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Software Verification of the Beam Element in LISA V8.0.0

LISA is a free to use finite element system and the current version (V8.0.0) may be obtained from:

http://www.lisafea.com/

The beam element in this software is a formulation based on Euler-Bernoulli theory which means that it is appropriate for slender beams where the shear deflection and rotational inertia, which are not considered in such a formulation, can be considered to have negligible effect on the results.

The Euler-Bernoulli beam theory is explained in:

https://en.wikipedia.org/wiki/Euler%E2%80%93Bernoulli_beam_theory

Theoretical solutions are available for the free vibrations of simple beam member configurations such as, for example, a cantilever configuration. For a beam of length, l, with elastic modulus, E, moment of inertia, I and mass per unit length, μ, the natural frequencies for the cantilever are:

$$\omega_i = \beta_i^2 \sqrt{\frac{EI}{\mu}} \tag{1}$$

The coefficients β_i are determined as the roots of:

$$\cosh(\beta_i l)\cos(\beta_i l) + 1 = 0 \tag{2}$$

The first coefficient in the series, corresponding to the fundamental bending frequency, β_1 was obtained using GRG non-linear solver within Excel with an initial guess of 1.8 as shown in Figure 1.

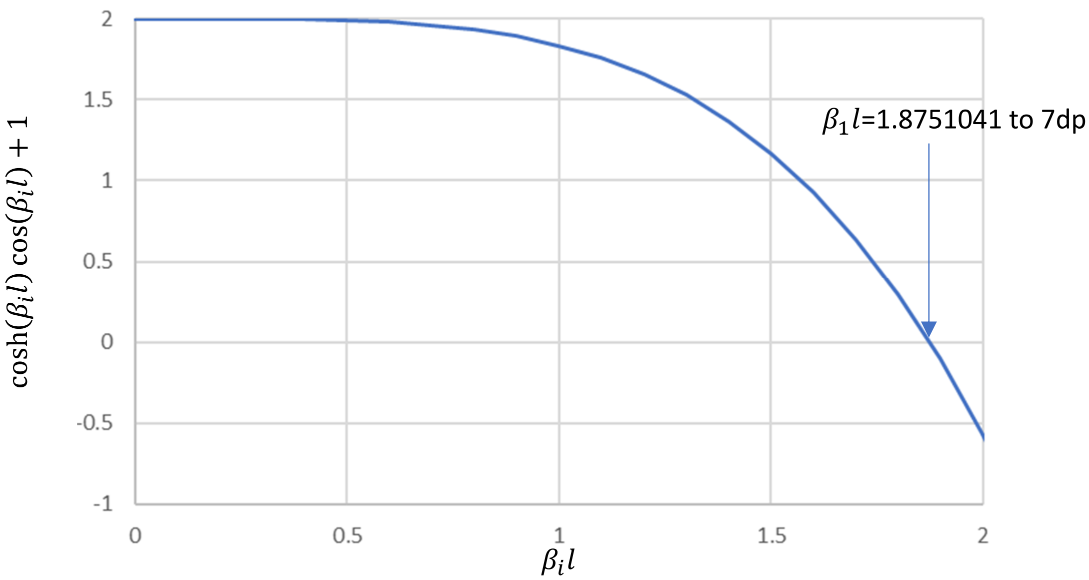

Figure 1: Determination of the first coefficient

A slender beam with a circular tube cross section will be used to compare the fundamental frequency from LISA with that of the theoretical solution.

Geometric Properties:

Length	l=118.6m
Inner Radius	r_i=1.47m
Outer Radius	r_o=1.48m
Moment of Inertia	$I = \frac{\pi}{4}(r_o^4 - r_i^4) \cong 0.100816$
Area	$A = \pi(r_o^2 - r_i^2) \cong 0.092677$

Material Properties:

Elastic Modulus	E=210GPa
Mass Density	ρ=7800kg/m^3
Mass per Unit Length	$\mu = \rho A \cong$722.8805kg/m

The coefficient β_1 is then determined as 0.01581 and the corresponding natural frequency is 1.35rad/sec or about 0.215Hz. This theoretical frequency (expressed in full accuracy) is used to determine the relative error in the finite element solution from LISA and this is plotted, as a percentage, against the number of elements in the mesh in Figure 2.

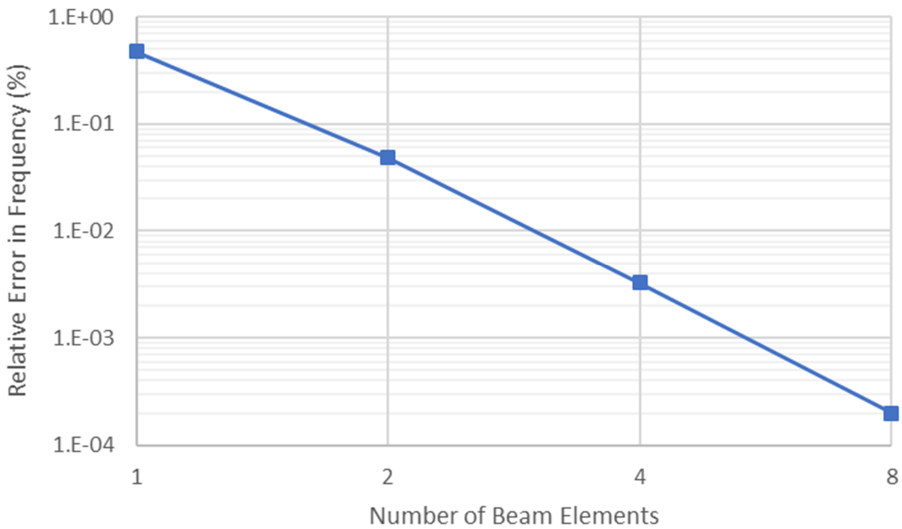

Figure 2: Convergence of fundamental frequency for LISA

The Euler-Bernoulli beam element models the displacements as a cubic polynomial and this cannot exactly match the theoretical solution which is trigonometric in form. As such, the single element cannot precisely recover the theoretical solution although it does so with an error of less than 1%. Mesh refinement in LISA leads to the theoretical solution as demonstrated by the monotonic reduction in error shown in Figure 2.

This study shows that the Euler-Bernoulli beam element in LISA V8.0.0 is capable of capturing the theoretical solution for the natural frequency of a slender cantilever beam configuration.