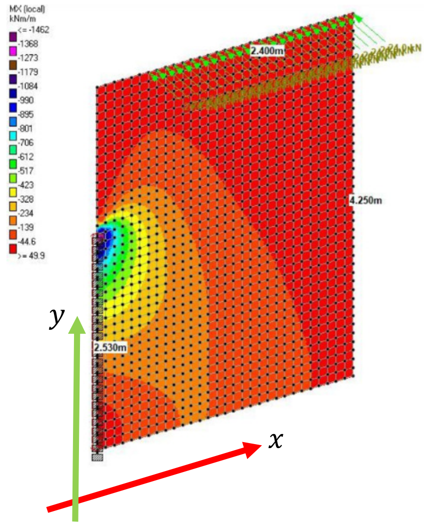
Ramsay Maunder ASSOCIATES *Finite Element Specialists and Engineering Consultants*

Limit Analysis of a Reinforced Concrete Wing Wall

This brief technical note looks at the following question posed on LinkedIn:

Dear all, I'm designing a wing wall with a crash barrier on top. Collision load is 600kN spread over 2.4m to be applied on the top. Total height is 4.25m and length of wingwall is 3m.The same I have modeled as plates in staad. plate size is 0.1mx0.1m. The fixity is along the vertical plane about the connection with abutment. How should I be considering the moment values for design/detailing? The moment seems to be varying quite rapidly from 1462kNm to 234kNm within 1.8m. How should I go about extracting the results and manual calculations to arrive at a suitable convergence. Thank you



There are a couple of minor points to note about the post. Firstly, the moments are actually moments per unit length and so should be specified as kNm/m. Secondly, the line load of 600kN spread over a length of 2.4m has been distributed evenly as 24kN on 25 nodes. Whilst this distribution of nodal forces is statically equivalent to the specified line load, i.e., the distribution of nodal forces has the same resultant force and moment as the specified line load, it is distributed in a manner that is inconsistent with the finite element formulation. To distribute consistently one first recognises that for a linear plate element with two nodes per edge, each node attracts one half of the uniform line load multiplied by the length of the element edge. The line load is 600/2.4=250kN/m and the length of an element edge is 2.4/24=0.1m. Thus, the total load on an element edge is 250x0.1=25kN with each node taking 25/2=12.5kN. As such, the two end nodes should attract 12.5kN whereas the intermediate nodes should attract 25kN so the total load is 2 x 12.5 + 23 x 25 = 600kN. This is a minor issue in this case since the peak moments are well away from the loaded region and Saint Venant's principle should mean that the moments away from the load are uninfluenced by the way in which the load is applied provided it is statically equivalent to the specified load. Whether or not this is a user or a software error is not certain but it is advised that this be checked and that in future loads are applied in a consistent manner lest one comes unstuck in a situation where the moments of interest are close to the applied load.

The peak elastic moments seen at the end of the supported edge are high and might well be singular which, as noted in the post, makes it rather difficult to be able to reliably specify a suitable reinforcement configuration for the wing wall. In essence, we are dealing with a design problem for which there are no sensible serviceability limit state, (SLS), conditions to be respected but where there is an important ultimate limit state (ULS) condition that needs to be covered. The load case given in the post is understood to be an extreme case where plastic deformation is to be accepted (barriers that have been impacted are likely to be replaced) but plastic failure is unacceptable. Thus, the question is to find the reinforcement for the wing wall that provides the required level of strength. To answer the question thus posed above is one that can be solved through limit analysis.

Limit analysis can be undertaken for reinforced concrete (RC) designs. Limit analysis assumes the behaviour of the structural component to be rigid, perfectly-plastic and also assumes infinite ductility. It is generally considered as a conservative approach because it does not include the strengthening phenomena of strain-hardening of the reinforcement bars or membrane action as the structure deforms under the applied load. Limit analysis holds to the principles laid down in the theory of plasticity. There it is stated that there are two broad approaches to solving the limit analysis problem. The first is the kinematic approach known as the yield line approach. Here, the engineer is required to postulate a collapse mechanism in terms of yield lines. The danger with this approach is that it is an upper-bound approach such that poorly postulated collapse mechanisms will lead to over-estimates of the strength of the slab. The complementary approach is the lower-bound approach which requires the engineer to postulate an equilibrating moment field for collapse and the danger of this approach is that the solution, whilst conservative, might be so conservative that the solution is uneconomical in term of the required reinforcement. However, with modern computational software problems such as the wing wall can be solver using both lower and upper bound techniques leading to two estimates of the collapse load. These estimates will bound the exact solution and are generally extremely close so that the uncertainty in the solution is minimal.

