Safe and Economical Simulation for the Built Environment

Angus Ramsay | Ramsay Maunder Associates (RMA)

In order for the UK and, indeed, the world to meet the targets agreed in the 2016 Paris Agreement, significant reductions in the current rate of harmful emissions need to be made as soon as is practically possible. Engineering, which is responsible for many of the benefits seen in our modern society that we now see as essential to our way of life, needs to embrace cleaner technologies in power generation and transportation. Perhaps surprisingly, however, this is not the only way in which significant reductions in emissions can be made. As far as the built environment is concerned, the operational energy usage has rapidly become secondary to what is termed the embodied energy used in the construction and transportation of the materials used in buildings.

The statistics are staggering, as will be noted in this article. This provides an exciting opportunity for structural design engineers to raise their game. Not only will they need to ensure the safety of their designs but also attempt to curb overt conservatism in order to ensure that their structures are designed to minimise the amount of material used in their construction. Structural engineering, like many engineering disciplines, has traditionally been a highly conservative discipline. No engineer wishes to see their structure fail in operation and— being aware at least of the presence if not the magnitude of the uncertainties in, for example, the applied load and/or the material strength— they will always tend to err on the side of safety, i.e., more strength. However, with Limit State Design (LSD), prescribed partial factors of safety have been calculated in a robust statistical manner to account for these uncertainties and should, so the guidance goes, be considered as reliable, i.e., there should be no reason for the design engineer to throw in any additional self-imposed layers of conservatism.

Copyright Nafems 2021 Licensed solely to Angus Ramsay (angus_ramsay@ramsay-maunder.co.uk) for single use only

When it comes to errors and uncertainties, one should, if one is to be meticulous, include those inherent in the stress resultants used for the design. These may, for simple structural models, e.g., rectangular plates, come from published data or they may come from the results of a simulation. Whilst published data is usually, but not always, conservative, the degree of conservatism is often variable over the range of dimensions considered for the member and is sometimes overtly conservative. This, then, is not appropriate if the engineer is attempting to minimise embodied energy. Equally, the simulation software adopted by the engineer generally requires him/her to undertake sophisticated code and calculation (solution) verification studies in order to estimate the error in the stress resultants. Such studies may not always be performed, and with the majority of commercial simulation tools where equilibrium is only satisfied weakly, this leaves the possibility that the design will be unsafe.

In writing this article I am hoping to provoke the simulation community to think of other, non-conventional, Finite Element (FE) formulations and design methodologies that will, I feel, be the new, future paradigm required as engineers necessarily become concerned not only with the safety of their designs, but also with minimising the embodied energy use in the production of the artefacts (structures or machines) that they design.

Minimising Energy in Construction (MEICON)

MEICON [1] is an EPSRC funded project involving researchers at the Universities of Cambridge and Bath looking at ways to minimise energy in construction of the built environment, also known as embodied energy.

The statistics motivating this research are staggering:
"The built environment is estimated to account for around 50% of all carbon emissions. About 10% of global GDP is generated by the construction industry, which creates and maintains our built environment. Recent success in reducing operational energy consumption and the introduction of strict targets for near-zero energy buildings mean that the embodied energy will soon approach 100% of total energy consumption.", [1]

There is a realisation that current design practice is inefficient:
"The importance of this fundamental shift in focus is highlighted by the analysis of recently constructed steel and concrete buildings, in which it was demonstrated that embodied energy wastage in the order of 50% is common. Inefficient over-design of buildings and infrastructure must be tackled to minimise embodied energy demand and to meet future energy efficiency targets.", [1]

There is an imperative to improve the design process:
"The UK Government has set out its ambition to achieve 50% lower emissions, 33% lower costs, and 50% faster delivery in construction by 2025. These ambitious targets must be met at the same time as the global construction market is expected to grow in value by over 70%. Achieving growth and minimising embodied energy will require a step change in procurement, design and construction that puts embodied energy at the centre of a holistic whole-life cycle design process." [1]

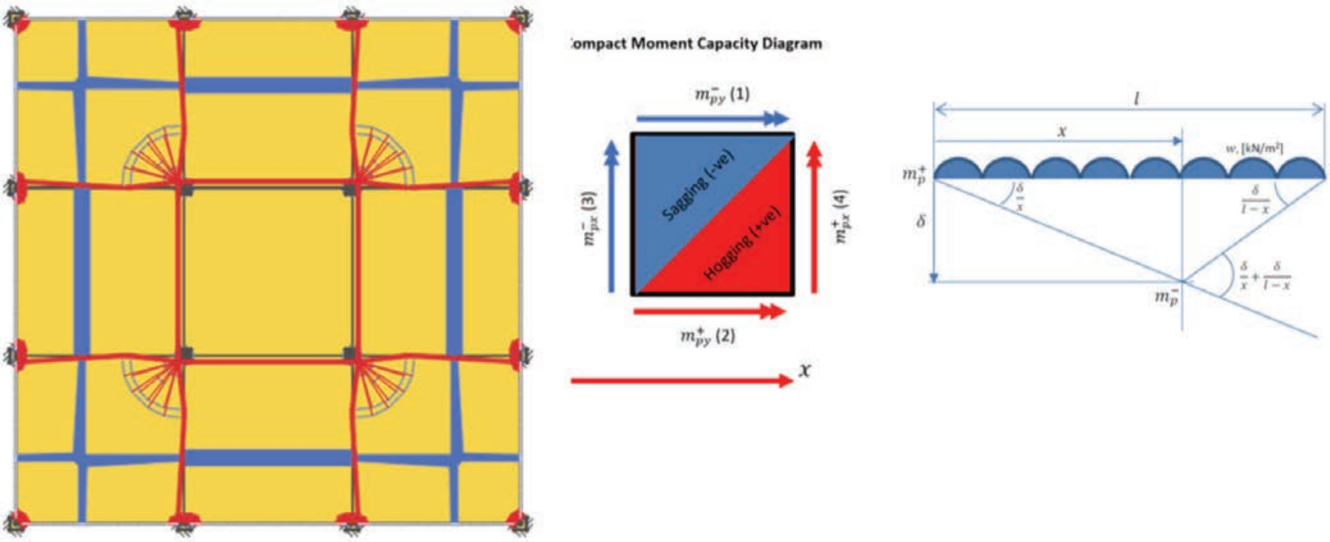
One of the Research Questions posed but seemingly not yet answered is:
RQ10: How can structural models be checked in an automated fashion? How can we reduce error rates in all structural engineering design? Should there be a partial safety factor for analytical errors in all structural design, and how might this change over time as automation increases? ' [1].

It will be seen in this article that the answer to this question depends to a certain extent on the manner in which the stress resultants used in the design are derived.

An RC Slab Project Confirming the MEICOM Findings

I was recently involved in a legal dispute where a heavily loaded reinforced concrete slab was designed by a civil engineering consultancy and built by a contractor for a particular client. Because of the way in which the design/build process is nowadays compressed, the slab was cast before the design had been finally signed-off and as a result when an error was noticed in the orientation of a patch of additional ULS reinforcement, the error could not be corrected. The error that occurred was that the additional reinforcement had been laid at right angles to the way it should have been thereby not giving the additional local strength that was intended.

