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NAFEMS Benchmarks for Plastic Collapse of Square Metal Plates

(NL7A and NL7B- Corrigenda)

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Introduction

The practising structural engineers' role in design is to ensure that his/her structure is fit for purpose. Normally the client, e.g., the owner of a new building, ensures this by prescribing that the design be undertaken to a particular code of practice. Modern codes of practice, such as Eurocode 3 (EC3) for steel structures and EC2 for concrete structures are based on a limit state approach. For the structure to be 'code compliant' a range of limit state conditions need to be satisfied and these are categorised as Serviceability Limit State (SLS) conditions and Ultimate Limit State (ULS) conditions. Of particular relevance to this article is the ULS condition of plastic collapse. Provided a structure or structural member is capable of redistributing the internal stress resultants then the external loads required to collapse the structure can be greatly increased beyond those to cause first yield. This requires the structural material to exhibit a decent degree of ductility so that strains beyond the yield strain can be accepted by the material. This condition is certainly satisfied for structural steels and for under-reinforced concrete.

As a result of the ductility of most structural steels at 'normal' operation temperatures, it becomes perfectly acceptable for the ULS condition of plastic collapse to be assessed using limit analysis and, indeed, this approach is deemed acceptable in the Eurocodes. There are relatively few commercial software systems available that offer a true limit analysis capability. As an alternative the engineer can perform an incremental analysis in conventional finite element analysis systems adopting an elastic, perfectly plastic material model together with the appropriate yield criterion. For reinforced concrete the appropriate yield criterion would be the maximum principal stress criterion whereas for steels and other ductile metals the von Mises criterion would be more appropriate.

The practising engineer wishing to test out his/her understanding of the plastic collapse of even simple structures is somewhat stymied by the lack of problems with know theoretical solutions. For example, as will be discussed in this article, even for the square plate there are no such solutions so that the best the engineer can do is to rely on numerically derived solutions hopefully of good provenance. This same problem exists for the developer of limit analysis software when he comes to undertake software verification. In the second of the NAFEMS Benchmark Challenges, [2], the author undertook a blind experiment in which he asked practising engineers for the collapse load of a uniformly loaded, simply supported rectangular steel plate. Two of the several solutions offered by the readers of the Benchmark Magazine agreed with each other and with the value obtained by the author's newly developed limit analysis software, RMA:EFE. On the basis that three different pieces of engineering software using different numerical approaches agreed with each other led the author to conclude that his software was producing the correct collapse load.

Using the same software, RMA:EFE, the author produced results for the uniformly loaded square steel plates presented in the NAFEMS document covering benchmark problems for material plasticity, [1] see Figure 1. The two cases presented by NAFEMS are for simple and for fixed supports, NL7A and NL7B respectively. He found here the published results to be incorrect with the collapse load offered for the fixed support case (NL7B) being some 16% greater than the more accurate prediction given by RMA:EFE.

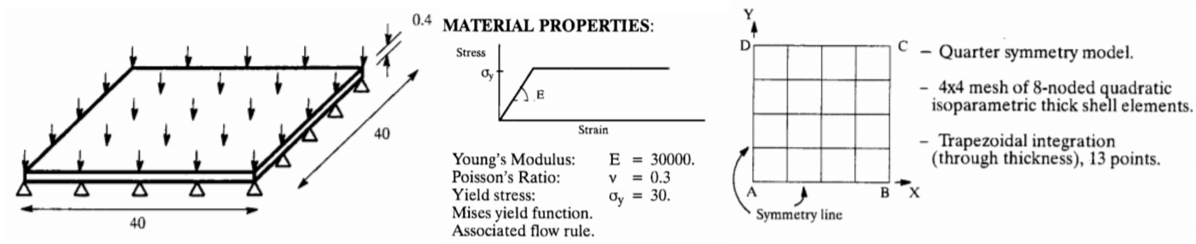

Figure 1: Geometry, material and mesh for NL7 Benchmarks from [1]

There is much more of interest to discuss than the space in this article allows and so here the author will simply present the results from RMA:EFE. However, the reader with time to explore further will find a more detailed article at RMA's website, [3]. The reader interested in exploring how his/her FE system performs on these benchmark problems is encourage so to do and the author would be interested to hear of the results produced by such an exercise.

RMA:EFE

The finer details of RMA's Equilibrium Finite Element (EFE) software can be found in [4]. But, put simply, the software adheres to the approach outlined in the lower bound theorem of plasticity, i.e., it finds a field of equilibrating stress resultants that does not violate the appropriate yield criterion whilst maximising the load carrying capacity of the structure. By definition the predicted collapse load is a safe one which lies below the true value.

In the case of thin plate members, the stress resultants are the three moment fields which equilibrate with the applied loads through Kirchhoff's equations. The yield criteria available in RMA:EFE are the square, maximum principal moment criterion and the elliptical criterion of von Mises.

The convergence of the collapse load for NL7A and NL7B are shown graphically in Figures 2 & 3 for uniform mesh refinement. Although the NAFEMS results are for the elliptical yield criterion, the results for the square criterion have also been included. The figures show contour plots of utilisation. This is a non-dimensional quantity formed by dividing the appropriate equivalent stress by the yield stress. Contours range from the minimum utilisation, shown as blue, to the maximum utilisation shown as red.

