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Large Thermal Strains in Finite Element Analysis

Introduction

The practising engineer will probably be familiar with the idea of thermal expansion or contraction of structural members or mechanical components. If this deformation is constrained then stresses will be induced in the member or component. In finite element (FE) analysis the idea of thermal expansion/contraction can be used as a rather straightforward device to model a number of phenomena. The interference fit between rotating components, for example, is a good example where the standard small thermal strain approximation is generally appropriate. There are other cases where the use of the small thermal strain approximation can lead to significant errors. For example, the extension/contraction of a hydraulic ram can be modelled as an axial element with temperature change used to control the ram length. In this case it is necessary to adopt the proper thermal strain equation in order to obtain accurate results in an FE model which includes large displacements and rotations. This technical note reminds readers of the difference between the small and large thermal strain theories and presents examples where small strain theory is appropriate and where large strain theory is required to obtain the correct solution.

Interference Fit using Small Thermal Strain Theory

It is common practice in finite element (FE) analysis to use thermal expansion/contraction to model the influence of a shrink or interference fit between a rotating component and the shaft onto which it is fitted. Such interference fits are used to ensure that the rotating component, e.g., a pulley wheel, and the shaft remain in contact at speed (when the rotating component is growing radially at a greater rate than the shaft) and that the appropriate torque can be transmitted between the two components; the torque being transmitted through friction between the two components. In this context, the interference is usually small compared to the nominal radius and a linear-elastic analysis is usually appropriate. The choice of the *coefficient of linear thermal expansion*, a term which is often shortened to *coefficient of expansion*, is, in this case, normally assumed to be independent of temperature and is somewhat arbitrary provided that the product of the coefficient of expansion and the temperature change produces the required interference.

In a recent project, RMA was asked to checked the fit for a pulley and supported on two bearings – see Figure 1.

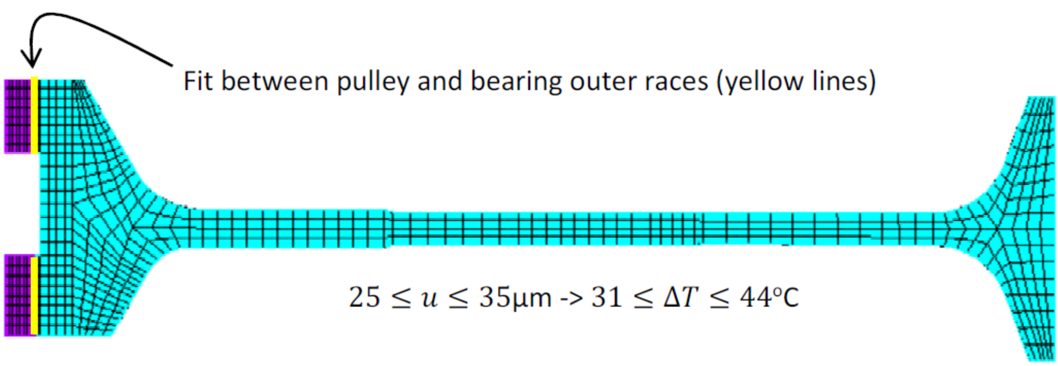


Figure 1: Axisymmetric model of pulley and the outer races of two bearings

The radial interference at the nominal radius, r_i, between the bearing and pulley is shown in Figure 2.

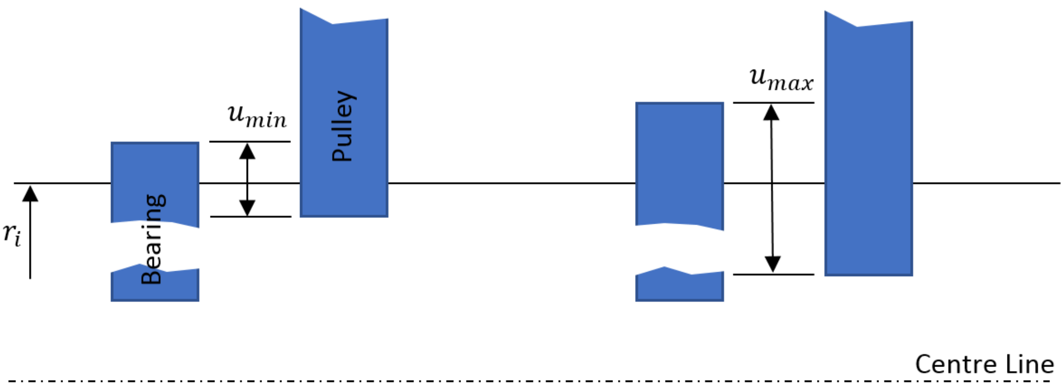


Figure 2: Bearing/Pulley interference fit

The symbol u is used for radial displacement and from the radial tolerances on the components lies in the range $u_{min} \leq u \leq u_{max}$. Small thermal strain theory gives the radial strain, ε_r, as shown in Eq. (1).

$$\varepsilon_r = \frac{u}{r_i} = \alpha \Delta T \tag{1}$$

The range of interference values for the pulley are shown in Figure 1. Note that these are in micro meters, i.e., they are small compared to the interface radius, which is 70mm, and lead to radial strains of 0.036% and 0.05% respectively. The standard understanding of small strain theory is that it is appropriate for strains that are significantly less than 1%, i.e., $\varepsilon \ll 1\%$ and it is seen that this condition is met. Using a coefficient of thermal expansion of 11.5μm/m/°C gives the temperature changes shown in Figure 1.