The design of slabs is generally undertaken through linear-elastic analysis. The reinforcement required for SLS (deflection and cracking) is determined and then additional patches of enhanced reinforcement are placed where required to cope with the ULS condition. These would typically be hogging reinforcement over the columns and sagging reinforcement patches in the middle sections between columns.



A 3x3 bay flat slab on a regular 4x4 array of columns. Yield line pattern showing collapse under a uniformly distributed load and with different moment capacities in sagging and hogging. The yield line pattern involves partial hogging fans around the internal columns and sagging yield lines (blue) through the outer bays. The yield line pattern was generated using LimitState:SLAB (http://acad.limitstate.com/slab/details). The simple yield line pattern used to calculate a collapse load by hand is shown. As seen in the actual pattern, the sagging line runs off-centre in the outer bays and the position of the sagging yield line in the hand calculation was retained as a variable, x, which was obtained through geometric optimisation.

Figure 1: Flat slab example (3x3 bays) - plastic solution.

Whilst the slab was clearly satisfactory for SLS, the client was concerned that it no longer satisfied the ULS conditions. As a result, the client put in a claim in order to strengthen the slab where the sagging reinforcement had been misplaced.

The lawyers representing the consultancy employed a highly experienced structural engineer as their Technical Expert who in turn requested RMA to undertake an assessment of the strength of the slab as cast using limit analysis. The basis for this selection was, amongst others, the papers RMA had published with, *inter alia*, NAFEMS on the limit analysis of RC slabs [2].

The outcome of my analysis was that even with the incorrectly laid enhanced reinforcement removed from the model, the load factor at ULS was 1.27. In other words, the slab was 27% stronger than required to pass the ULS condition. A sanity check was made using a hand generated yield line model which gave a load factor of 1.25. The opposing technical expert, working for the Client, essentially agreed with my finding and the case will be decided at a mediation hearing in the coming months.

Had a formal limit analysis been performed during the design then it is likely that a more economical design could have been achieved.

Lower-Bound Limit Design – a Paradigm for a Sustainable Future?

The conventional FE formulation used in the majority of commercial FE systems is the Conforming Finite Element (CFE) formulation. The non-conventional FE formulation I refer to at the end of the introduction is the Equilibrium Element Formulation (EFE). Models using either formulation produce approximate solutions which should converge to the theoretical solution (known or unknown) with mesh refinement. Demonstrating this, where the theoretical solution is known is, of course, the *raison d'étre* of code verification; we wish to see the exact error between FE and theory converge towards zero as the mesh is refined. In practical engineering the theoretical solution is normally unknown and with the faith generated by appropriate code verification that the FE system can recover a known theoretical solution, we use mesh refinement, known as solution verification, and, perhaps, extrapolation techniques such as that of Richardson [3], to estimate the theoretical solution and, therefore, give us an approximation of the error for our particular FE mesh.

Whilst the aim of verification is to ensure that the error in an engineer's FE model is sufficiently small so as not to affect the engineering decisions to be made with the FE results, this is not always the case. Although NAFEMS has a remit to ensure, through education, that the simulation engineer is aware of good practice, it is common to see in engineering reports scant regard being given to verification and demonstration to the reader of the report that the errors in the models are within reasonable limits. In extreme cases such errors have caused and will continue to cause failures in the final design. One thinks here of the Sleipner Platform [4] that collapsed catastrophically causing very significant financial and reputational damage to the companies involved although thankfully no loss of life or injury occurred.

CFE and EFE based systems, whilst both producing approximate but, hopefully, convergent FE solutions do so in different manners. The errors seen in a CFE model generally involve a lack of equilibrium between the FE stresses and the applied loads. The design process generally involves ensuring that the structure is sound under both service and ultimate loads. In Limit State Design philosophy, these are termed the Serviceability Limit State (SLS) conditions and the Ultimate Limit State (ULS) conditions. Generally, for particular structural forms, one finds that one of these conditions dominates. For example, thin simply supported plates are generally controlled by SLS conditions, i.e., once the thickness that satisfies the stiffness requirements is obtained, the ULS condition of strength is also satisfied. Thick plates, particularly those with clamped supports, work the other way so that once the thickness has been established to satisfy the ULS condition, then the SLS condition is also satisfied. For the SLS condition it is generally expected that the stresses remain elastic and can, therefore, be taken from a linear-elastic analysis. For the ULS condition, however, it is generally acceptable (provided the material is deemed to have sufficient ductility) to allow the stress (resultants) to redistribute plastically. In this manner additional strength can normally be obtained from a structure with the plastic limit load, i.e., that at which the structure collapses, being greater, and often significantly greater than the elastic limit load.

The process for designing a structure that is governed by the ULS condition involves distributing material with sufficient resistance to withstand the stresses and to support the applied load. Since the plastic limit load is normally greater than the elastic limit load, a first stab at a ULS design might be to use an elastic stress field. It is immediately clear to the thoughtful engineer that using stresses from a CFE model in such a manner leads to something of a quandary; how can the designer possibly ensure that the design is safe if the stresses given don't balance the applied loads? The only satisfactory way to resolve this for a CFE model is to ensure that the errors are sufficiently small so as not to influence the design process, i.e., to undertake solution verification.

This quandary is not present when using an EFE based system. The reason for this is that one can invoke the lower-bound theorem of plasticity. For readers not familiar with the plasticity theorems I would strongly recommend that these be investigated – they will change the way you think about engineering design! The lower-bound theorem simply states that your design will have sufficient strength if a stress field can be found for it which is in equilibrium with the applied loads and for which, at all points, the equivalent stresses do not exceed the yield stress. Any EFE stress field will do and the question of whether or not the model is sufficiently refined is of secondary importance at least in the initial design stage.

If the design of a structural member or mechanical component is governed by the ULS condition, then there is great potential benefit in terms of structural efficiency or usage of material in designing to the plastic limit state.