In this short post I will not discuss in any detail the computational methods. There is an informative article on RMA's website which readers might like to look at:

https://www.ramsay-maunder.co.uk/knowledge-base/publications/yield-line-analysis-of-reinforced-concrete-slabs-is-the-10-rule-safe/

The upper bound technique leads to a yield line pattern describing the collapse mechanism and a load factor, λ_u ,which is greater than the exact value, i.e., $\lambda_u \geq \lambda$. The lower bound technique leads to a moment field which may be illustrated as principal moment trajectories and a load factor, λ_l, which is less than the exact value, i.e., $\lambda_l \leq \lambda$.

The wing wall was analysed using an isotopic and homogeneous moment capacity of 100kNm/m for both front and rear layers of steel and the results are shown in Figure 1.

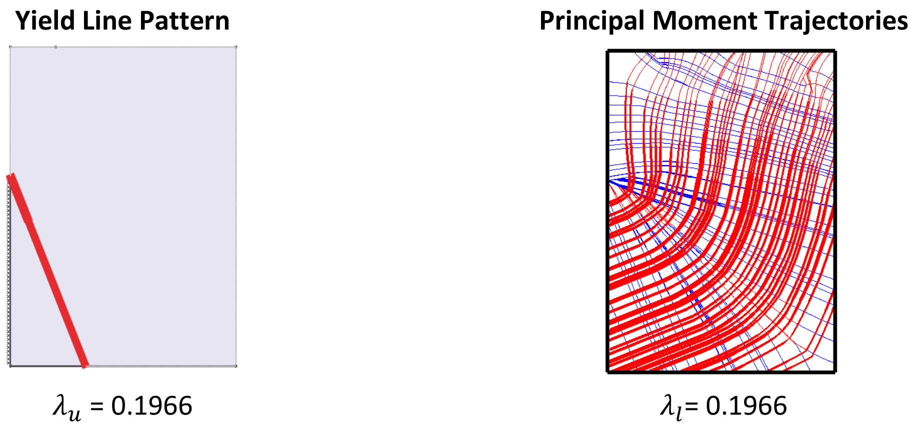


Figure 1: Limit analysis solutions for original wing wall configuration

The load factor for both upper and lower bound solutions agree to four significant figures. The moment capacity required to provide a wing wall with sufficient strength to take the applied load is then 100kNm/m ÷ 0.1966 = 509kNm/m.

There is some dispute in the comments to the post regarding whether or not the bottom edge of the wing wall is supported. If the bottom wall is fixed then the solutions given in Figure 2 are obtained.

Yield Line Pattern **Principal Moment Trajectories**

λ_u = 0.2376 λ_l= 0.2375

Figure 2: Limit analysis solutions for wing wall configuration with additional support on bottom edge

The load factor for both upper and lower bound solutions agree to three significant figures. The moment capacity required to provide a wing wall with sufficient strength to take the applied load is then 100kNm/m ÷ 0.2375 = 521kNm/m.

One of the advantages of the lower bound approach is that the moment field at collapse is obtained. This enables the designer to understand the flow of principal moments through his/her structure and to begin to recognise how the reinforcement might be rationalised so as to minimise the amount of reinforcement required for the wall. Such design optimisation is possible which can significantly reduce the amount of reinforcement required. This might be an important step with the wing wall which, will presumably, be manufactured in large quantities. An example of how a simple rationalisation leads to a 50% reduction in the required reinforcement is presented here:

https://www.ramsay-maunder.co.uk/knowledge-base/publications/equilibrium-finite-elements-for-rc-design/

In this example referenced above the reinforcement was rotated in order that two of the rows of rebars could be completely removed without influencing the load capacity of the slab. For the wing wall a different strategy can be used. A clue to how the reinforcement can be reduced is to examine the way in which it is utilised. With the lower bound approach, the moment fields at collapse are known and contours of utilisation, which is the ratio of the moment demand to the moment capacity, can be plotted. Utilisation of the complete reinforcement can be considered as can the utilisation of the hogging and sagging reinforcement individually. Utilisation contours for the initial assumed reinforcement is shown in the second row (100%) of Figure 3. The average utilisation is calculated by integrating over the slab and is reported in the figure (U). The key point to note is that the average sagging utilisation is only 14%. As such it would be sensible to reduce the amount of sagging reinforcement. Four levels of sagging reinforcement are considered as shown in the figure.

