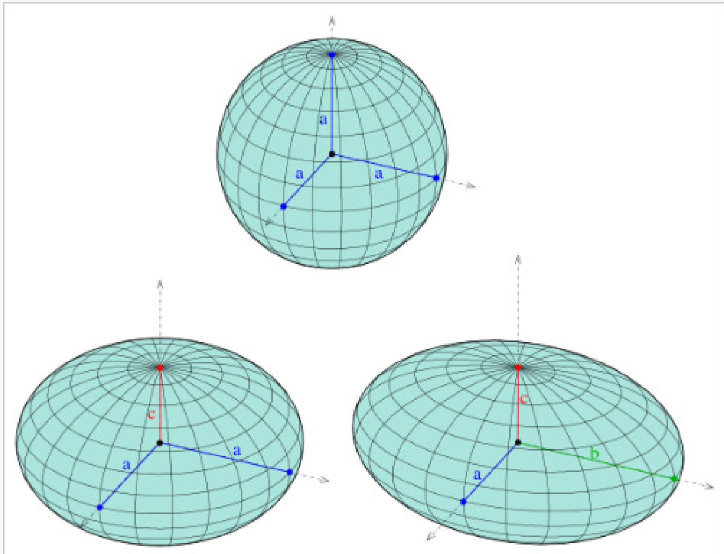
The Inertial Ellipsoid

The practising engineer might find it useful, particularly when dealing with skeletal structural models, beams, plates, point masses etc, to be able to visualise the inertial properties of his/her structure. For a three-dimensional structure the ellipsoid provides an opportunity to satisfy this desire. Three examples of ellipsoids are shown in Figure 1.



Examples of ellipsoids with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$:

sphere (top, a=b=c=4),
spheroid (bottom left, a=b=5, c=3),
tri-axial ellipsoid (bottom right, a=4.5, b=6, c=3)

https://en.wikipedia.org/wiki/Ellipsoid

Figure 1: Ellipsoids – taken from Wikipedia

The principal moments of inertia for the ellipsoid are given in terms of its mass, m, and the length of the semi-axes a, b and c as shown in Eq. (1).

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{Bmatrix} a^2 \\ b^2 \\ c^2 \end{Bmatrix} = \frac{5}{m} \begin{Bmatrix} I_{xx} \\ I_{yy} \\ I_{zz} \end{Bmatrix} \tag{1}$$

The mass is also given in terms of these values and the mass density ρ as shown in Eq. (2).

$$m = \frac{4}{3}\pi abc\rho \tag{2}$$

For a given finite element (FE) model the following mass/inertia properties will be known and reported by the FE software:

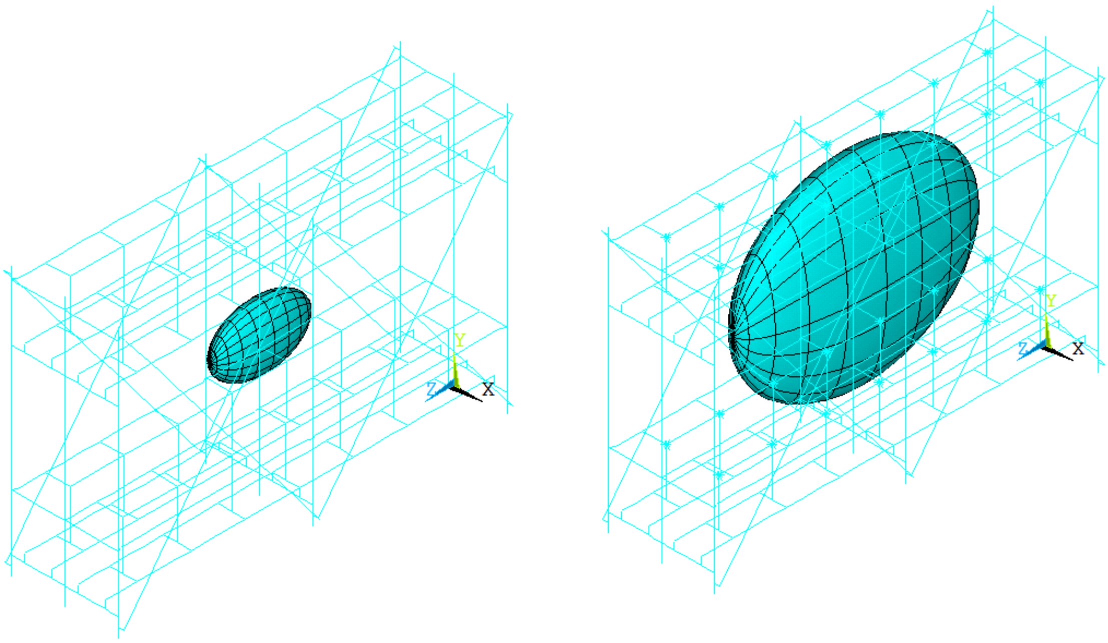
(a) mass
(b) location of the centre of mass (CoM)
(c) principal moments of inertia
(d) orientation of these with respect to some global coordinate system

Whilst it would be ideal if we could form an ellipsoid that represents all the above data, this is not possible. By inverting Eq. (1), as shown in Eq. (3), the semi-axis lengths may be determined from given FE inertias. It will be noted however that this expression involves the mass of the ellipsoid which, from Eq. (2), is also dependent on the semi-axis lengths.

$$\frac{5}{2m}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}\begin{Bmatrix} I_{xx} \\ I_{yy} \\ I_{zz} \end{Bmatrix} = \begin{Bmatrix} a^2 \\ b^2 \\ c^2 \end{Bmatrix} \qquad (3)$$

Nonetheless, an ellipsoid representing the relative magnitude of the principal moments of inertia may be determined and the origin of the ellipsoid could be placed at the CoM and orientated parallel to the principal moment axes. This representation covers three of the four properties identified above and is still of value to the engineer as a quick visual confirmation.

