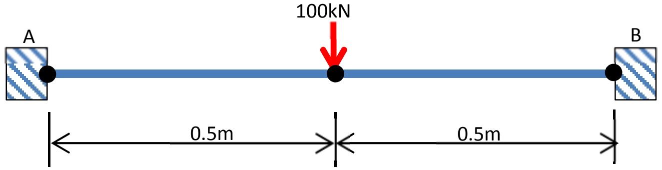
NAFEMS The NAFEMS Benchmark Challenge

Modelling Beam Members with Finite Elements

Commercial finite element systems generally offer a range of beam finite elements for the engineer to model his/her beam type structure. Different finite element formulations are often provided to cope with the standard Euler-Bernoulli beam, which does not include the influence of shear deformation, and with Mindlin and/or Hybrid formulations, which do account for shear deformation as in Timoshenko's theory. The Mindlin type element, which uses the same shape functions to approximate all displacements (translations and rotations) is often offered as a variable degree element, e.g., two-noded linear shape functions or three-noded quadratic shape functions, providing the engineer with the opportunity to adopt a *p*-type mesh refinement strategy in addition to the usual *h*-type approach. The Mindlin element with both linear and quadratic shape functions was used to model the problem shown in figure 1(a) where the beam is fixed at both ends with a concentrated load at the centre of the span. A two-element mesh was used with a node at the centre of the beam where the load is applied. The finite element reactions agree for both degrees of shape function (p=1 and p=2), and with the verification documentation supplied by the software vendor.

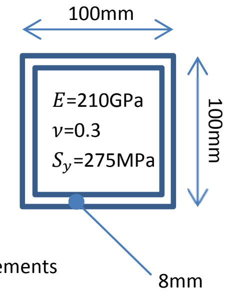


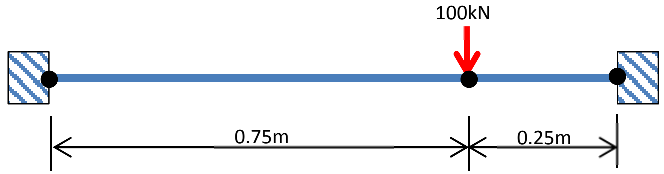
FE Reactions

Shape Function	V_A (kN)	M_A (kNm)	V_B (kN)	M_B (kNm)
Linear	50.00	-12.50	50.00	12.50
Quadratic	50.00	-12.50	50.00	12.50

● End Nodes of Beam Elements

(a) Software verification problem





FE Reactions

Shape Function	V_A (kN)	M_A (kNm)	V_B (kN)	M_B (kNm)
Linear	20.83	-7.29	79.17	11.46

Element Table Reactions

Shape Function	V_A (kN)	M_A (kNm)	V_B (kN)	M_B (kNm)
Linear	20.83	-0.52	79.17	1.56

(b) Design problem

Figure 1: Beam problems and finite element reactions

On the basis of the quality of the results for the problem of figure 1(a) achieved with a minimal two-element mesh, the engineer might be led into thinking that the same quality would be achieved if the concentrated load were moved to three quarter's span as shown in figure 1(b). The finite element reactions certainly satisfy vertical and rotational equilibrium in the global sense. However, since moment equilibrium involves an unknown statically indeterminate constant moment one cannot be sure that the moment reactions are the correct values. Evidence that they might be incorrect is provided by the so-called element table results provided by the software used to produce these results, which shows moments at the supports that have been extrapolated from the integration points.

The Challenge

The reader is asked to investigate the two beam problems presented in figure 1 using the various beam elements offered in their finite element system (including and excluding shear deformation) with the aim of providing accurate results for the moment reactions, the maximum displacement and the maximum bending moment. Evidence of solution verification should be provided in terms of the convergence of the quantities of interest with mesh refinement and a prediction of the minimum number of elements required to obtain a accuracy of 1% in all quantities should be provided.