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Finite Element Specialists and Engineering Consultants

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Shades of Equilibrium

Edward Maunder & Angus Ramsay

Institution of Structural Engineers
(Devon & Cornwall Branch – Technical Meeting)
University of Exeter
14th October 2010 (6.30pm)

EFE Equilibrium Finite Elements *for safe structural analysis and design optimisation*

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Contents

- Equilibrium
- Early experience with NWB
- Sleipner A
- Equilibrium in FEM
- EFE – an alternative FEM
- EFE Examples

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Equilibrium

- **In perfect balance – quality of life;**
- **Free bodies – from the finite to the infinitesimal;**
- **Sir Christopher Wren;**
- **Professor Jacques Heyman.**

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- Sir Christopher Wren (1660s)
  - The design must be regulated by the art of **staticks**, or invention of the centers of gravity, and the duly poising of all parts to **equiponderate**; without which, a fine design will fail and prove abortive. Hence I conclude, that all designs must, in the first place, be brought to this test, or rejected.

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Palma Cathedral (1300-1600), nave height 44m, column slenderness ratio 14.4

N.B. Widecombe church (Cathedral of the Moor) would fit inside, tower and all!

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elliptic “cantilever” stone staircase, Palma, 19th century

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- Professor Jacques Heyman
  - The Master Safe Theorem (1960s)
  - If any **equilibrium** state can be found, that is, one for which a set of internal forces (***a stress field***) is in **equilibrium** with the external loads, and, further, for which every internal portion of the structure satisfies a **strength criterion**, then the structure is **safe**.

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Palace of Arts, Valencia, 1990s (Calatrava) – another cantilever

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- **Early experiences with FEM – NWB tower (1970s)**

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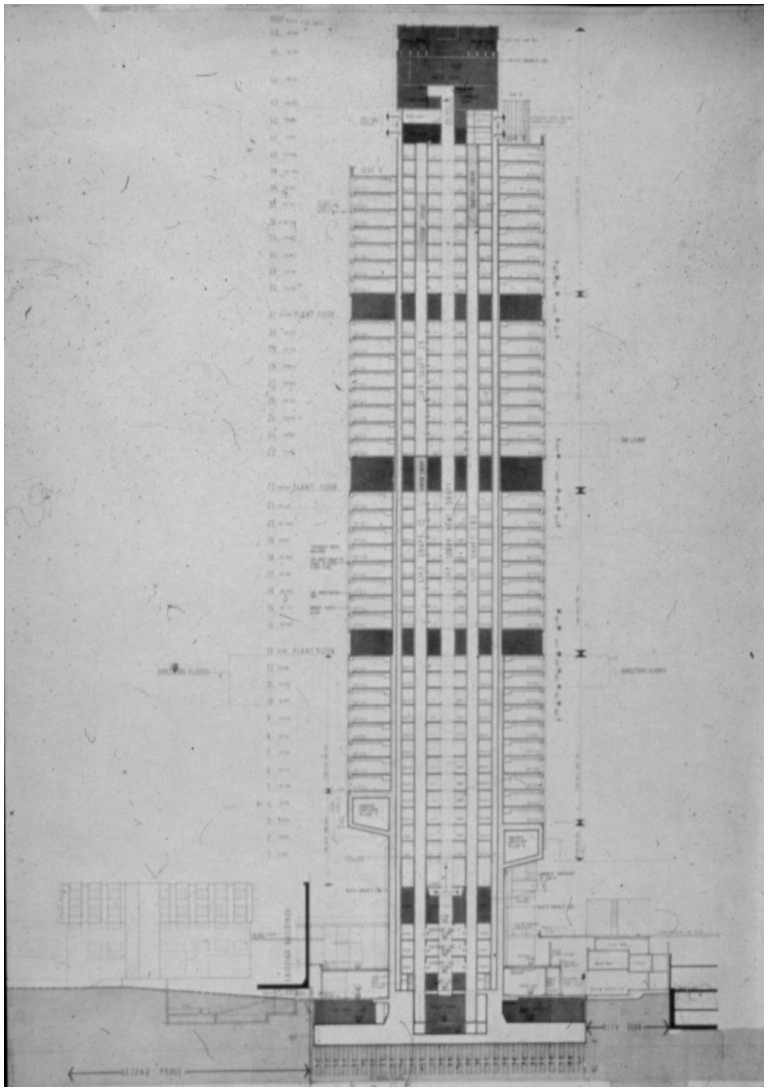

The design

The construction

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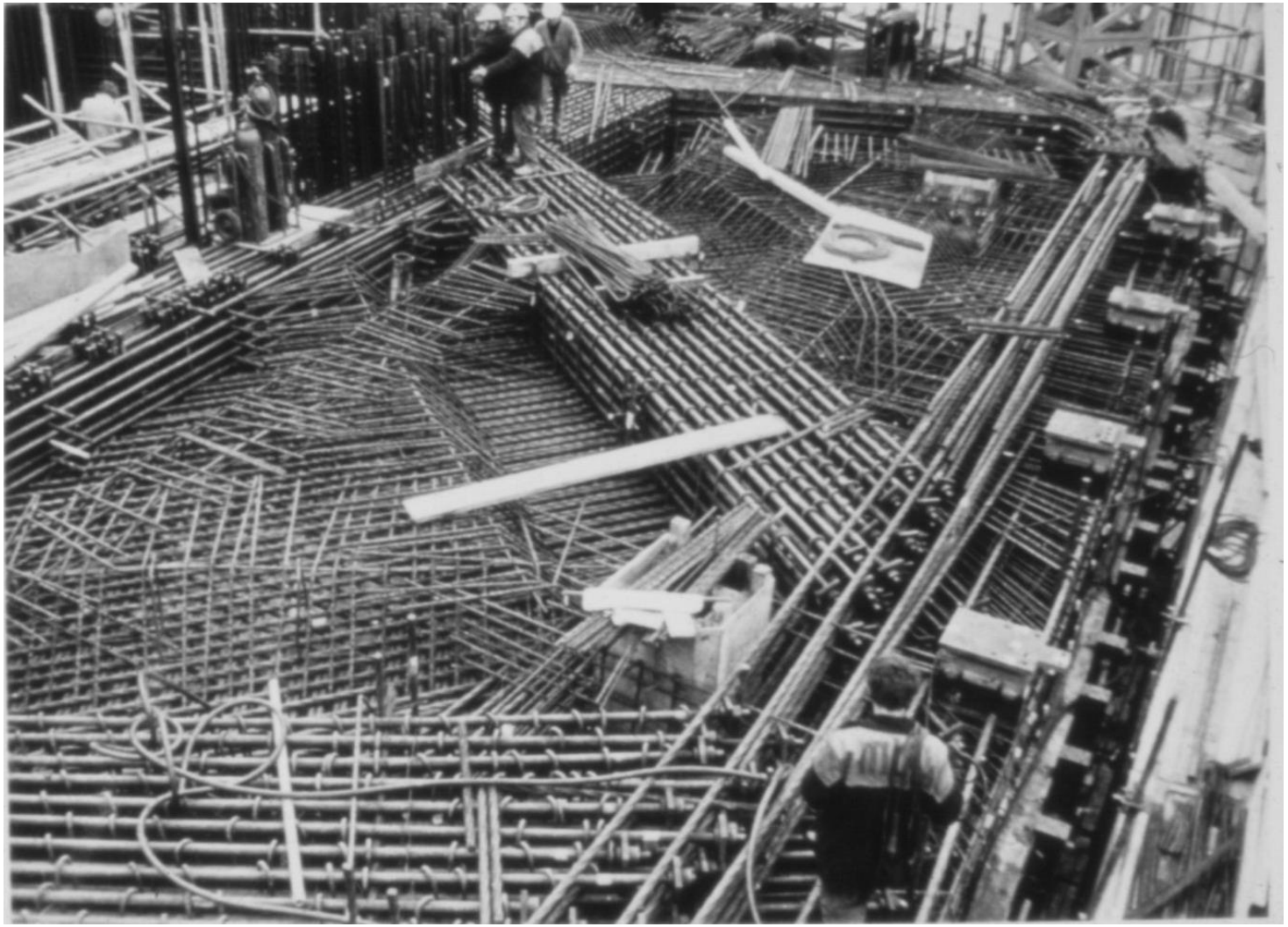