A small macro or script has been produced by RMA which discretises the inertial ellipsoid into areas which may then be plotted with the FE model. The only user input is a scaling factor on size of the ellipsoid. The macro was used in a recent project involving the seismic assessment of a standby power battery rack for a nuclear power station in the UK. The rack is symmetric and holds four rows of batteries (considered as point masses) at two height levels along the longitudinal direction. The inertial ellipses for the rack without and with batteries is shown in Figure 2.

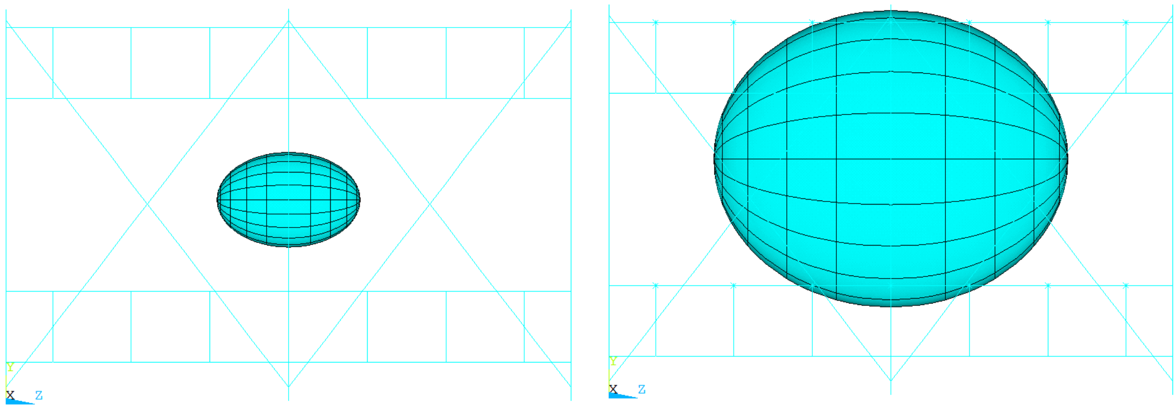


(a) Without Batteries

(b) With Batteries

Figure 2: Inertial ellipses for battery rack – same scale factor used for both plots

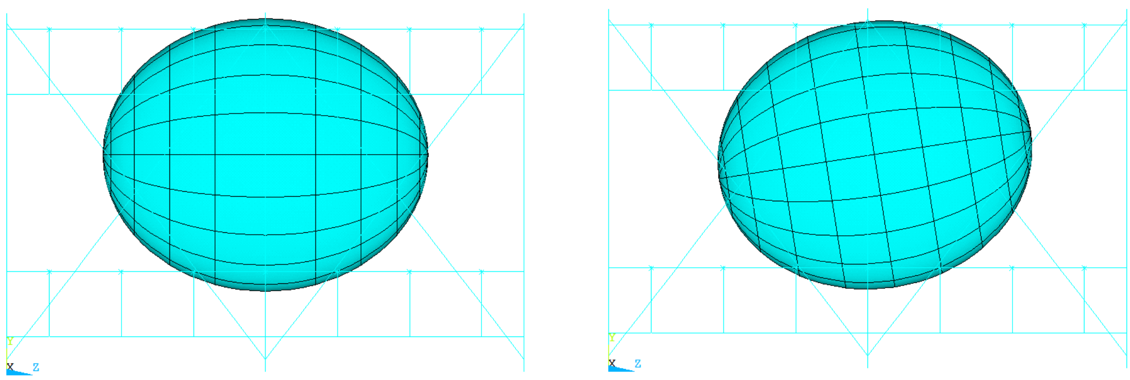
The models of Figure 2 are shown in Figure 3 viewed along the X axis. This view shows clearly that the addition of the batteries increases the height of the CoM and also that the principal moment of inertia about the X axis, this being proportional to the sum of the squares of the two principal dimensions of the visible ellipse, is significantly greater when the batteries are included. Because the same scale factor was used to produce the two plots then the factor increase for the inertia can be determined by measurement from the diagram.



(a) Without Batteries (b) With Batteries

Figure 3: Inertial ellipses for battery rack viewed along X axis

The inertial ellipsoid shown in Figure 4 is a result of not including one of the batteries in the model. This leads to a skewed ellipsoid when viewed along the X axis as shown.



(a) With Batteries (b) Missing one Battery

Figure 4: Inertial ellipses for battery rack viewed along X axis

When working with complicated FE models it is rather easy to make a mistake in the modelling and for this not to be picked up unless scrupulous checking is undertaken. Visualisation techniques such as the inertial ellipsoid presented above make this task easier and reduces the likelihood that mistakes will pass through to the final results.

The script/macro used to generate the ellipsoid was written in ANSYS APDL and is available from RMA.