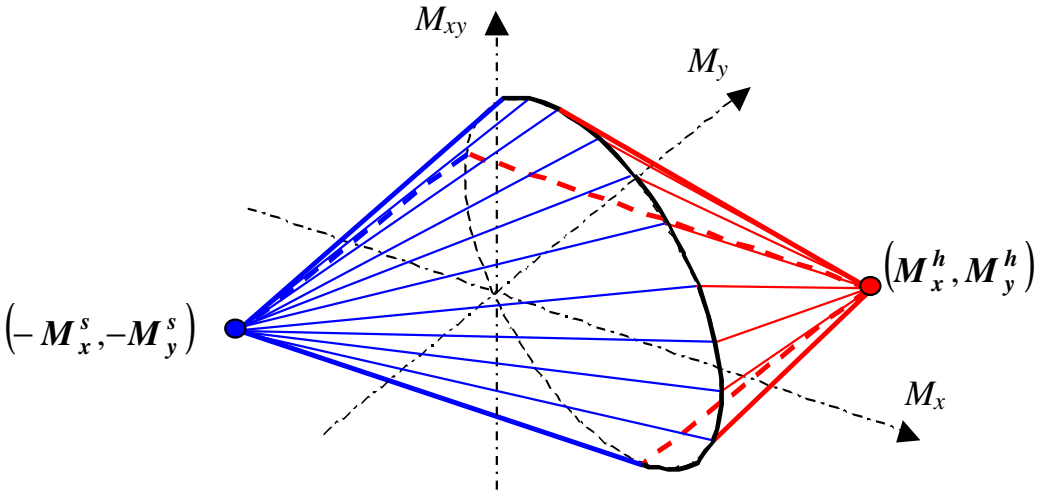
a sample of 3 load paths from a loaded element

- **hyperstatic force systems are balanced with ZERO load**;

- we seek the combination of hyperstatic force systems and load paths that enable the structure to support the **GREATEST LOAD** without stresses going outside the material yield surface.

- another optimisation problem for the computer.

- **What do yield surfaces look like?**



Biconic yield surface for reinforced concrete slabs.

Yield line examples/demonstrations

- Animation of a single basic fan mechanism

- A 2-way spanning square slab simply supported on 3 sides with a UDL
  - Fine unstructured mesh – multiple yield lines cf crack patterns at collapse
  - Simplified yield line pattern with geometric optimisation

- Collins anisotropic bridge deck problem
  - Comparison with Cam Middleton's solution;
  - Comparison with experimental yield line patterns.
  - Fine unstructured and simplified meshes to represent a collapse mechanism

- Verulam floor slab problem with isotropic/anisotropic properties
  - Demonstration of upper bound yield line solutions, and
  - Lower bound equilibrium solutions – moment redistribution from elastic solution, and yield utilisation ratio plots.
  - Observe "gaps" between upper and lower bound load factors, and **compare with the load factor implied by the "elastic" lower bound solution**.

Further information, examples and resources can be found on RMA's website: www.ramsaymaunder.com .