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The NAFEMS Benchmark Challenge

Calculation of the Hoop Burst Speed for Rotating Discs

In high speed turbomachinery, the discs which support the blades are highly loaded structural members. The kinetic energy in discs is considerable and whilst designed to avoid burst failure, if this occurs the surrounding structure is often not strong enough to contain the disc fragments, e.g, :

http://www.telegraph.co.uk/news/2016/10/30/american-airlines-plane-engine-flung-debris-in-rare-risky-failur/

The primary loading faced by a turbomachinery disc is that induced by rotation. This includes centrifugal self-loading and a radial load at the periphery due to the centrifugal loading of the attached blades and such loading induces stresses in the disc that are proportional to the square of the angular velocity ω. Although, the detailed design of a disc might include consideration of a range of potential failure mechanisms like, for example, low cycle fatigue induced by thermal and mechanical cycling, the basic disc shape is governed by the basic strength requirement to resist circumferential or hoop burst. Whilst hoop burst of a rotating disc is a phenomenon involving material plasticity and large displacements and strains, a rather simple approach, based on a linear-elastic analysis of an axisymmetric model of the disc, has been shown to provide sufficiently accurate predictions of the burst speed for the purposes of initial design. The method is attributed to Robinson and is discussed on p50 of Chianese Stefano's 2011 Master's thesis:

Robinson, E.: Bursting tests of steam-turbine disk wheels. Trans. ASME 66, page 373, 1944. & http://indigo.uic.edu/handle/10027/10185

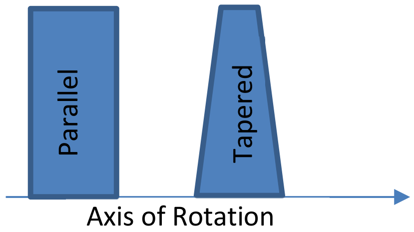
Hoop burst involves the circumferential or hoop stress σ_h which varies over the generator plane of the disc. Robinson's method requires the area weighted average hoop stress $\widetilde{\sigma_h}$ to be calculated for a given angular velocity ω. The angular velocity to cause the disc to burst is given in terms of the ultimate tensile strength, σ_{UTS}, of the material.

Inner radius = 0

Outer radius = 0.1m

Inner axial thickness = 0.03m

Outer axial thickness = 0.03m and 0.015m (parallel and tapered)



$\omega_b = \omega\sqrt{\frac{\sigma_{UTS}}{\widetilde{\sigma_h}}}$ where the average hoop stress is given by $\widetilde{\sigma_h} = \frac{\int \sigma_h dA}{\int dA}$ and A is the area of the disc generator plane.

Elastic Modulus = 210GPa

Poisson's Ratio = 0.29

Density = 7800kg/m^3

UTS = 1000MPa

Figure 1: Generator planes for the axisymmetric finite element analysis of two rotating discs

The Challenge

The reader is asked to use his/her finite element system to determine the angular velocity to cause the two discs of Figure 1 to burst based on a UTS of 1000MPa. For this study, no radial loads due to blades are considered. For the parallel sided disc the theoretical solution based on the Lamé equations can be used for software verification – see for example, Hearn, E.J., Mechanics of Materials, 3rd Edition, Butterworth-Heinemann, 2000. The second, tapered, disc has no theoretical solution and solution verification will need to be used to ensure that the mean hoop stress calculated by the axisymmetric finite element model has converged sufficiently.