reality at the top of a cantilever

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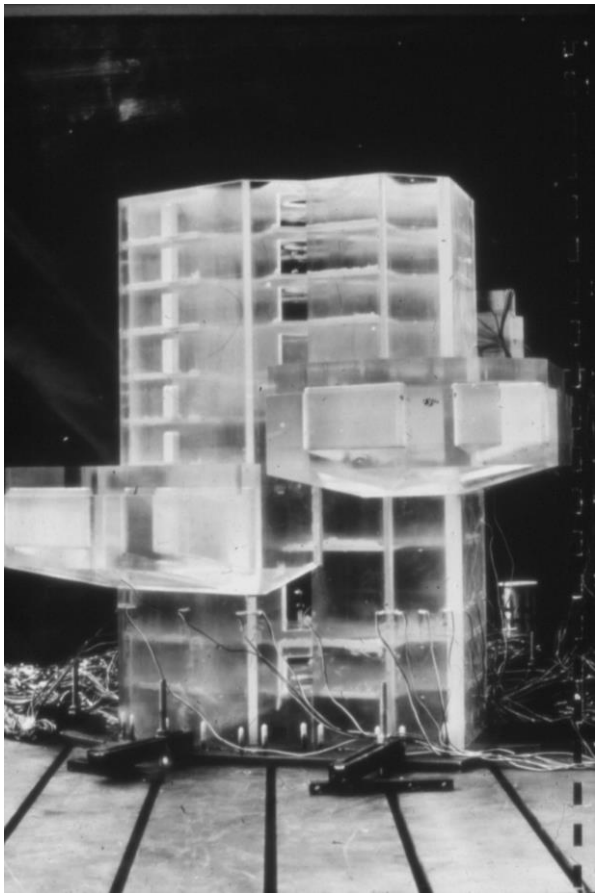
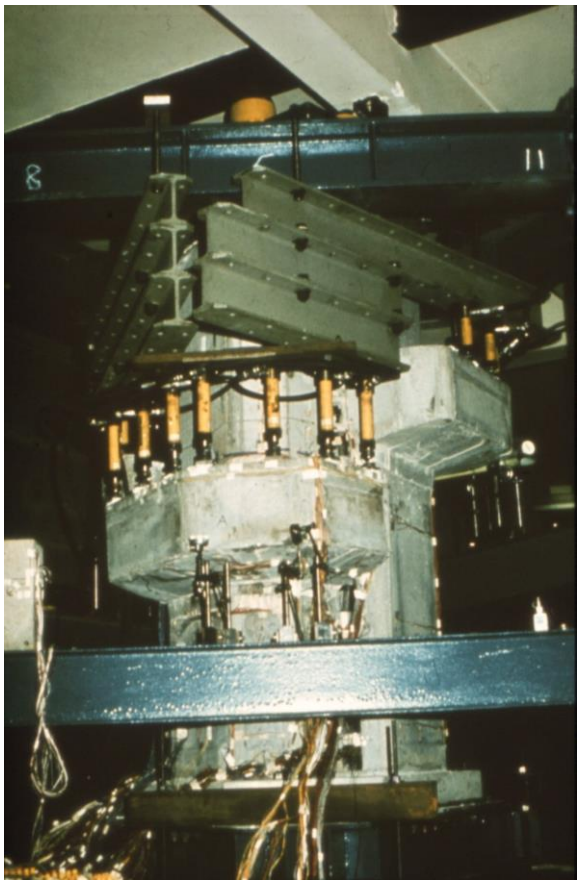
Two physical models to validate the FEM



perspex



micro-reinforced concrete

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- Basic questions raised re equilibrium
  - Strategies for detailed design in the absence of detailed equilibrium?
  - What forms of equilibrium do we want?
  - How do we achieve them?

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- Stresses or stress-resultants?
  - Different viewpoints
  - A “classic” example of error in deriving stress-resultants from stresses: SLEIPNER

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Sleipner collapse 1991 Computer Assisted Catastrophe