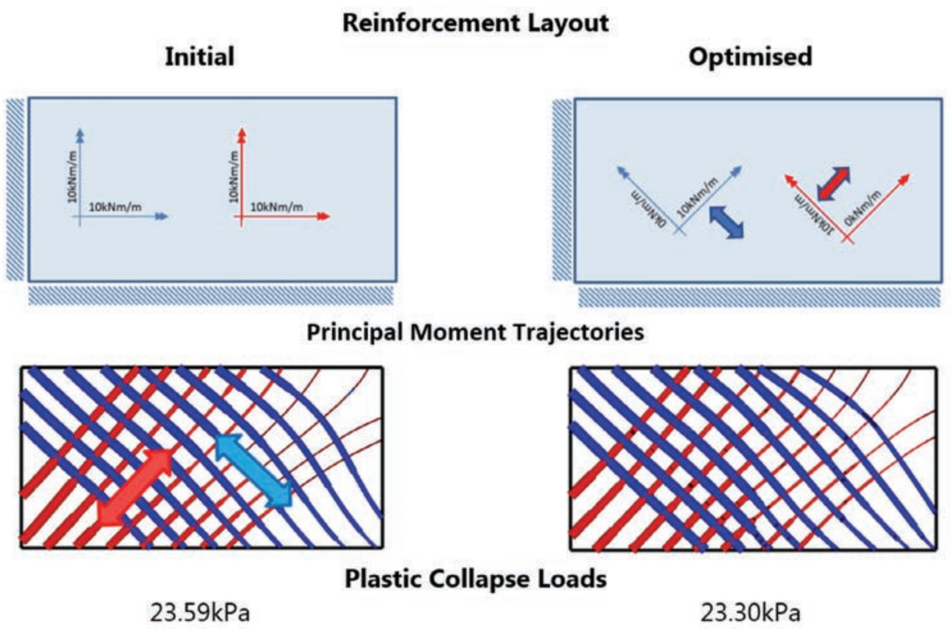


Figure 2: Lower-bound limit design optimisation of reinforcement layout

In conventional FE systems an incremental approach to plasticity is generally adopted. The load is applied incrementally and at each increment the stress resultants are redistributed according to the material laws. The incremental process proceeds until the statical indeterminacy of the structure is exhausted and it becomes a plastic mechanism. This load is then the plastic limit load. An alternative approach is to use limit analysis which is a form of optimisation which may start from an elastic solution. In the lower-bound form of limit analysis, the elastic solution could come from a linear-elastic EFE analysis. Additional self-balancing or hyperstatic stress fields are then added with the aim of maximising the load carrying capacity of the structure whilst ensuring that the stresses remain at or below the yield stress. Whereas the material laws in the incremental approach can be quite generally defined, in limit analysis perfect-plasticity is assumed. The yield stress for the material would typically be used here and the plastic limit load thus obtained would normally be considered to be a conservative one since the strengthening phenomenon of strain-hardening is not considered. Limit analysis achieves the plastic limit solution in a single step and is considerably more efficient than the incremental approach in terms of computational effort.

If elastic analysis is used in the design of a structure that is governed by the ULS condition then, by definition, the additional reserve in the structure due to plastic redistribution is ignored leading to an inefficient use of material.

It is a relatively simple matter, at least in principle, to reformulate limit analysis software in a limit design form. Reinforced concrete slabs offer a prime example of structural members where limit design can lead to massive savings in the amount of reinforcement required for a given ULS strength condition. RMA presented an example of limit design of a balcony slab simply supported on two adjacent sides and with a uniformly distributed load of 25kPa [5].

Reinforcing steel often comes in orthogonal meshes with equal moment capacity in the two orthogonal directions. Slabs need to resist both hogging (red) and sagging (blue) moments and so as an initial reinforcement layout, orthogonal meshes of reinforcement running parallel to the edges of the slab to resist these moments were considered. Mats positioned towards the top of the slab thickness resist hogging and those placed towards the bottom of the slab thickness resist sagging moments. This reinforcement layout led to a plastic collapse load of 23.59kPa, i.e., just under the required 25kPa load value, but sufficiently close to it as to be acceptable – see first column of Figure 2.

The principal moment trajectories plotted by EFE for the initial reinforcement layout indicated that an optimal layout might be achieved by rotating the reinforcement mats through 45 degree. Further, it indicated that one of each of the hogging and sagging bars were not being utilised. On this basis a limit analysis was undertaken with what was termed the optimised reinforcement layout as shown in the second column of Figure 2. Whilst the amount of reinforcement was reduced by 50%, the collapse load was virtually unchanged!

Although there is an additional cost in construction associated with the bespoke rather than standard layout of reinforcement, i.e., it is clearly cheaper for contractors to lay orthogonal mats of reinforcement over the entire slab rather than to have to adopt optimised layouts in different regions of the slab, the cost benefit in terms of minimising the amount of reinforcing steel required for a project is significant and, I suspect, will be impossible to ignore as we move into a more sustainable tomorrow.

A Case Study: Design of an Internally Pressurised Pipe

Currently available simulation tools, whilst highly sophisticated, are very much aimed at analysis rather than design. Of course, the two processes, whilst distinct, are often linked in so-called design-by-analysis processes. As an engineer who has for many years worked on the development of an alternative finite element (FE) formulation, it has become apparent that such non-conventional formulations offer great potential for efficient design that is not present in the conventionally adopted conforming finite element (CFE) formulation used in the majority of commercial FE systems. I am currently involved in writing a chapter for a NAFEMS book on industrial case studies using FEA. My chapter involves the analysis and design of rotating discs. For such components, the engineer is interested in both the elastic and plastic limit speeds and whilst these can be obtained using CFE software, the available elements are not best suited to the task and require significant mesh refinement before sensible or safe predictions of these speeds can be made. As such, I developed a software tool for the task using an element which solves the elastic solution for the Lamé equations exactly. It is a form of EFE which I have called the Lamé Finite Element (LFE) and it has the useful property that, irrespective of the level of mesh refinement, it produces safe lower-bound approximations of the two limit speeds. I have presented the theoretical developments in a paper which is currently under review. Such software fits well with themes from previous NAFEMS World Congresses, i.e., Simulation Governance and Democratisation of Simulation [6].

I thought, therefore, that it would be useful in this article to demonstrate firstly the code verification of the LFE and then compare its performance characteristics in the design of a component with those of a CFE element. The component I have chosen is a simple one, namely, a pipe with internal pressure where the design variable is the outer radius of the pipe. Like the rotating disc, the engineer is interested in establishing safe values for the elastic and plastic limit loads (pressures in this case) for the pipe and these are cast, respectively, as Serviceability

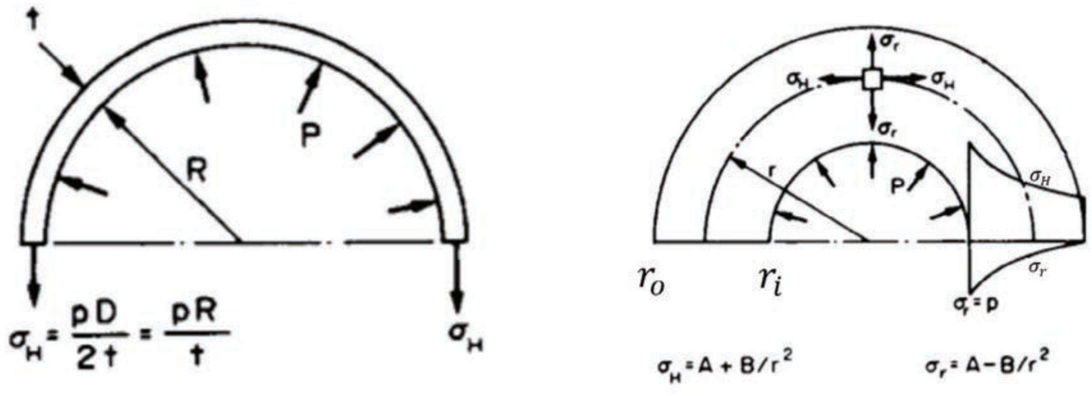


Figure 3: Distinction between thin-walled and thick-walled pipes [7].

Limit State (SLS) and Ultimate Limit State (ULS) conditions as per the modern Limit State Design process also known as Load Resistance Factor Design in the US. To simplify the text, I have not included any partial safety factors in the exposition. These would typically increase the load applied to the structure and reduce the material strength or resistance and, of course, would be essential in any practical assessment of the structural integrity of the pipe in order to account for uncertainties in loads and resistances.

Code Verification for LFE

The essential ingredient for code verification is a known theoretical solution with which to test the software. There are many such practical engineering solutions for the Lamé equations involving pressurised cylinders and rotating discs. The one used here is the pressurised cylinder. The closed-form theoretical elastic solution to this problem can be found in most strength of materials texts and some also include a closed-form plastic solution based on the Tresca yield criterion [7].