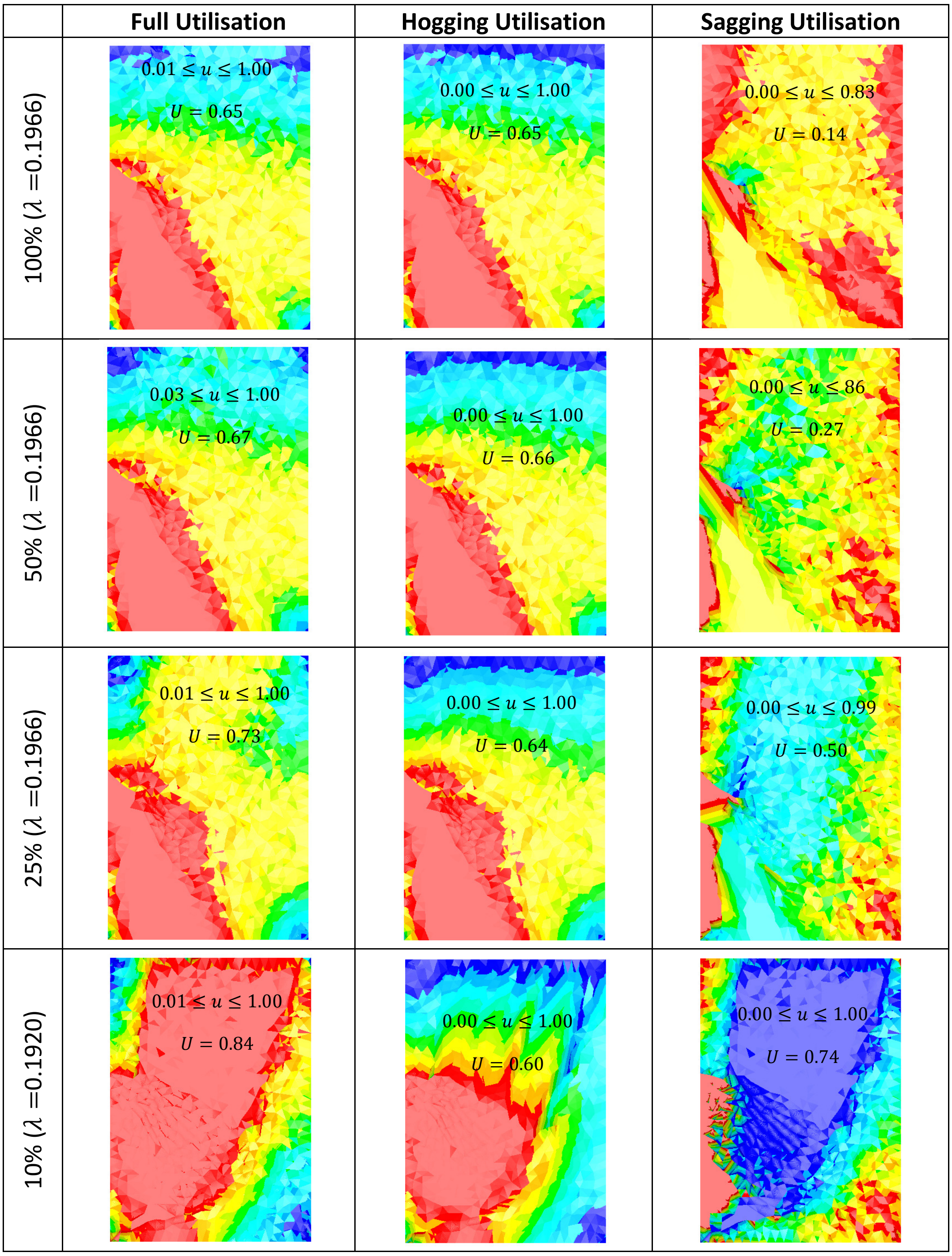
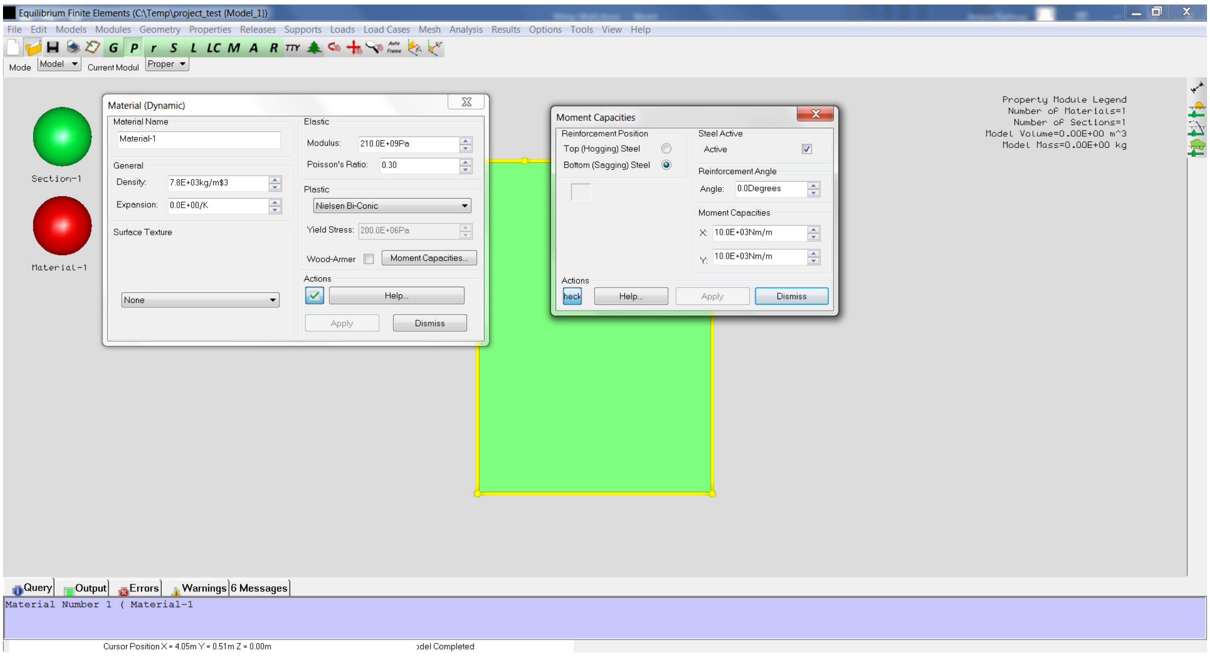