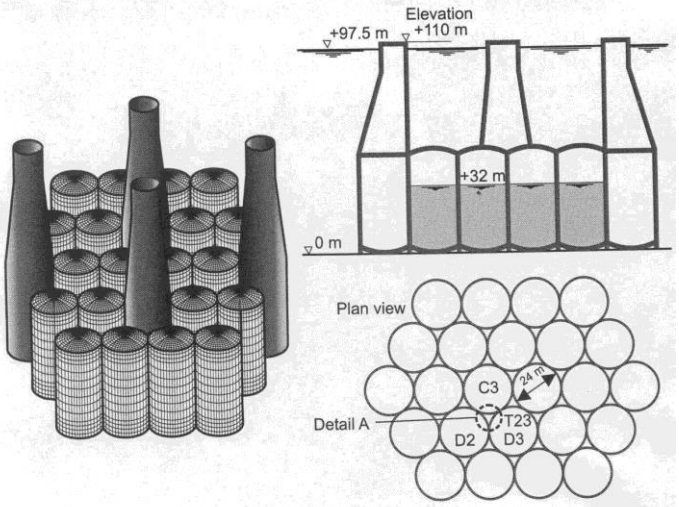


Fig. 5.2 'Sleipner A' platform

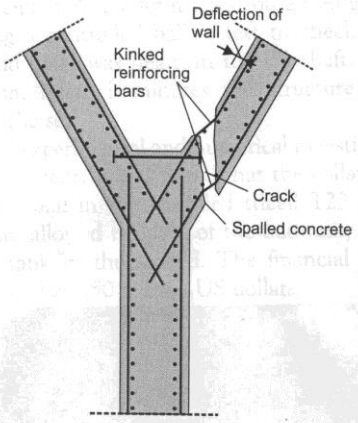
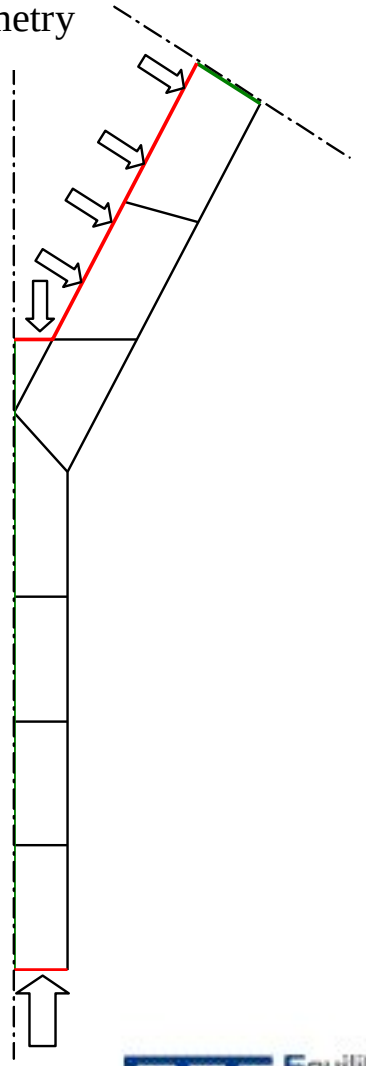


Fig. 5.4 Failure mode

axes of symmetry



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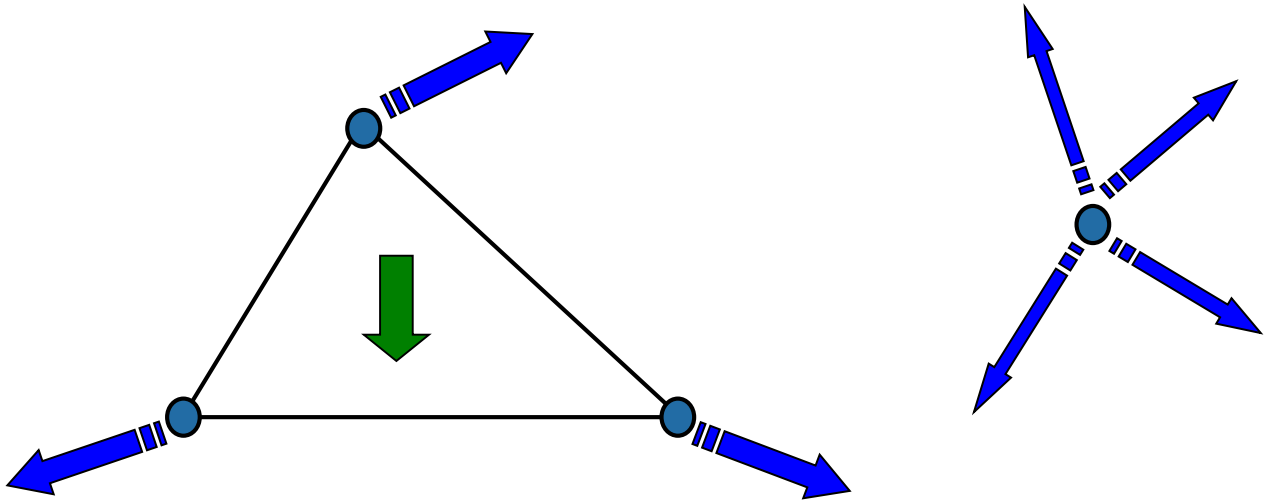
The conventional view of an element



Equilibrium of elements and nodes – nodal forces

“knotted handkerchiefs” model

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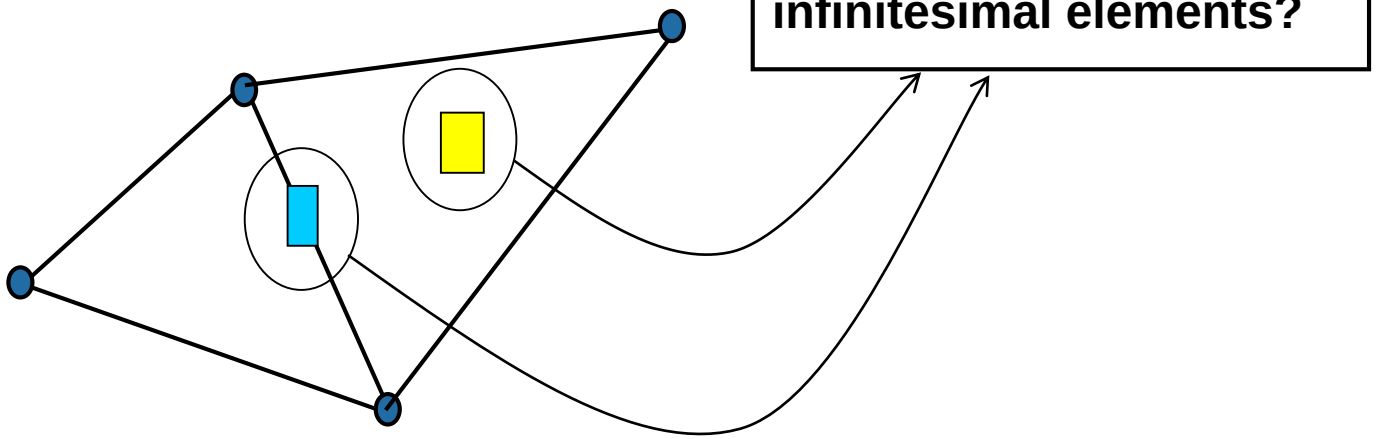

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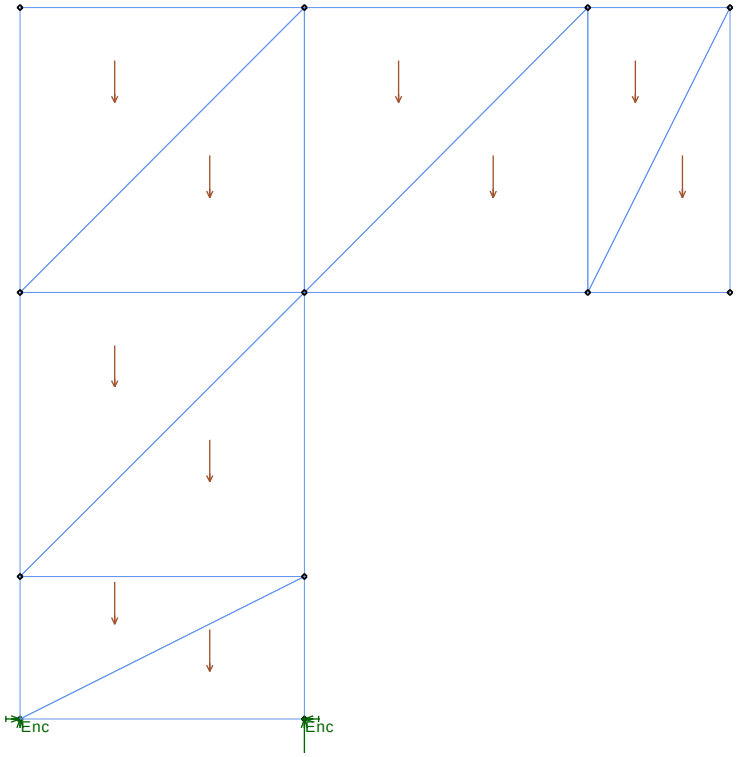