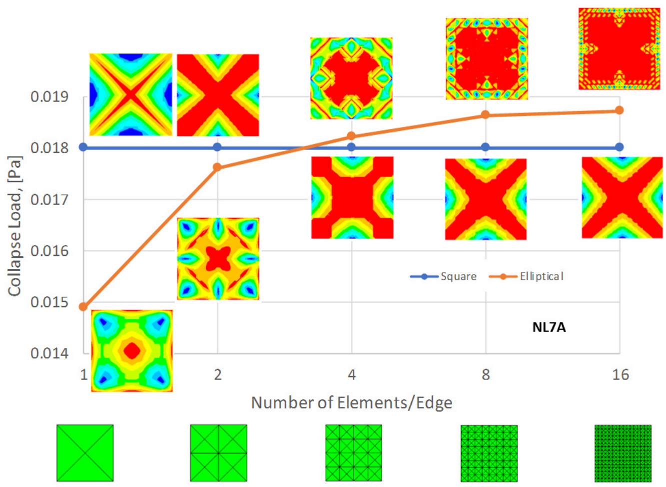

Figure 2: Convergence of collapse load for NL7A

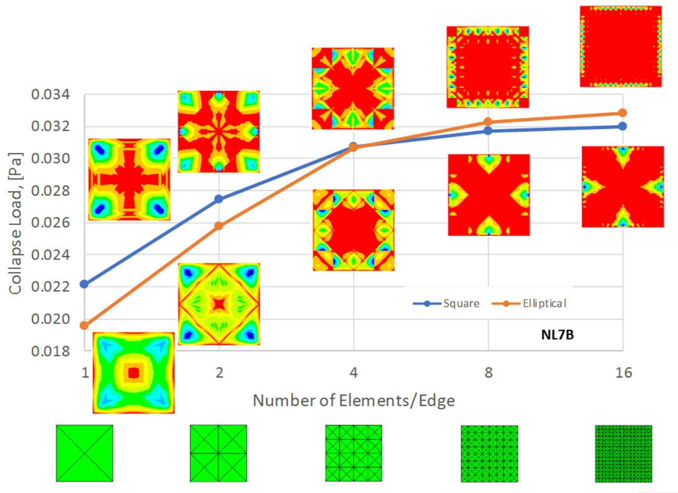

Figure 3: Convergence of collapse load for NL7B

The numerical results from this convergence study are presented in Table 1. For NL7A with the square yield criterion the result is independent of mesh refinement. This is because even with the coarsest mesh the exact collapse load is recovered. The stress resultant fields do change with mesh refinement as indicated by the contour plots of utilisation. This is a nice example of one of the properties of a limit analysis solution, i.e., that whilst there is a unique solution in terms of the collapse load, the moment fields are not unique.

For both problems, there are known theoretical solutions when using the square yield criterion and these are shown in the table. Richardson Extrapolation (RE) has been used to extrapolate the results for the number of elements/edge = 4, 8 and 16 to provide an estimation of the exact solution and the quality of this extrapolation can be judged by comparing the exact and estimated exact values for NL7B with the square yield criterion; the difference is less than 0.04%. Thus, it is possible to use with confidence the extrapolated collapse loads for the elliptical yield criterion as surrogates for the exact solutions – these are highlighted in red in the table.

	Support Condition	**NL7A**		**NL7B**	
	Yield Criterion	**Square**	**Elliptical**	**Square**	**Elliptical**
Number of Elements/Edge	**1**	0.018000	0.014895	0.022131	0.019578
	2	0.018000	0.017609	0.027459	0.025765
	4	0.018000	0.018228	0.030729	0.030665
	8	0.018000	0.018628	0.031672	0.032269
	16	0.018000	0.018732	0.031979	0.032784
	RE	/	**0.018768**	0.032126	**0.033027**
	Exact	0.018000	/	0.032138	/

Table 1: Numerical results from convergence study

In [1], *reference* solutions are provided from the literature together with *target* solutions obtained by finite element (FE) analysis. The reference solutions provided are incorrect but it is of interest to see how the target solutions compare with those presented in this paper. This comparison is made in Table 2.

Plate Configuration	Target from [1]	RMA:EFE	%age Difference
NL7A	0.01877	0.01877	0
NL7B	0.03852	0.03303	16.6

Table 2: Collapse loads (Pa) using the elliptical yield criterion

Thus, whilst the target solution for NL7A is very close to the value from RMA:EFE, the solution for NL7B is some 16% greater than the estimated exact value.

Closure

This short article has provided an updated and reliable pair of collapse loads for the NL7A and NL7B problems laid out in the NAFEMS documentation, [1]. It is understood that NAFEMS will be producing a new version of [1] to account for this and to account for any other updates that might be required in this publication.

References

[1] NAFEMS, ***Selected Benchmarks for Material Nonlinearity***, Vol. 2, R0030e (1993).

[2] Angus Ramsay, ***NAFEMS Benchmark Challenge Number 2; Assessment of a Simply Supported Plate with Uniformly Distributed Load***, NAFEMS Benchmark Magazine.

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[3] Angus Ramsay, ***NAFEMS Benchmarks for the Plastic Collapse of Square Metal Plates; the Complete Story***, RMA, 2020

[4] Edward Maunder & Angus Ramsay, ***Equilibrium Models for Lower Bound Limit Analyses of Reinforced Concrete Slabs***, Computers & Structures, 108-109, 100-109, 2012.