There is a distinction between thin-walled and thick-walled pressurised cylinders which is shown in Figure 3. For the thin-walled cylinder the radial stress is ignored and the hoop stress is taken as an average value over the wall thickness. For the thick-walled cylinder radial and hoop stresses vary through the wall thickness in the manner shown. In the absence of any shear stresses, these are principal stresses. The stresses are determined from the Lamé equations where the coefficients A and B are determined from the static boundary conditions at the inner radius, r_i, and the outer radius, r_o.

The pressure, p, to cause yield (Tresca) up to some radius, r_p, is given in Eq. (1) where S_y is the uniaxial yield stress for the pipe material.

$$p = S_y \left\{ \frac{1}{2r_o^2}\left(r_o^2 - r_p^2\right) - \ln\left(\frac{r_i}{r_p}\right) \right\} \qquad r_i \le r_p \le r_o \tag{1}$$

For first yield, $r_p = r_i$, whereas for plastic collapse, $r_p = r_o$. The limit pressures are given in Eq. (2).

Elastic Limit Pressure **Plastic Limit Pressure**

$$p_e = S_y \left\{ \frac{1}{2r_o^2}\left(r_o^2 - r_i^2\right) \right\} \qquad p_p = S_y \left\{ -\ln\left(\frac{r_i}{r_o}\right) \right\} \tag{2}$$

It is, of course, far more general and therefore valuable, to consider matters in terms of non-dimensional parameters. The results and findings are then no longer specific to a single pipe geometry, in this case, but to a family of pipe geometries. The dimensionless parameters used in this text are defined in Eq. (3).

$$P = \frac{p_p}{p_e} \quad P \ge 1 \qquad R = \frac{r_i}{r_o} \quad 0 < R < 1 \qquad L = \frac{p_e}{S_y} \qquad K = \frac{p_p}{S_y} \tag{3}$$

Substituting these dimensionless quantities in Eq. (2), the pipe dimensions, R, for a given operating pressure and material yield stress, L, can be determined from the SLS condition and the pressure load factor, P, from the ULS condition as shown in Eq. (4).

SLS **ULS**

$$R = \sqrt{1 - 2L} \qquad P = \frac{2\ln(R)}{(R^2 - 1)} \tag{4}$$

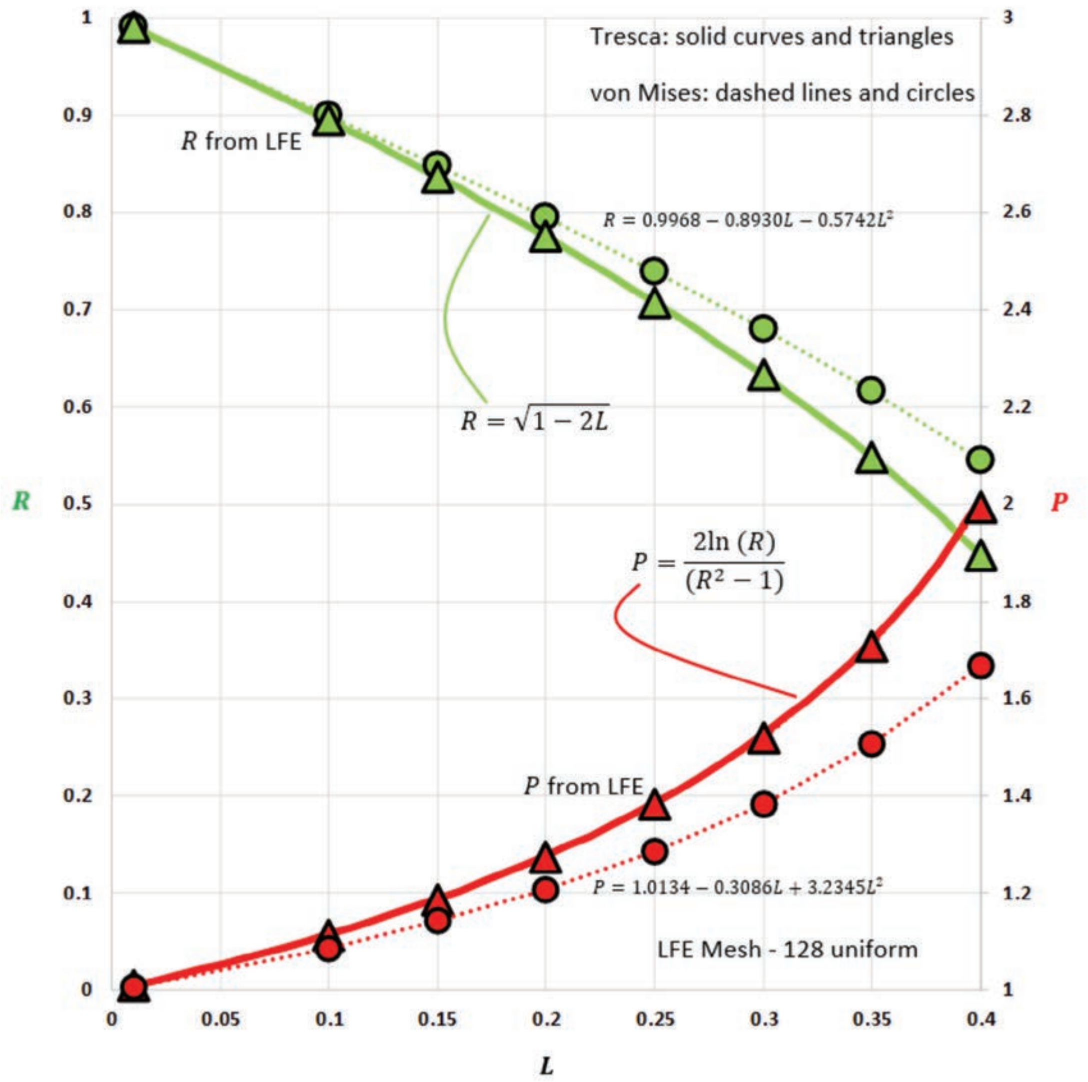


Figure 4: Design chart for internally pressurised pipes (Tresca & von Mises).

These design equations are appropriate for a pipe where the partial factors on both load and resistance have been taken as unity and they have been used to produce the graph in Figure 4.

The point values of R come from an elastic LFE analysis and a single element mesh could have been used to determine these values since a single element can capture the exact solution. As can be seen from the graph, these points lie exactly on the theoretical curve. The point values of P involve the plastic limit pressure and require a refined LFE mesh to produce accurate values. Meshes with 128 uniform length elements have been used here which gives values with an error of around 1%. Note that as the elastic limit pressure from LFE is exact and the plastic limit pressure is a lower-bound then P retains this lower-bound property. In order to determine the R value for a given L, the LFE software needed to be run in a *design-by-analysis* mode. The inner radius of the pipe was fixed at an arbitrary prescribed value and the outer radius was then sought to obtain an elastic limit pressure equal to the applied pressure. The simple bisection method was used for this problem and convergence to six or more digits was achieved with 25 iterations.