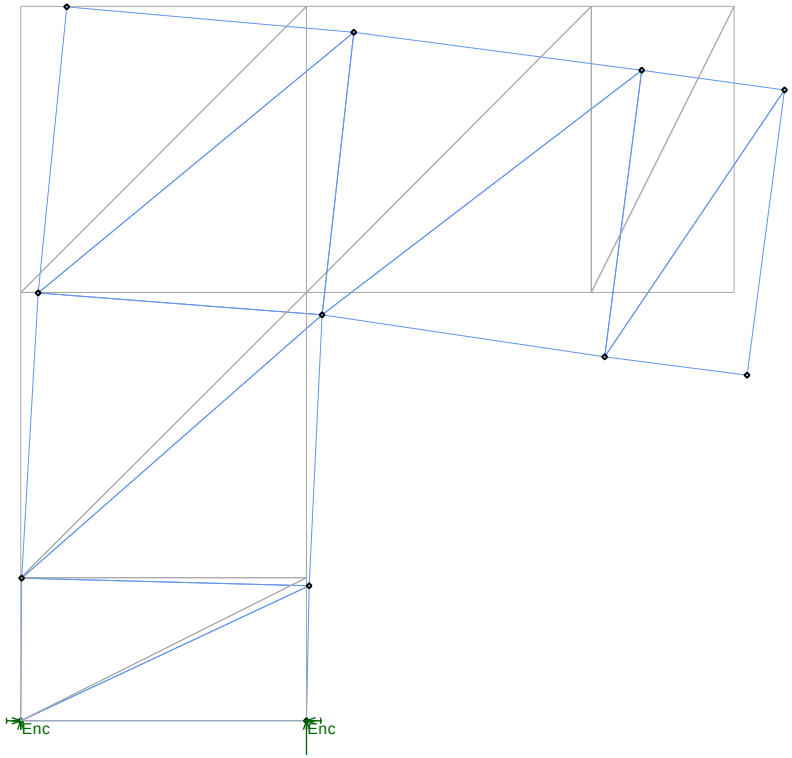

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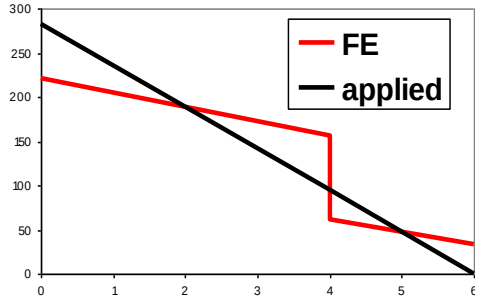
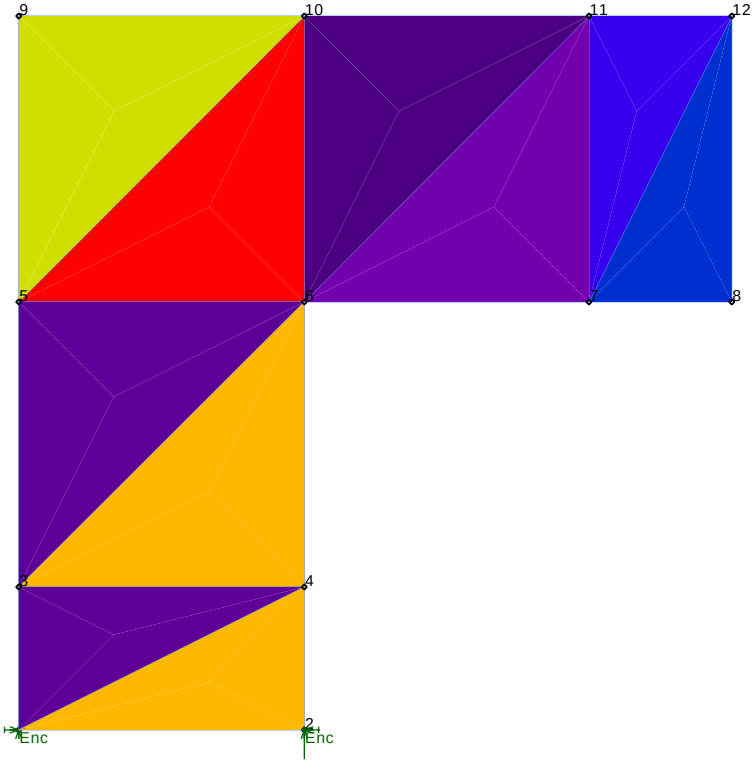
Shear force diagram





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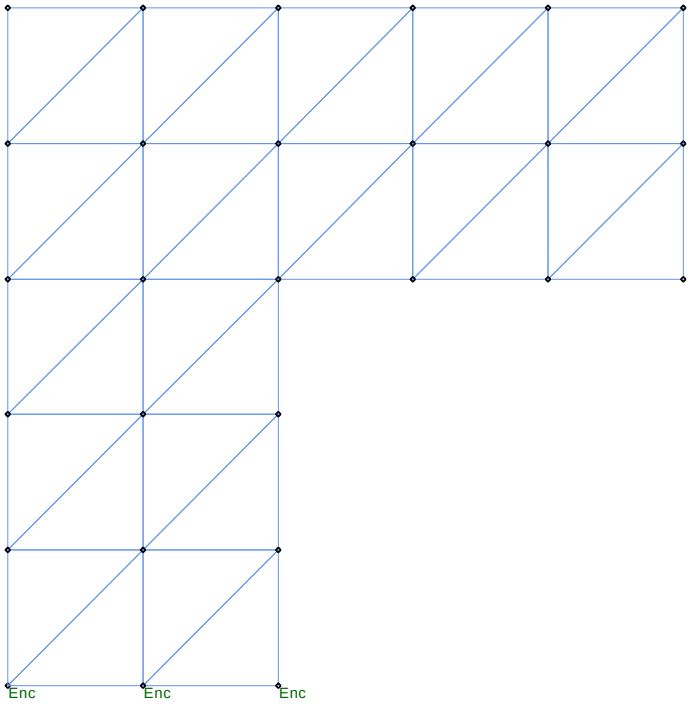