Figure 3: Utilisations for four levels of sagging reinforcement

As the sagging reinforcement is decreased from 100% to 25% the load factor and therefore the capacity of the slab remains unchanged as does the simple hogging collapse mechanism as predicted by the hogging utilisation. If the sagging reinforcement is decreased further to 10% then a small reduction is seen in the load factor and the predicted collapse mechanism now looks to include a significant region of sagging.

From the engineering perspective the required hogging reinforcement would be 100 ÷ 0.1920 = 521kNm/m with a sagging reinforcement requirement of 0.1 x 521 = 52.1kNm/m. This is, of course, a significant reduction in the steel, 90% for the sagging steel or 45% overall.

It should be noted that although this sort of analysis can be undertaken in conventional finite element software, it is far simpler and quicker to run it in a limit analysis program. In RMA's software (EFE) the only change required is the moment capacities which is easily performed in a simply dialogue as shown in Figure 4. The limit analysis then only takes a few seconds to complete enabling the engineer to rapidly assess different reinforcement configurations.



It is hoped that this brief Technical Note is of value to the original poster of the LinkedIn article and to a wider audience. The wing wall example appears to have a singularity in the elastic moment field which makes determination of the moment capacity rather unreliable. Since this problem is governed by ULS considerations then the most appropriate and simplest approach to design of reinforcement is to adopt limit analysis techniques. Modern software is now available for such purposes and, as shown, provide extremely reliable predictions of the collapse load for a given reinforcement layout. Should any reader require more information about the limit analysis approach then please contact RMA at:

https://www.ramsay-maunder.co.uk/contact-us/