Uniformly Loaded Rectangular Plate on Three Corner Supports

Part 1: Error in Reported Maximum Deflection

Abstract

Numerical solutions to a theoretical solution for the uniformly loaded rectangular plate on three corner supports published in the Journal of Structural Engineering provides, *inter alia*, tables of maximum displacements and maximum principal moments for use by the practising engineer. The maximum displacements are assumed to occur at the free corner of the plate and whilst this is true for small plate aspect ratios, the maximum displacement moves away from this position as the aspect ratio increases. Finite elements are used in this technical note to provide an independent numerical solution which agrees with the published values for the corner displacements and is then used to provide the maximum displacement. For the largest aspect ratio considered (2.5) it is seen that the maximum displacement is underestimated by some 10% in the published results.

Introduction

During a recent project the author required, for verification of a Finite Element (FE) model, the theoretical solution for a rectangular plate on three corner supports and loaded by a point load at the unsupported corner. The theoretical solution for this plate configuration is not presented in standard reference texts such as Timoshenko's 'Theory of Plates and Shells' but a rather elegant solution was found in [1] which also included the case when the plate is uniformly loaded as shown in Figure 1.

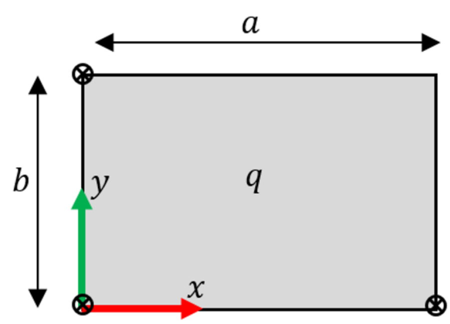

Figure 1: Uniformly loaded rectangular plate on three corner supports

For the uniformly loaded plate configuration the theoretical displacement at the unsupported corner, w, was presented as in Eq. (1) where q is the uniformly distributed load, D is the flexural rigidity and ν is Poisson's ratio.

$$w = \frac{qa^2b^2}{8D(1-\nu)} \tag{1}$$

Results were then presented for this plate configuration in terms of non-dimensional maximum displacements and maximum principal moments for a range of aspect ratios and Poisson's ratios. The non-dimensional displacement is expressed as the alpha parameter in Eq. (2) using $s = \min(a, b)$ as opposed to a in the denominator to make the expression more general than in the original publication.

$$\alpha = \frac{wD}{qs^4} \tag{2}$$

Using FE analysis with a highly refined mesh of four-noded, quadrilateral Kirchhoff elements the author was able to check the published results. Whilst for small aspect ratios the maximum displacement in the plate occurred at the unsupported corner, as the aspect ratio increased the point of maximum displacement began to move away from the corner and along the upper free edge and for the largest aspect ratio considered, i.e., 2.5, the actual maximum displacement was some 10% greater than that at the corner of the plate.

The results of the author's study are shown in Figure 2 for a Poisson's ratio of 0.3. The lower curve uses Eq. (1) for the displacement and the published results, which were derived from and agree with Eq. (1), are shown as squares in the figure. The upper curve represents maximum displacements from FE obtained by the author and are seen to diverge from the corner displacements as the aspect ratio increases beyond about 2.

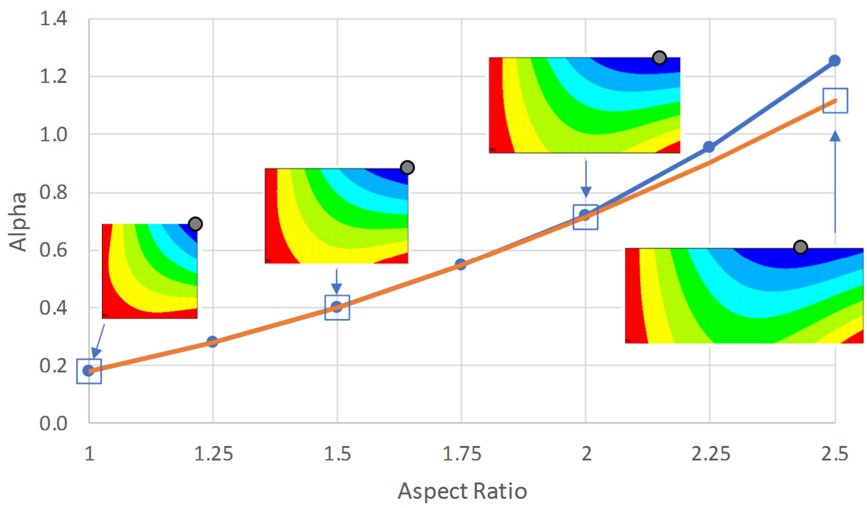

Figure 2: Displacement at unsupported corner compared to maximum value

Conclusions

It is, of course, good practice for the engineer to seek verification for FE results if he/she is going to be able to use them with confidence for a design or assessment project. Commercial FE systems do contain bugs and all engineers are capable of perpetrating *finite element malpractice*! Whilst theoretical solutions seem an ideal way in which to obtain verification, the engineer needs to be cautious that these solutions are accurate. In other words, the theoretical solution also needs verification! If there is agreement between theoretical and FE solutions then, whilst not impossible, it is highly likely that both results are correct. In the case shown in this technical note, the author obtained almost perfect agreement between theory and FE for the corner displacement and was, thus, able to present the maximum displacement results with confidence that they were correct.

Whereas a 10% underprediction of the maximum deflection might not threaten the structural integrity of the plate, it could lead to a structure that is more responsive than anticipated to, say, pedestrians.

In checking the maximum displacements for this plate configuration, the author also checked the published maximum principal moments and found these to be correct.

The error exposed in this article is not the first that the author has uncovered – see for example, [2]. The advice offered to practising engineers might well thus be *caveat emptor* when it comes to accepting numerical results even from seemingly valid results presented by reputable workers in authoritative publications.

References

[1] Azarkhin, A. (1992). “Bending of Thin Plate with Three-Point Support” Journal of Structural Engineering, 118(5), 1416-1419.

[2] Ramsay, A.C.A. and Maunder, E.A.W. (2016). “An Error in Timoshenko’s Theory of Plates and Shells” The Structural Engineer, June, 36-39.