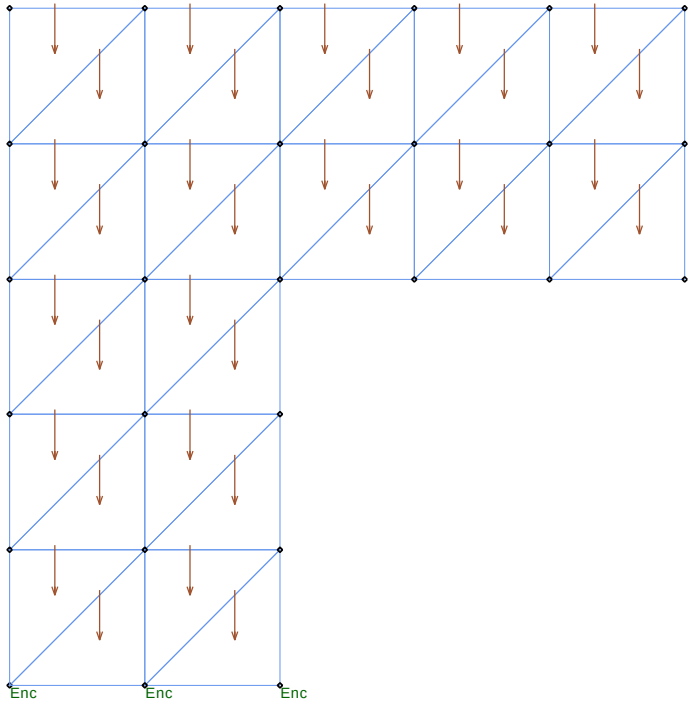

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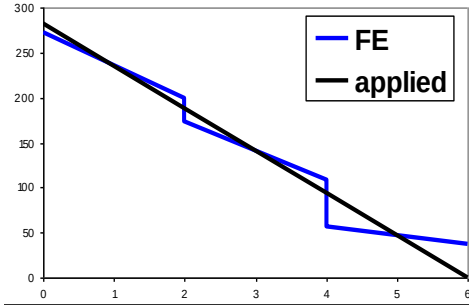
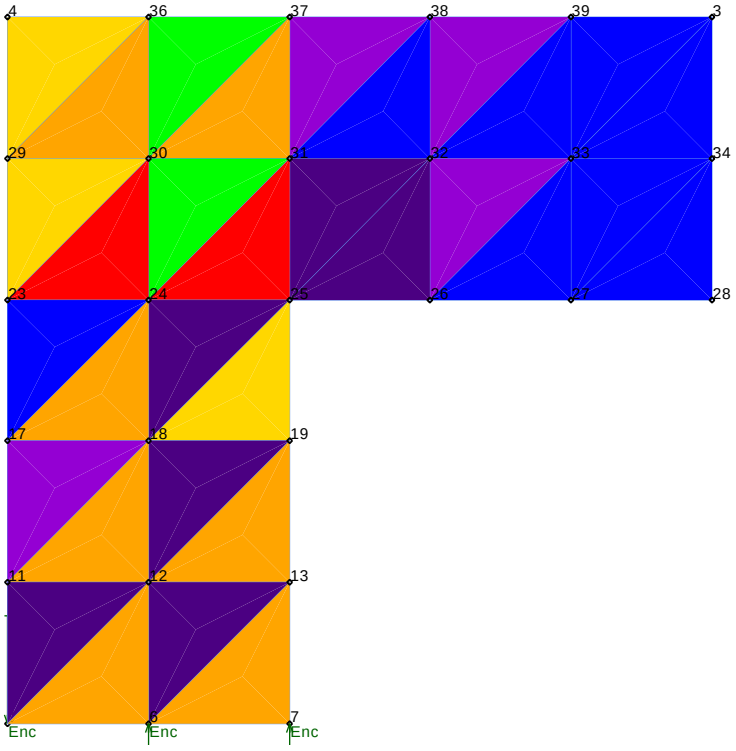
Shear force diagram





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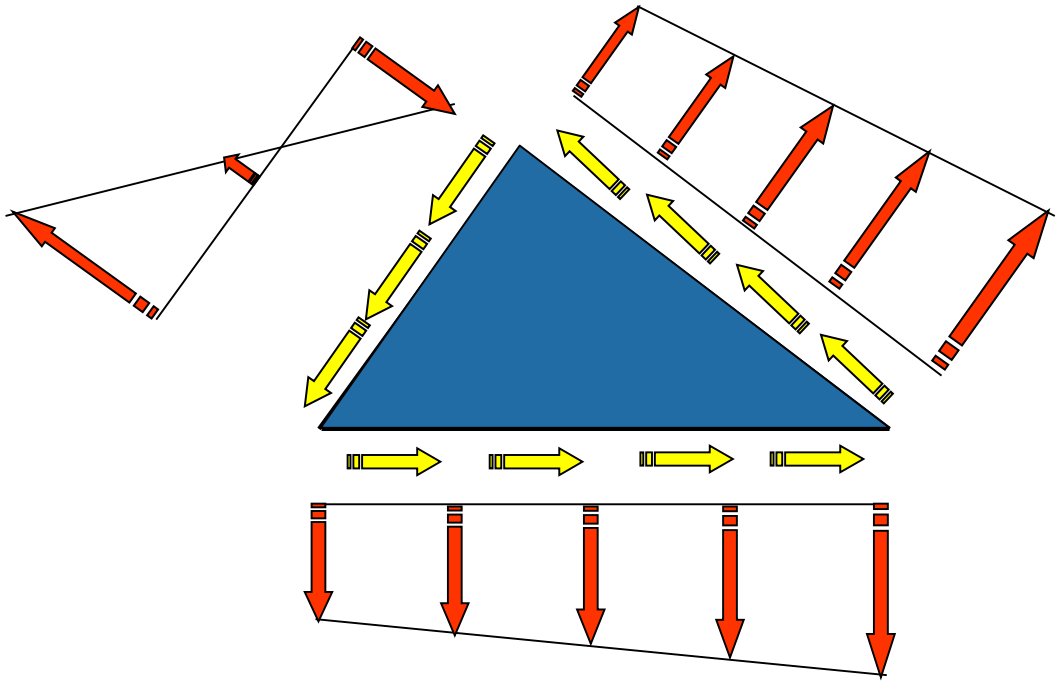
EFE - an alternative FEM



EFE: equilibrium of elements – tractions and stresses

“patchwork quilt model”

