Finite Element Specialists and Engineering Consultants

Benchmark Problem: Stress in a Thin, Solid, Parallel-Sided Rotating Disc

The problem considered is that of a rotating disc, the geometry of which is defined by the radius, R, and the (axial) thickness, t as shown in Figure 1.

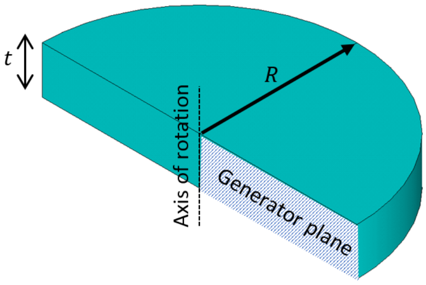

Figure 1: Geometric of a solid, parallel-sided disc – half disc with axisymmetric generator plane

The theoretical solution for the linear elastic stresses in a solid parallel-sided disc, of density, ρ, and Poisson's ratio, ν, with small thickness/radius ratio (t/R) and rotating at a speed of ω, is given by the plane stress form of the Lamé equations as shown in Eq(1).

Elastic

	Hoop	**Radial**	
Elastic	$\sigma_h = \frac{\rho\omega^2}{8}[(3+\nu)R^2 - (1+3\nu)r^2]$	$\sigma_r = (3+\nu)\frac{\rho\omega^2}{8}(R^2 - r^2)$	(1)

The stresses vary quadratically with radius, r, and the maximum stresses occur at the centre of the disc (r=0) where the hoop and radial stresses are equal as shown in Eq(2).

Stress at Centre

$$\sigma = \sigma_h = \sigma_r = \frac{\rho\omega^2}{8}(3+\nu)R^2 \qquad (2)$$

There are five variables involved in this problem and these can be expressed as two non-dimensional groups:

Demand/Capacity	Thickness/Radius
$\frac{\rho\omega^2 R^2}{S}$	$\frac{t}{R}$

For thin discs, where the thickness/radius ratio is small, the stress at the centre leads to a demand/capacity ratio as given in Eq(3); the numerical value is that when ν=0.3.

Demand/Capacity

$$\frac{\rho\omega^2 R^2}{S} = \frac{8}{3+\nu} = 2.4242' \qquad (3)$$

Eq(3) may be used to determine the stress at the centre of a given disc as might be required, for example, in a software verification exercise. As the thickness/radius ratio increases the stress at the centre of the disc decreases and moves more towards a plane strain situation. However, the result is within 1% of the theoretical value for thickness/radius ratios up to about 0.4.