Large Thermal Strain Theory

The incremental expression for the thermal strain is given in Eq. (2) in terms of the change in length dl, of an element of length l subject to an incremental temperature change, dT. The constant of proportionality is α, the coefficient of linear thermal expansion and this is normally a function of temperature.

$$\frac{dl}{l} = \alpha dT \tag{2}$$

For a finite rather than infinitesimal temperature change from an initial temperature T_i to a final temperature T_f the change in length from the initial length l_i to the final length l_f is obtained by integrating Eq. (1) as indicated in Eq. (3).

$$\int_{l_i}^{l_f} \frac{dl}{l} = \int_{T_i}^{T_f} \alpha dT \tag{3}$$

Since the thermal strain is being used as a device for creating a prescribed displacement then we can reasonably adopt a coefficient of expansion that is independent of temperature, i.e., $\alpha \neq \alpha(T)$. Performing the integration of Eq. (3) leads to the expression given in Eq. (4) where the symbol Ln is the natural logarithm of the function in brackets.

$$Ln\left(1+\frac{\Delta l}{l_i}\right) = \alpha\Delta T \qquad \Delta T = T_f - T_i \text{ and } \Delta l = l_f - l_i \tag{4}$$

Eq. (4) may be rearranged to determine the overall thermal strain as shown in Eq. (5).

$$\frac{\Delta l}{l_i} = e^{\alpha\Delta T} - 1 \qquad \text{Large thermal strain (exact)} \tag{5}$$

When the product αΔT is small then Eq. (5) tends to the usual linear expression given in Eq. (6).

$$\frac{\Delta l}{l_i} = \alpha\Delta T \qquad \text{Small thermal strain (approximation)} \tag{6}$$

The relative error in using the linear expression of Eq. (6) is shown in Eq. (7) and has been plotted on log-log axes in Figure 3.

$$error = \frac{e^{\alpha\Delta T} - 1 - \alpha\Delta T}{e^{\alpha\Delta T} - 1} \tag{7}$$