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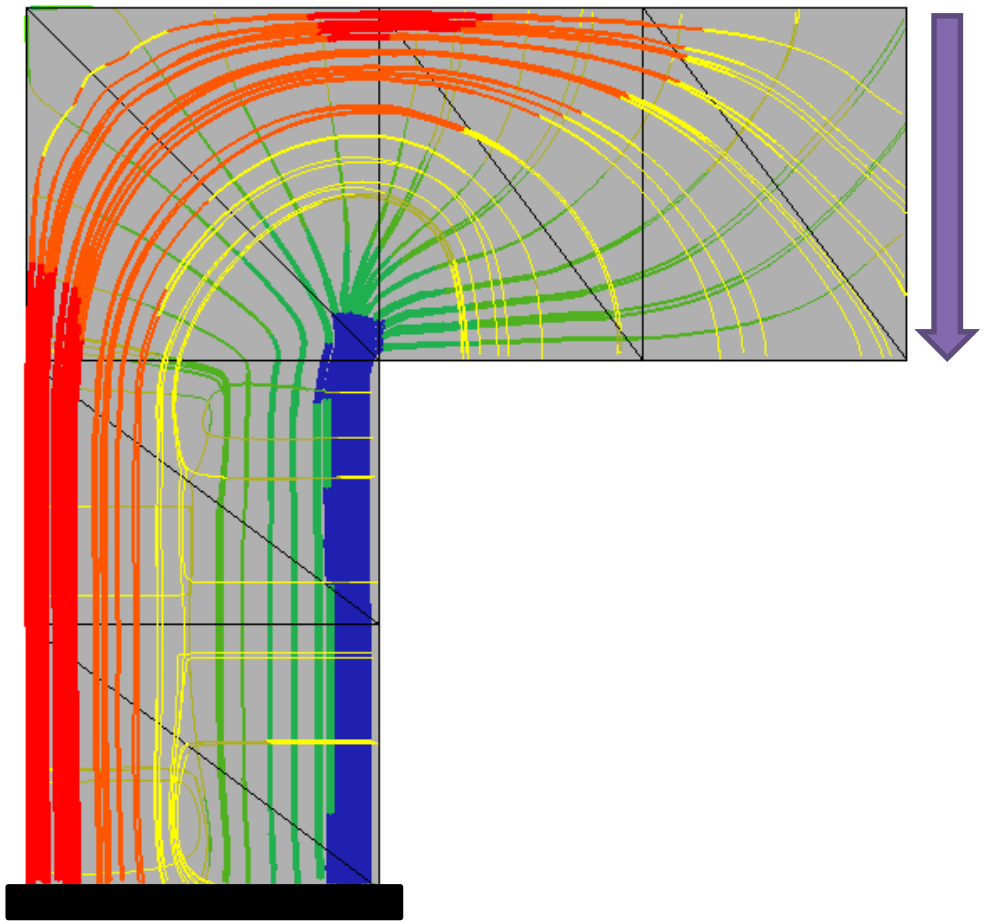
Stress trajectories help to identify load paths and where to place reinforcement



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EFE Examples

Example 1 – Steel Plate
- Problem Description
- Elastic solution in EFE
- Lower Bound Limit solution in EFE

Example 2 – Concrete Slab
- Problem Description
- Comparison of Upper and Lower Bound Results

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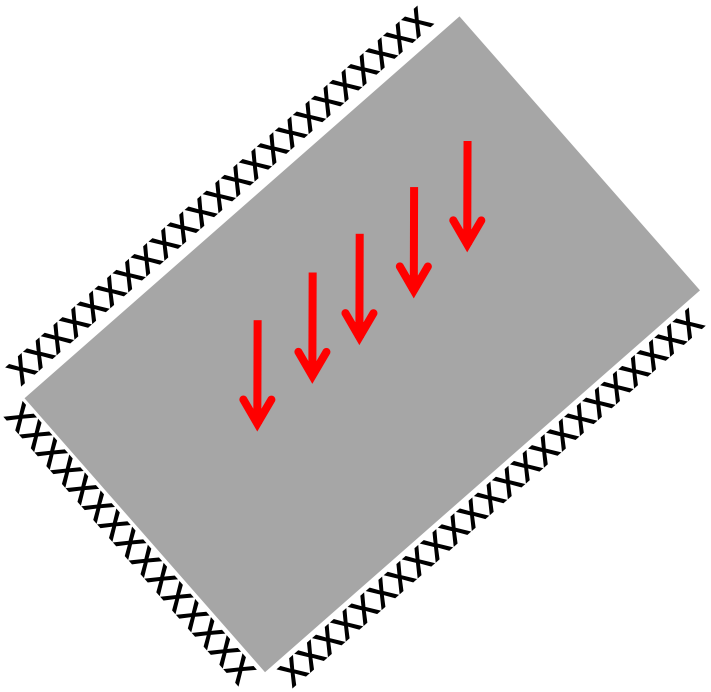
Example 1: Steel Plate



Rectangular Steel Plate

Uniform Yield Strength

Fixed Edges

Uniform Line Load

Elastic solution in EFE

Lower Bound Limit solution in EFE

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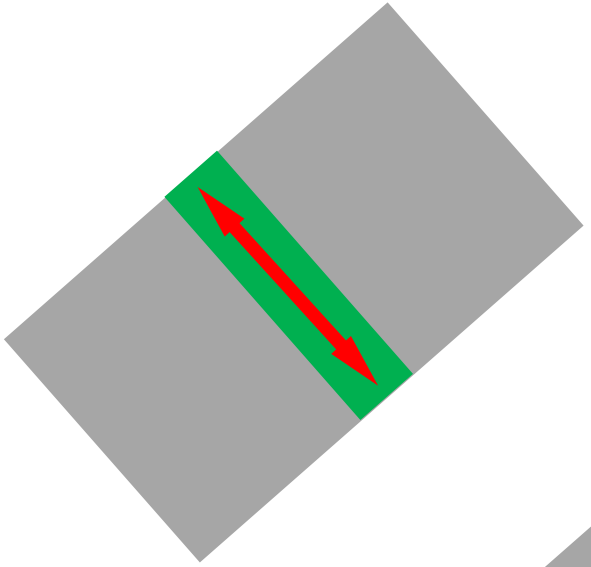
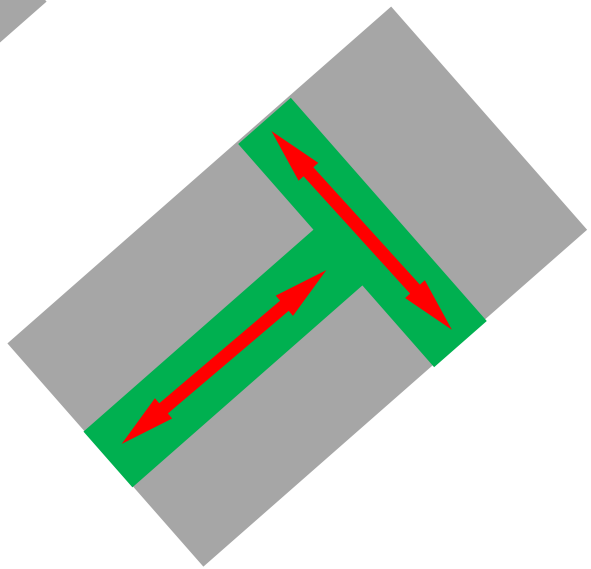
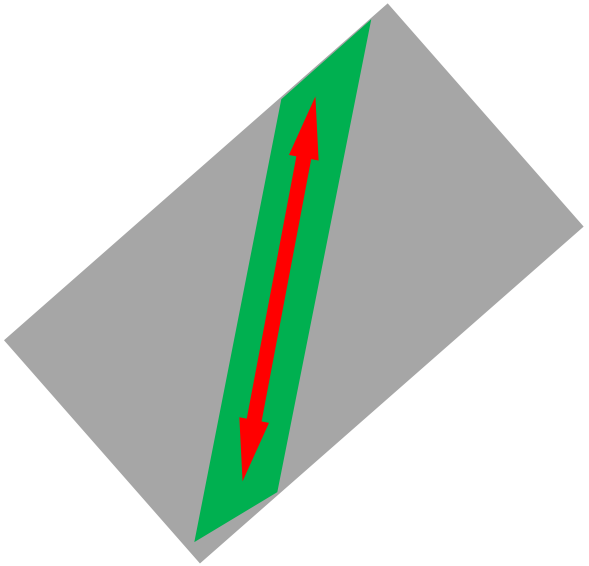
Alternative Load Paths in the Steel Plate



Elastic Analysis produces one particular solution





Lower Bound Limit Analysis produces another solution that maximises the load carrying capacity of the plate. Exploits the Master Safe Theorem.

