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Equilibrium Finite Elements

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The hybrid-stress or equilibrium finite element formulation offers the practicing engineer a number of advantages over the conventional displacement element used almost exclusively in proprietary finite element software. These advantages stem from the nature of the finite element approximation which, for equilibrium elements, satisfies, *a priori*, the equations of equilibrium in a strong, point wise, sense. The engineer using equilibrium elements can rest assured that for ductile structures or components, the stress field obtained, irrespective of the level of mesh discretisation, satisfies equilibrium and, in conjunction with an appropriate strength criterion, leads to a safe or lower-bound estimate of the limiting load. This property, which exists for the elastic solution, can be exploited further in limit analysis where the elastic solution may be optimised to maximise the lower-bound.

RMA have produced a finite element program called EFE which has been written specifically for the commercial exploitation of equilibrium finite elements. The program currently offers a range of two-dimensional elements including membrane, plate and axisymmetric. Further planned work will lead to inclusion of line and solid element types. It currently performs linear-elastic analysis for these element types and also includes the automated yield-line for the upper-bound limit analysis of plate structures and a subsequent lower-bound optimisation based on the yield-line results.

An important feature of the software is that it is real-time analysis enabled. This means that, having set up an analysis, the user may manipulate model data, e.g. geometry, material properties, loads etc, and immediately obtain graphical and numerical feedback on structural response. This feature is a prerequisite for automated design optimisation and EFE can be used, in this manner, for the generalised optimisation of structural response.

The presentation will summarise the theoretical differences in conventional displacement and equilibrium elements and show how these differences manifest themselves to the practising engineer. To illustrate design optimisation a problem taken from the turbomachinery industry will be presented and discussed. Recent research and development work at RMA has concentrated on the limit analysis of reinforced concrete plates and this work will be discussed with results presented from both upper- and lower-bound techniques.

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EFE Equilibrium Finite Elements

for safe structural analysis and design optimisation