The LFE results for the von Mises yield criterion are also presented in the figure. Whilst there may well be a closed-form theoretical solution for this yield criterion, it didn't jump out of the reference literature and so the theoretical curve is not included in the figure and the LFE points are joined with a least-squares fit with quadratic polynomials. As the yield curve for Tresca inscribes that for von Mises it is to be expected, for a given L, that von Mises will lead to a greater R value, i.e., a smaller wall thickness of pipe. The meaning of a lower P value is not easily gleaned since it is the ratio of two limit pressures both of which, for a given pipe geometry, will be greater when considering von Mises as opposed to the Tresca yield criterion. Clarification is given through the following example.

Yield Criterion	p_e, [MPa]	p_p, [MPa]	L	P
Tresca	70.13	97.76	0.26	1.39
von Mises	77.92	105.18	0.28	1.35

Table 1: Pressures and dimensionless parameters for example.

A pipe with geometry r_i = 0.7m and r_o = 1.0m (R = 0.7) and material yield stress of S_y = 275MPa was analysed in LFE to determine the elastic and plastic limit pressures, p_e and p_p respectively, for both the Tresca and von Mises yield criteria. These pressures are presented in Table 1 together with the parameters L and P.

The values of L, P and R form a coherent set that agree, as they should, with the curves in the design chart. Both limit pressures are greater for von Mises than for Tresca but the ratio P is smaller. This leads to the conclusion that for a given load and material, L, a more economical design will be achieved using the von Mises criterion and for a given pipe geometry, R, designs based on the von Mises criterion can take greater load in terms of elastic and plastic limit pressures.

An example of the design of a pipe using the theoretical design chart of Figure 4 is now presented.

Design from Theory

Consider a pipe for an application where the flow rate of product requires an inner radius of r_i = 0.7m, and needs to operate at a pressure between 60 and 70MPa. The pressure is invariant with time and so fatigue is not an issue and the pressure can safely go up to that to cause yield at the inner radius. The pipe is to be manufactured from stainless steel with a yield stress of S_y = 275MPa. Under certain operating conditions the operating pressure can increase by 25% before a pressure relief device activates. The client wants a conservative design but not an overly conservative one. As such the more appropriate von Mises yield criterion will be adopted and calculations of the plastic limit pressure will be based on the yield stress rather than the higher ultimate tensile stress. Any reduction in wall thickness due to possible corrosion either internally or externally can safely be ignored.

The design problem involves finding the required outer radius for the pipe using the empirical equations for the von Mises yield criterion shown in Figure 4. The design must satisfy both SLS and ULS conditions and the design for an operating pressure of 60MPa is shown in Table 2.

60MPa	**SLS Condition**		**ULS Condition**	
	L	R	P	
SLS Design	60/275 = 0.2181	0.775	1.235 $\ngtr$ 1.25	Fails ULS Condition
ULS Design	0.227	0.765	=1.25	

Table 2: Design for 60MPa operating pressure – ULS governed.

The design for the 70MPa pressure condition is shown in Table 3.

70MPa	**SLS Condition**		**ULS Condition**	
	L	R	P	
SLS Design	70/275 = 0.2545	0.732	1.30 > 1.25	
ULS Design	0.2269 $\ngtr$ 0.2545	0.765	=1.25	Fails SLS Condition

Table 3: Design for 70MPa operating pressure – SLS governed.

The radius ratios required for the 60 and 70MPa pressures are, respectively, 0.765 and 0.732 and for the internal radius considered (0.7m) this gives outer radii of 0.92 and 0.96m.

As mentioned earlier, where the example of different plate configurations was used, one or other of the SLS and ULS conditions will govern a particular design. This is seen with this current example where the lower-pressure design is ULS governed whilst the higher-pressure design is SLS governed.

Dimensionless Parameters

Plastic Limit Pressure (Estimate)	***K* ratio (Estimate)**	***F* Parameter**	
$\tilde{p}_p = 2S_y(1-R)$	$\tilde{K} = \dfrac{\tilde{p}_p}{S_y} = 2(1-R)$	$F = \dfrac{K}{\tilde{K}} = \dfrac{p_p}{\tilde{p}_p} = \dfrac{\ln(R)}{2(R-1)}$	[5]

Whereas the design chart of Figure 4 presented a theoretically exact solution for the pressurised pipe, there are design formulae available which, whilst useful in their day, now need to be used with caution. In the next section I will look at a commonly adopted formula for the plastic limit pressure of internally pressurised pipes, compare it with the theoretical solution, and make some comments in the context of this article.

Barlow's Formula for Plastic Limit Pressure.
According to Wikipedia, Peter Barlow FRS (1776-1862) was an English mathematician and physicist who spent many years at the Royal Military Academy in Woolwich. Barlow's formula provides an estimate for the plastic limit pressure as given in Eq. (5). It would appear to have been derived from the hoop stress for a thin-walled pipe with a factor of safety of two applied for conservatism when used for thick-walled pipes.

The quantities K, $\tilde{K}$ and F have been plotted against the radius ratio, R in Figure 5. A semi-log plot is used for K, $\tilde{K}$ and it is seen that Barlow's curve lies below the theoretical solution for radius ratios greater than about 0.2. The ratio F is less than unity when Barlow is conservative and greater than unity when Barlow is unsafe.

The level of conservatism in Barlow's formula is 100% for very thin-walled pipes and this comes from the safety factor of two applied in his formula. However, the conservatism reduces in a non-linear fashion as the wall thickness of the pipe increases until a radius ratio of about 0.2 beyond which the formula becomes unsafe to use. Barlow's formula has, presumably, been of practical use to design engineers for over 150 years. However, if engineers are going to make more efficient utilisation of materials, then such approaches, which are overly conservative for some pipe geometries and unsafe for others, need to be updated. Even if theoretically derived design charts cannot be determined, design curves derived from simulation form a useful surrogate particularly if they come from simulation software that can be guaranteed to produce a safe result, e.g., EFE.

The reader may be surprised to find out that some companies selling pipes still recommend the use of Barlow's formula to their customers for the design. It might be unjust to suggest that in doing so their motivation is that they are going to sell more expensive pipes with thicker wall thicknesses when smaller thicknesses would suffice. It could also be the case that the companies are unaware of the issue or, alternatively, are aware of it but would prefer to guide their customers in what they judge to be a conservative manner. Whatever the reason, and this occurs not only for pipes but other structural members, if the world is to meet its aims on carbon reduction then I would suggest that this is an area that needs to be addressed.