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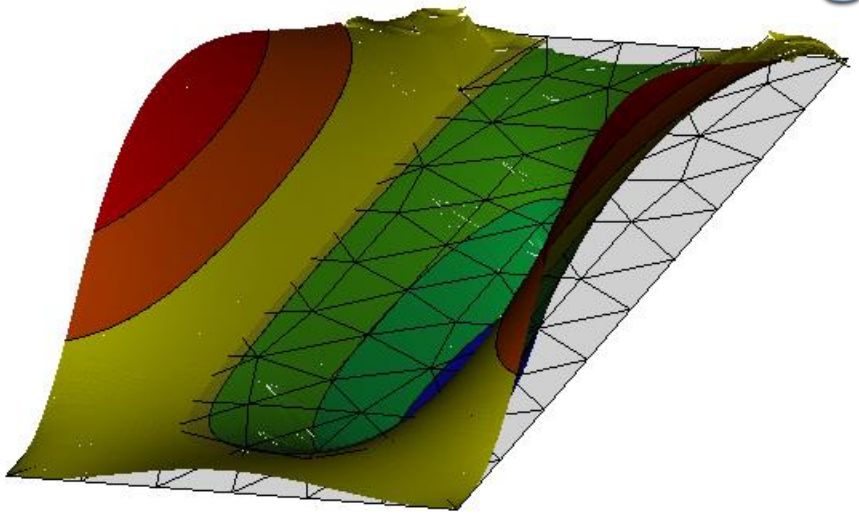
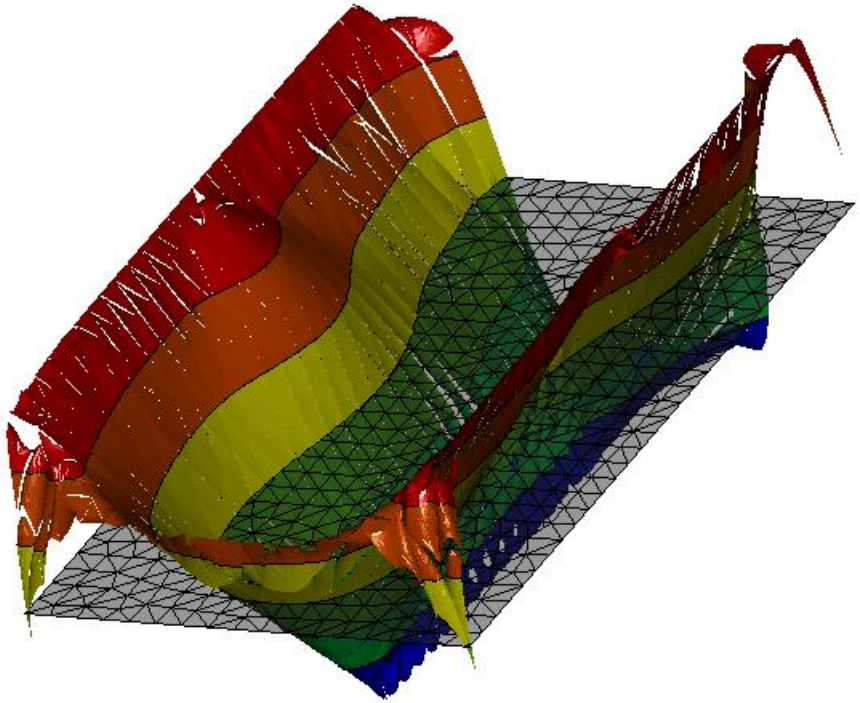
Bending Moments



Elastic



Plastic

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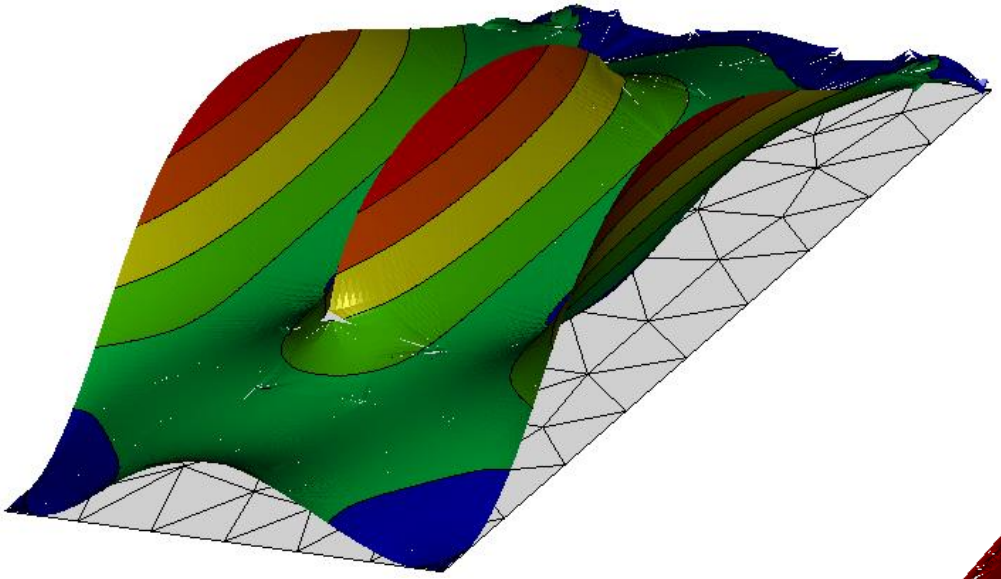
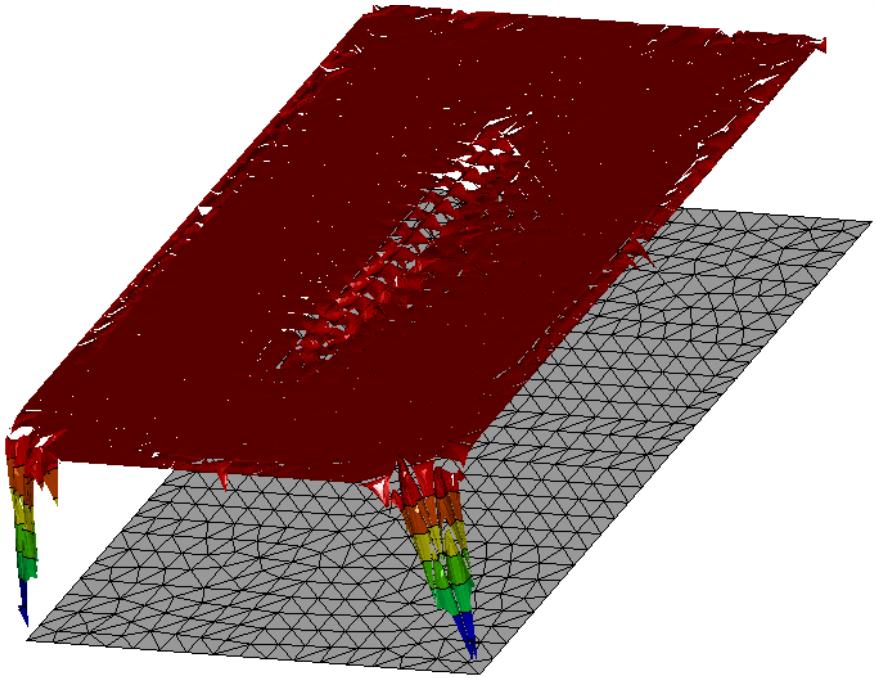
Von Mises Moments