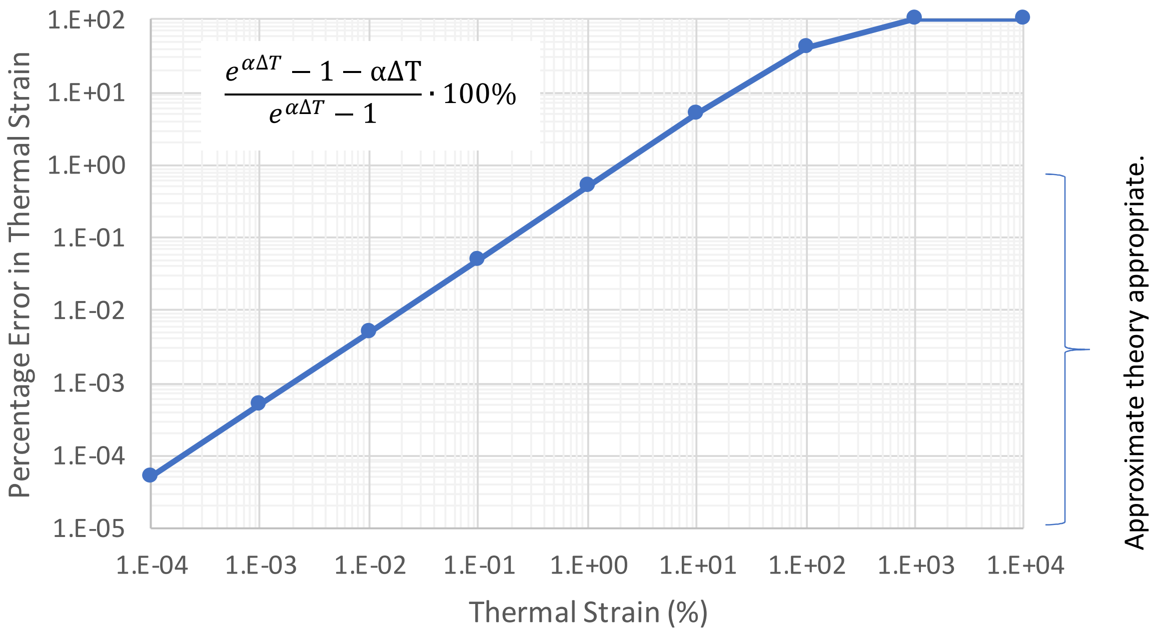


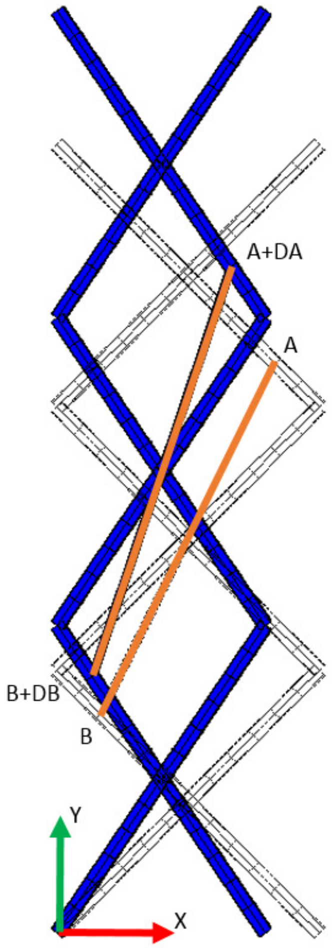
Figure 3: Error in thermal strain when using the linear approximation

An engineer might be prepared to accept a 1% error in his calculation and to ensure that this is not exceeded then the product $\alpha\Delta T$ should not exceed about 0.01, i.e., the thermal strain should not exceed about 1%. Thus, for the case of the interference fit considered earlier, the thermal strain was about 0.05% which leads to an error in of between 0.02 and 0.03%, i.e., a trivial error in engineering terms.

Hydraulic Ram Displacement using Large Thermal Strain Theory

RMA was recently involved in a project where a thermal expansion/contraction was used to drive a mechanism. The mechanism in question was a scissor lift the height of which is controlled by a hydraulic ram. The length of the ram, which might reasonable be modelled as a single axial element, can be adjusted by thermal expansion or contraction. There is, however, a significant difference here compared to the use of this approach for the interference fit between rotating component and shaft. The difference is that in order to drive the scissor mechanism from it lowest to its highest position requires a significant extension of the hydraulic ram. The members of the scissor lift translate and rotate by a significant amount, as does the ram, and this may only be considered using a non-linear FE approach which includes the effects of large displacements and rotations.

The scissor lift mechanism is shown in Figure 4 and shows the lift in the position where the FE model was built (member angles of 45^o) and in a raised position (blue) resulting from a temperature increase of 100^{o}C applied to the ram.



	X (m)	Y (m)
A	2.9467	7.6613
B	0.5893	2.9467
DA	-0.6060	1.3028
DB	-0.1212	0.5011

$\alpha = 0.001\text{m/m°C}$

$\Delta T = 100\,\text{°C}$

$l_i = 5.2711\text{m}$

$\Delta l = 0.5544\text{m}$

$l_f = 5.8255\text{m}$

Figure 4: Hydraulic ram driving scissor mechanism

By considering the geometry or kinematics of the mechanism, a relationship between the height, h, of the lift and the length of the hydraulic ram, l, may be obtained, i.e., $l = f(h)$. Thus, for a given lift height the ram length can be determined. The scissor lift under consideration had members that were about 5m in length and for this lift the (non-linear) relationship between ram length and lift height is shown in Figure 5.