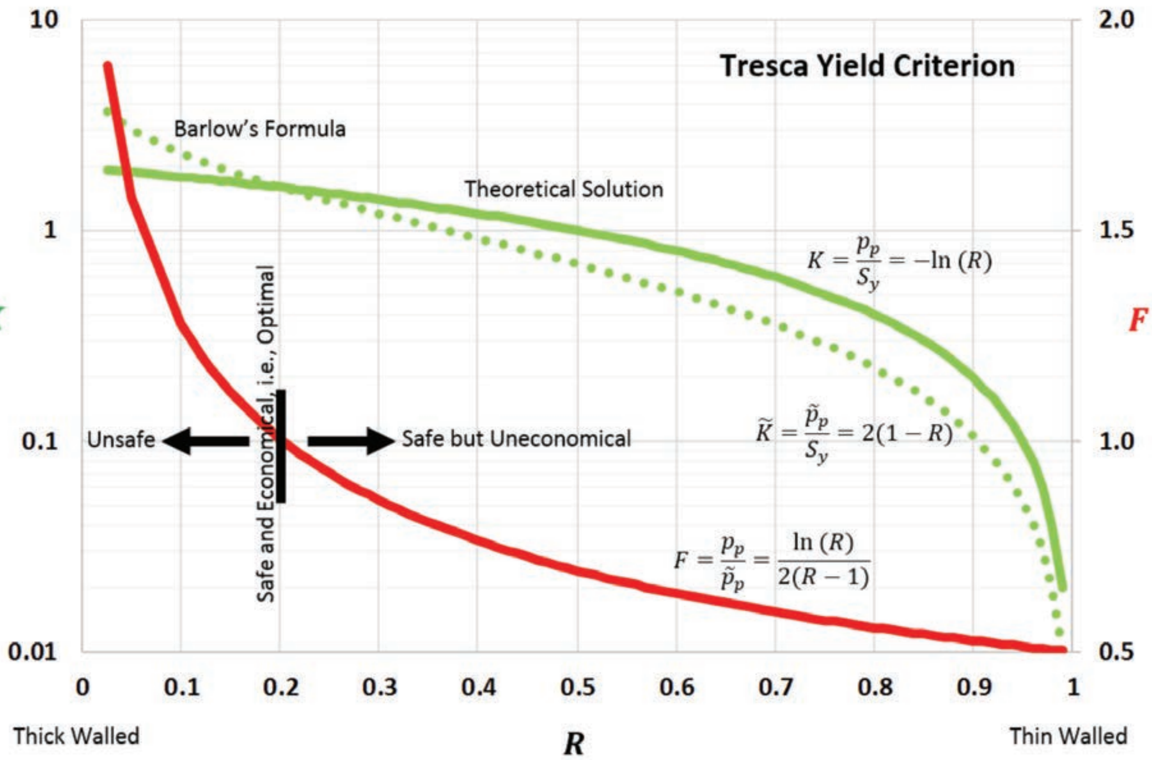


Figure 5: Barlow's approximation compared with exact solution (Tresca).

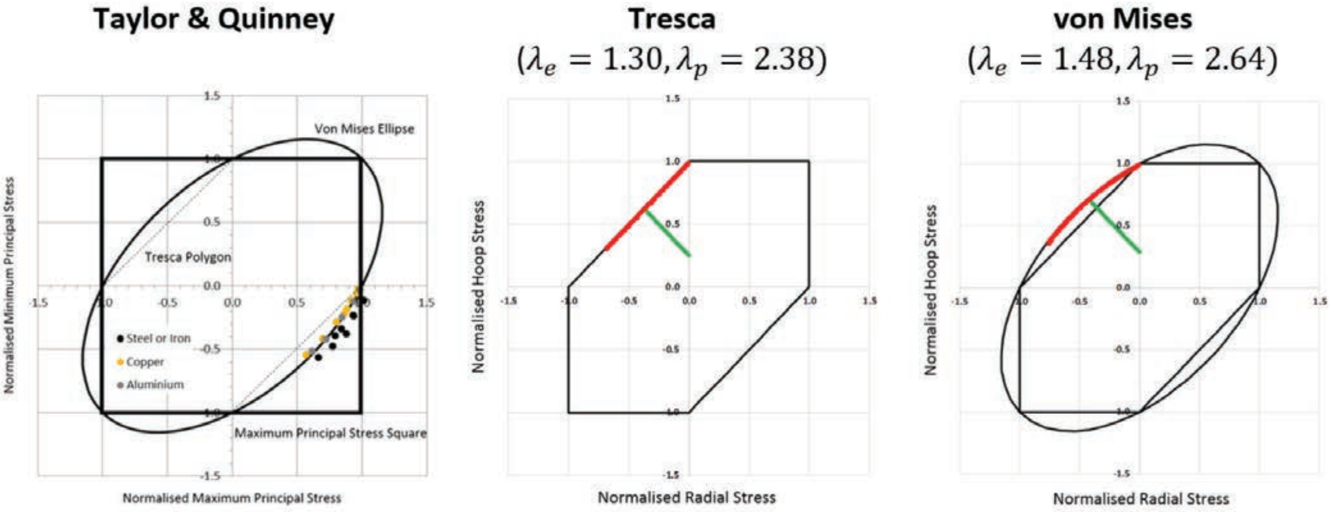


Figure 6: Different yield criteria.

Tresca versus von Mises

Another way to avoid potentially unnecessary conservatism in simulation is to ensure that the most appropriate yield criterion is being used. It has long been established that for ductile metals the von Mises criterion is more appropriate than Tresca. Taylor and Quinney's experimental results of 1931 [8], which are reproduced in the first column of Figure 6, show this clearly.

For a pipe with a radius ratio of R = 0.5, the radial and hoop stresses have been normalised by dividing by the yield stress and plotted along a radial line for Tresca (column 2) and von Mises (column 3) of Figure 6. The green lines represent the elastic solutions whereas the red lines are for the plastic solutions. The outer radius of the pipe is easily recognised as it lies on the line of zero radial stress and first plasticity develops at the inner radius of the pipe. As the Tresca curve inscribes that of von Mises and given the state of stress at the inner radius, it is clear that the limit pressures for the von Mises criterion will be greater than or equal to those for Tresca. The same applied pressure was used in these analyses and is multiplied by an elastic load factor, λ_e, or a plastic load factor, λ_p, to obtain the elastic and plastic limit pressures. The increase in elastic limit pressure when the yield criterion is changed from Tresca to von Mises can be represented by a factor which, for the pipe considered, is 1.48 / 1.30 = 1.14. The value of this factor is close to the maximum possible which is $2/\sqrt{3}$ = 1.1547. Thus, given that von Mises is the more appropriate yield criterion, use of the Tresca criterion adds an unnecessary additional conservatism which can approach 16%.

Similar situations can arise in the design of reinforced concrete slabs and metallic plates. The most appropriate yield criterion for reinforced concrete is the maximum principal stress theory and, as seen in the first column of Figure 6, this curve (square) intersects the von Mises ellipse a number of times. Thus, depending on the state of stress, the prediction of the collapse load based on one criterion can be greater or less than that based on the other, and so use of an inappropriate yield criterion can, in general, lead to overt conservatism (lack of economy) or lack of structural safety.

Design from Simulation

Let us assume, as will generally be the case for an arbitrary problem, that the engineer has no theoretical solution for use in the design process. As such, it would be necessary to use simulation results. Recognising design as an iterative process, the engineer would typically make an intelligent guess at the required pipe geometry, perform a simulation and then modify the geometry accordingly to satisfy the SLS and ULS conditions.

Let us assume the engineer is looking for a pipe geometry with an inner radius of 0.7m which is to be manufactured from a steel with a yield stress of 275MPa and requires an operating pressure of 82MPa with a requirement to be able to withstand 1.25 this pressure during an extreme event, i.e., a plastic limit pressure of 82 x 1.25 = 103MPa is required. If a conventional CFE system were used for the simulation then the engineer would have the choice of a lower-order (two-noded) element or a higher-order (three-noded) element. The results from a mesh convergence study using the lower-order axisymmetric membrane are presented in Figure 7. The results from LFE are also shown in the figure where NE is the number of elements in the mesh.

The convergence characteristics of the CFE and LFE elements are typical of such elements. The CFE element over-predicts both the elastic and plastic limit pressures for coarse meshes whereas the LFE element under-predicts these quantities. Actually, for this problem, the LFE predicts the elastic limit load exactly as was the intent when formulating the LFE.

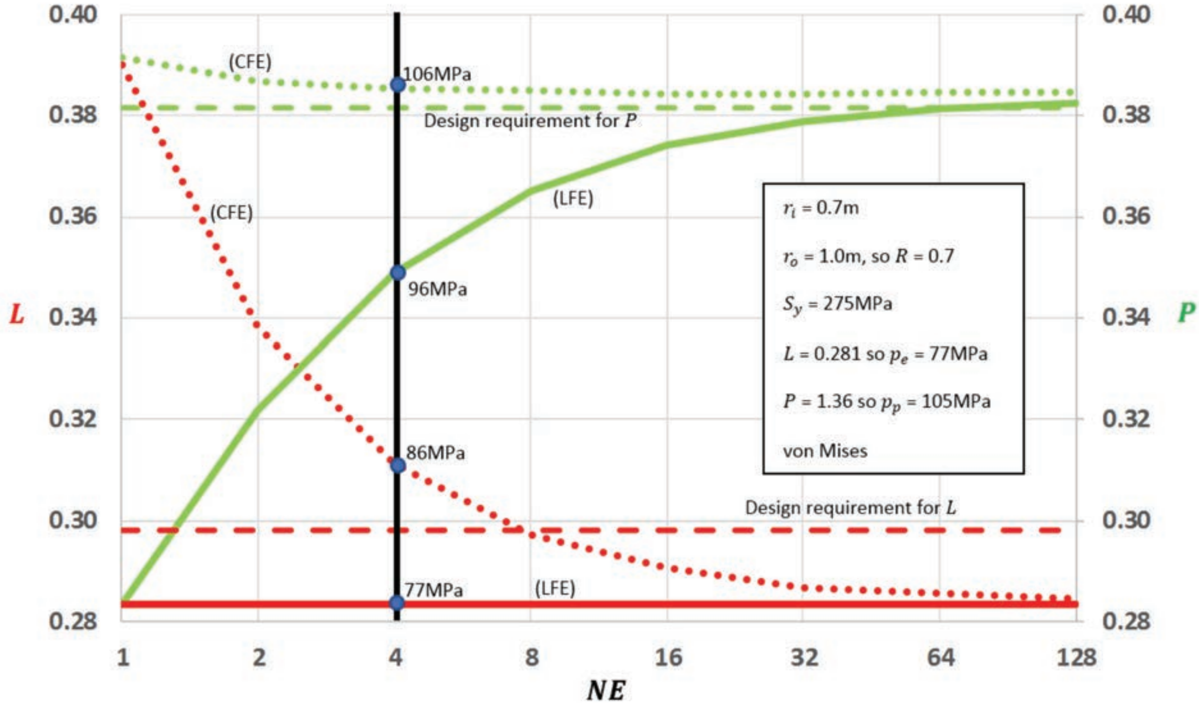


Figure 7: Mesh convergence or solution verification results.

Let us assume that the engineer is unaware of the necessity for solution verification and does not undertake a mesh convergence study. Let us further assume that the engineer used a mesh of four elements to produce results. These results are summarised and compared with the design requirements in Table 4.

The conclusions drawn using coarse models from different simulation packages lead to different guidance as noted in the table. However, when compared to the exact solution, it is the conclusion and guidance drawn from the LFE solution that are correct, i.e., that the outer radius needs to be increased. The engineer adopting the conventional CFE simulation process would have been guided incorrectly and may have expended effort in optimising the design only, hopefully, later to find that the solution was unsafe!

As seen from this example, conclusions drawn from simulation results using a coarse mesh are often incorrect when using the conventional CFE formulation whereas they are correct when using an EFE formulation. CFE indicates a reduction in the outer radius whereas LFE indicates an increase in this geometric parameter.

$NE = 4$	**SLS** p_e, [MPa]	**ULS** p_p, [MPa]	**Conclusions & Guidance**
CFE	$86 > 82$	$106 > 103$	SLS and ULS conditions are satisfied. The design could be optimised by reducing the outer radius.
LFE	$77 \ngtr 82$	$96 \ngtr 103$	SLS and ULS conditions are not satisfied. The outer radius needs to be increased.
Exact	$77 \ngtr 82$	$105 > 103$	SLS condition is not satisfied but the ULS condition is satisfied. The outer radius needs to be increased.

Table 4: Summary of results, conclusions and guidance from an unconverged simulation.

Although we have treated SLS and ULS conditions equally in this example, it should be noted that whilst violation of the SLS condition, for the pipe, might only lead to the pipe suffering from some limiting yield at the inner radius, violation of the ULS condition could mean that the pipe actually bursts! The LFE formulation unlike the CFE formulation, safely underpredicts the ULS condition when a coarse mesh is used. As has been seen, designs based on coarse LFE meshes might predict the need for more material, e.g., an increase in the outer radius of the pipe. This might lead to an uneconomical design in terms of an outer radius greater than is actually required, but at least the design will be safe which is something that cannot be said for the CFE result.

In terms of the formulation adopted by the engineer for the design of pipes, then it is clear that the LFE formulation is more appropriate than the CFE formulation in terms of providing safe solutions. In terms of the concept of optimising material usage then, again, one is best advised to adopt an LFE formulation. Solutions are always safe, even the single element in our pipe example where the elastic and plastic solutions are identical will lead to a safe design, and mesh refinement can be seen as a way of reducing conservatism in the design whilst maintaining safety.

It should, of course, be noted that the LFE is a higher fidelity element than its corresponding CFE element. This was seen in the problem considered, where even a single LFE element recovered the exact elastic solution whereas the corresponding CFE failed so to do. This higher fidelity property is an obvious one to include in EFEs. If, for example, continuum problems generally require elements that can cope with bending, then why should this not be included? It is also the case that the majority of the mathematical vagaries which cause so much of an issue with young inexperienced engineers, i.e., hour-glassing, shear and volumetric locking etc., can be by-passed.

The LFE is an exact element for elastic solutions to the Lamé equations. CFE element models need to be refined before they can recover the exact elastic solution. The plastic solutions for axisymmetric membrane problems, however, develop differently with CFE elements converging at a higher rate than the LFE element, albeit from an upper bound. The LFE element currently adopts a very simple plasticity scheme based on the elastic compensation method [8], and uses peak elemental stress values. It is likely that the approach can be improved by adopting formal limit analysis techniques and that the rate of convergence for plastic solutions will, thereby, improve significantly. This development work, which requires the definition of hyperstatic stress fields and implementation into a second-order cone programme, is a current research project at RMA Initial studies undertaken using a spreadsheet indicate that extremely accurate plastic solutions can be achieved without the current need for the sort of mesh refinement shown above.

Closure

The MEICON project specification and early findings illustrate just how essential it is that the structural design engineer raises his/her game to minimise the embodied energy in the built environment. Similar arguments are likely to exist in other industries. The case study presented in this article has been used as a vehicle to illustrate how, in the author's view, the current simulation tools available to the design engineer are not best suited to the task. A new paradigm for design is required and this might include the use of non-conventional equilibrium finite elements (EFEs) in conjunction with lower-bound limit analysis/design. Whilst still important when using EFE models, the necessity for solution verification becomes of secondary importance in that any equilibrium solution will lead to a safe prediction of strength. This is sufficient for the engineer to establish a feasible design which can then later be optimised in order to minimise embodied energy.