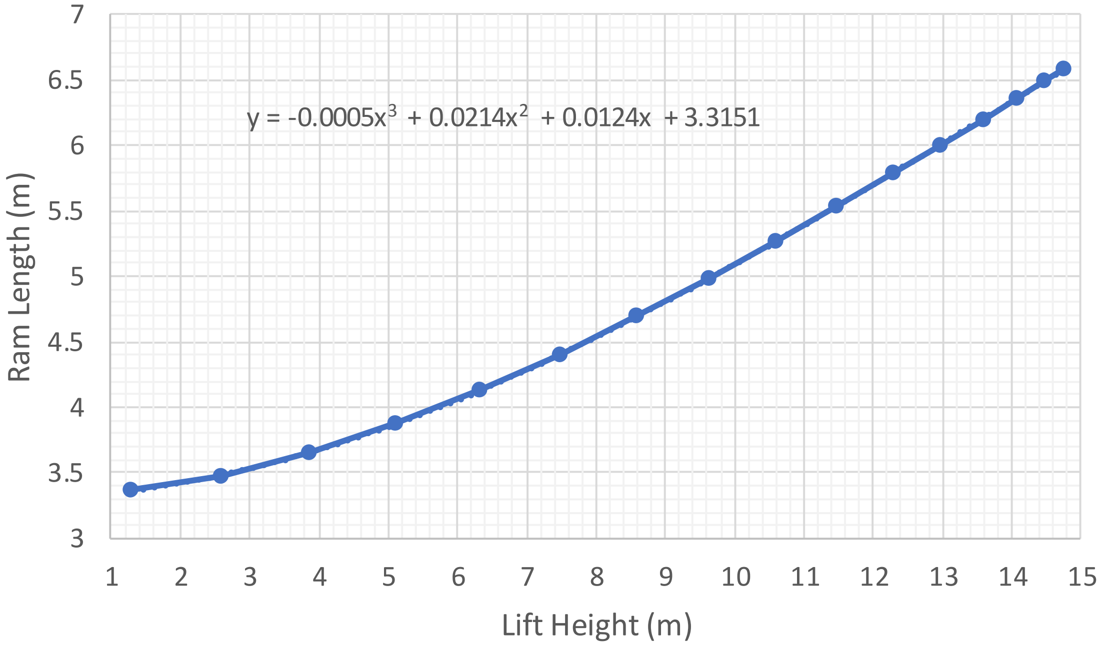


Figure 5: Ram length as a function of lift height

The temperature change required to achieve a given lift height calculated from Eq. (4) and Eq. (5) are shown in Figure 6 with the orange line corresponding to the exact value and the grey line to the approximate value.

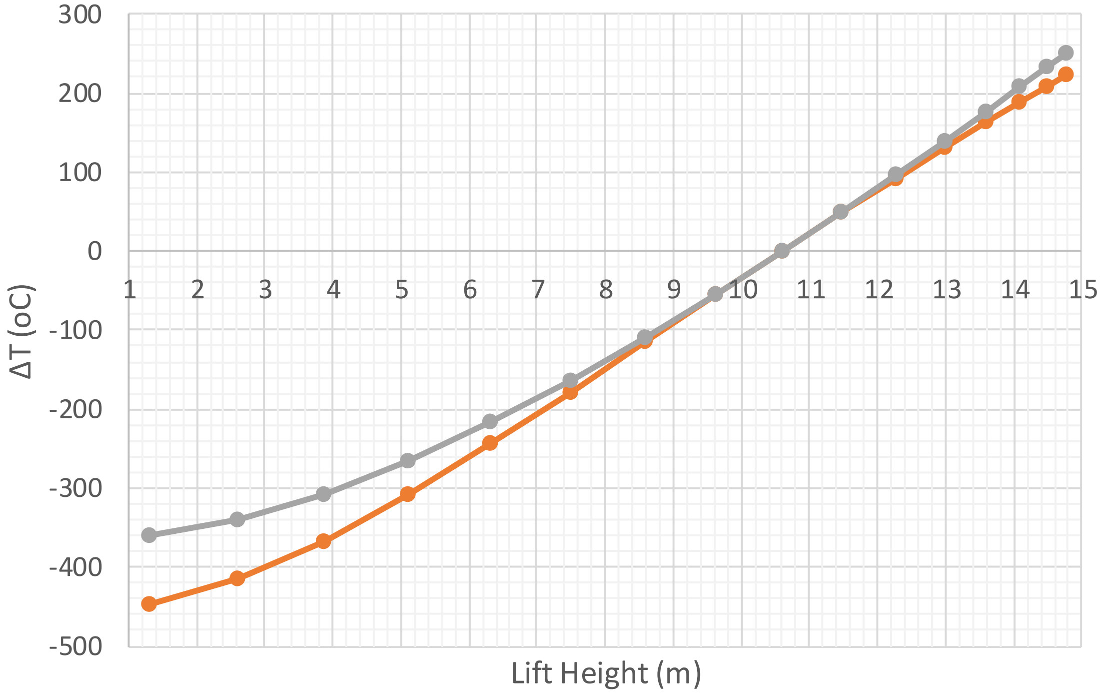


Figure 6: Temperature change as a function of lift height

The FE model was constructed so that the members are angled at 45° and in this configuration the height of the lift is 10.61m and the ram length is 5.2711m. If we were interested in raising the lift by 1.80m to a height of 12.41m then the required ram length is 5.8255m which can be achieved with a temperature change of 100°C for a thermal expansion coefficient of 0.001 µm/m/°C.

Closure

Large strain theory gives a thermal strain greater then small strain theory! The rule then might be to calculate the thermal strain using small strain theory and if it is greater than 1% the engineer should adopt large strain theory.

This technical note reports a case where it is necessary to account for large displacements and rotations in a FE analysis. Thermally induced expansion is used to model the change in length of a hydraulic ram which raises or lowers the scissor lift. The change in length of the ram is significant when compared to the length of the ram and so the thermal strains required are large, i.e., much greater than 1%. Since the FE model includes large displacements, the accurate thermal strain theory must be adopted when working out the temperature change to apply to the element representing the hydraulic ram.