One point that is strongly made by the MEICON team is that the load factors presented in codes of practice, e.g., the Eurocodes EC2 and EC3 can be considered as reliable and, for the ULS condition the engineer should try and design the structure so that these are not exceeded unnecessarily. The case mentioned earlier of the RC slab designed using linear-elastic methods showed that the codified load factor was exceeded by some 27% and that had a formal limit analysis not been undertaken, not only would the structure have been constructed with an unnecessarily large amount of embodied energy, but additional strengthening would also have been added, thereby exacerbating the energy usage.

If the MEICON ambitions are to be achieved then, in my view, the following points need to be addressed:

- Use of appropriate methods of analysis, e.g., plastic rather than elastic analysis for ULS
- Use of simulation technology that is inherently safe and therefore encourages the engineer to feel unconstrained in undertaking design optimisation, e.g., EFE.
- Development of appropriate software tools that automate the design process for particular structural forms, e.g., limit analysis/design. RC slabs are a prime example where significant savings in reinforcement can be achieved.
- Given that coarse, unrefined meshes of CFEs normally lead to unsafe solutions, should there not be a partial safety factor included in LSD to account for this fact?
- Published design rules and data which are often overtly conservative and sometimes plain wrong need to be reviewed and updated.

On this last point — and this fixes in my mind the fact that simulation engineers do not generally consider plastic collapse for their ULS calculations, either that or they do

Copyright Nafems 2021 Licensed solely to Angus Ramsay (angus_ramsay@ramsay-maunder.co.uk) for single use only

not undertake code verification — the NAFEMS benchmark problems for non-linear material behaviour concerned with the plastic collapse of metallic plates adopt erroneous theoretical solutions. I have written a corrigendum to this benchmark [10], as well as a more detailed explanation [11].

Conclusion

If, as engineers, we are going to live up to the expectations put on us strongly by society to urgently reduce emission levels, then a new simulation paradigm is required for the analysis and design of, amongst others, the built environment. The behemoth simulation software that engineers are currently faced with is not the solution. No doubt such software can solve a great variety of increasingly sophisticated physics and even multi-physics problems when cajoled so to do by experts, yet despite the seeming ease of use that modern simulation software exhibits, these packages remain traps for the unwary engineer to commit finite element malpractice. Such malpractice led to the Sleipner catastrophe and, one suspects, untold other unpublished failures to match simulation results with the performance of the real engineering artefact.

An exemplar software would be one which any competent undergraduate engineer can use and produce safe results without having to undergo vast amounts of unnecessary academic or industrial post-graduate training in order to understand the mathematical vagaries of what goes on under the bonnet of the simulation software. This should be unnecessary with a properly formulated simulation package which allows the engineer to concentrate on the engineering rather than the underlying mathematics. The modern term describing such software would be democratised software. Whilst many software vendors increasingly offer software to engineering designers, i.e., those without a specialist knowledge of FE, these are mostly inadequate and often unsafe. I know of one honourable exception to this trend, this being ESRD [12] who produce CFE software with the capability for both h-type and p-type refinement and also with inbuilt error estimation of quantities of interest. With such a software tool the practising engineer is unlikely to fall into the finite element malpractice trap.

As previously noted, partial factors of safety adopted in Limit Design Codes do not account for uncertainties due to the simulation technique used to obtain the stress resultants used in the design process. This article has illustrated that this is a risky approach when using conventional simulation software which will undoubtedly influence the reliability of the design. In general, the solution from a CFE simulation will not be feasible, i.e., it will be unsafe and, therefore, an unreliable candidate for design optimisation, e.g., to minimise the material requirements.

This article has also demonstrated, at least as far as the safety critical ULS condition is concerned, that non-conventional equilibrium finite elements can provide feasible solutions which are safe and can be optimised with an appropriate mathematical program.
The practising engineer needs software that is democratised and which can then lead the engineer to safe and economical designs. This requires a different paradigm to the current simulation software and it might also need, at least initially, the development of bespoke tools utilising EFEs and limit analysis/design principles for particular structural forms.

References

[1] Minimising Energy in Construction (MEICON) [Online] Available: https://www.meicon.net/

[2] Ramsay, A.C.A., Maunder, E.A.W. & Gilbert, M., "Yield Line Analysis of Reinforced Concrete Slabs: is the 10% Rule Safe?" *NAFEMS Benchmark Magazine*, January 2015. [Online] Available: https://www.ramsay-maunder.co.uk/knowledge-base/publications/yield-line-analysis-of-reinforced-concrete-slabs-is-the-10-rule-safe/

[3] Richardson, L.F., "The Approximate Arithmetical Solution by Finite Differences of Physical Problems including Differential Equations, with an Application to the Stresses in a Masonry Dam," *Philosophical Transactions of the Royal Society A.* 210 (459-470): 307-357, 1911.

[4] Ramsay, A.C.A., "The Sleipner Incident: A Computer-Aided Catastrophe Revisited," *NAFEMS Benchmark Magazine*, April 2016.
[Online] Available: https://www.ramsay-maunder.co.uk/downloads/nbr06.pdf

[5] Ramsay, A.C.A. & Maunder, E.A.W., "Equilibrium Finite Elements for RC Slab Design," *Structure Magazine* (US), July 2017.
[Online] Available:
https://www.structuremag.org/?p=11774
https://www.ramsay-maunder.co.uk/knowledge-base/publications/equilibrium-finite-elements-for-rc-design/

[6] Ramsay, A.C.A. & Maunder, E.A.W., "Avoiding Finite Element Malpractice through Simulation Governance," I*ET, Engineering & Technology Reference,* July 2016.

[7] Hearn, E. J., *Mechanics of Materials,* Volume 1, Chapter 10, 3rd Edition, Butterworth-Heinemann, 2000.

[8] Taylor, G.I., Quinney, H., "The Plastic Distortion of Metals," *Proceedings of the Royal Society, London A,* Vol. 230, pp. 323-362, 1931.
[Online] Available: https://www.ramsay-maunder.co.uk/knowledge-base/publications/avoiding-finite-element-malpractice-through-simulation-governance/

[9] Mackenzie, D., Nadarajah, C., Shi, J., Boyle, J.T., "Simple Bounds on Limit Loads by Elastic Finite Element Analysis," *Journal of Pressure Vessel Technology,* Vol. 115, pp. 27-31,1993

[10] Ramsay, A.C.A., "NAFEMS Benchmarks for Plastic Collapse of Square Metal Plates (NL7A and NL7B – Corrigendum)," *RMA,* 2020.
[Online] Available: https://www.ramsay-maunder.co.uk/downloads/nl7ab-short-article.pdf

[11] Ramsay, A.C.A., "NAFEMS Benchmarks for Plastic Collapse of Square Metal Plates," *RMA,* 2019.
[Online] Available: https://www.ramsay-maunder.co.uk/downloads/nl7ab.pdf

[12] Engineering Software Research and Development, Inc. (ESRD) [Online] Available: https://www.esrd.com/

Acknowledgements

I am grateful to Dr Edward Maunder for undertaking a technical review of this article, to the Rev. Max Ramsay for proof reading the text and to my colleagues at NAFEMS for their support of this article.