Elastic



Plastic

Limit load 2.7 times load
to cause first yield

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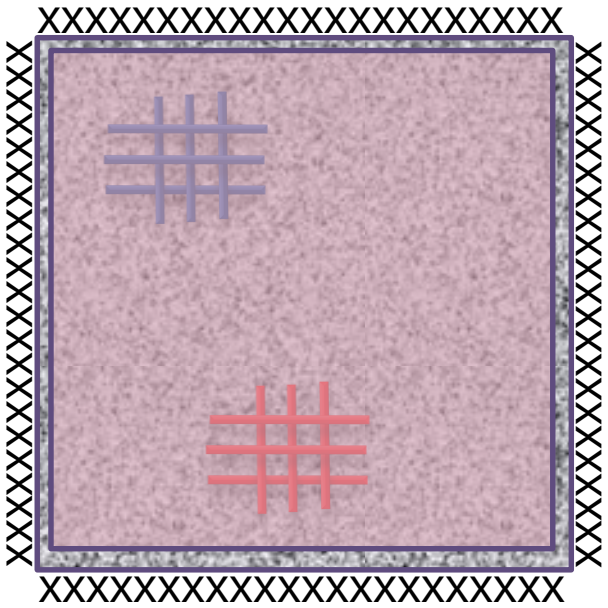
Example 2: Reinforced Concrete Slab



Square Concrete Slab

Uniform Strength (Hogging)

Uniform Strength (Sagging)

Fixed Edges

Uniform Area Load

Upper Bound Limit solution in EFE

Lower Bound Limit solution in EFE

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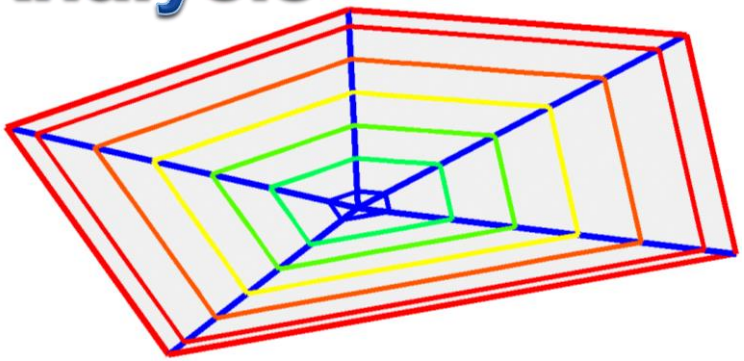
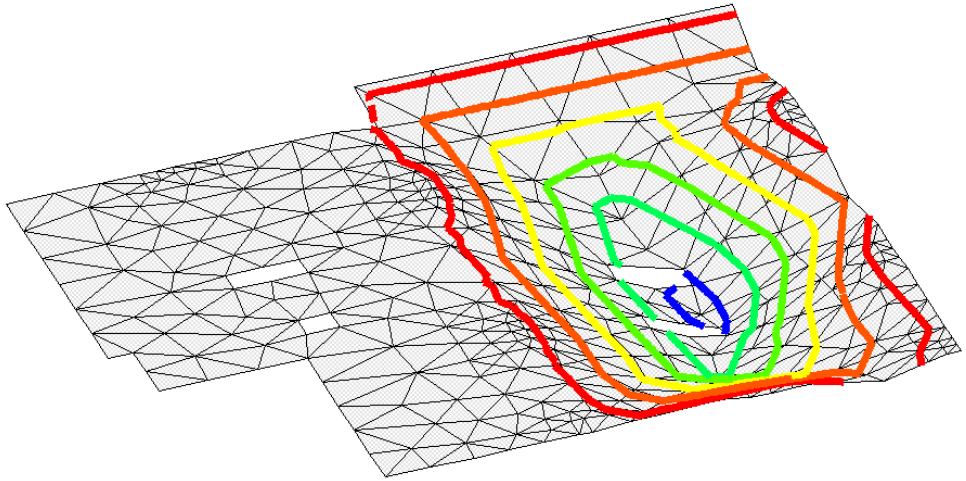
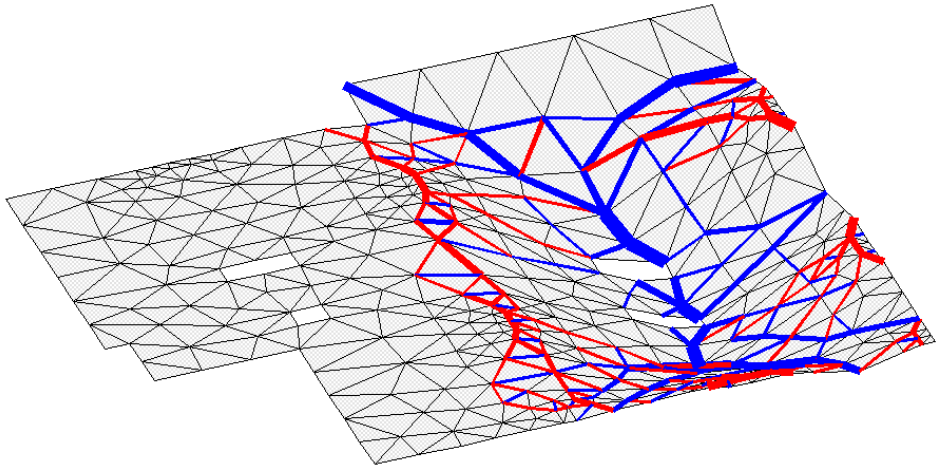
Yield-Line Analysis

basic mechanism based on rigid Morley elements





contour lines of a collapse mechanism



yield lines of a collapse mechanism

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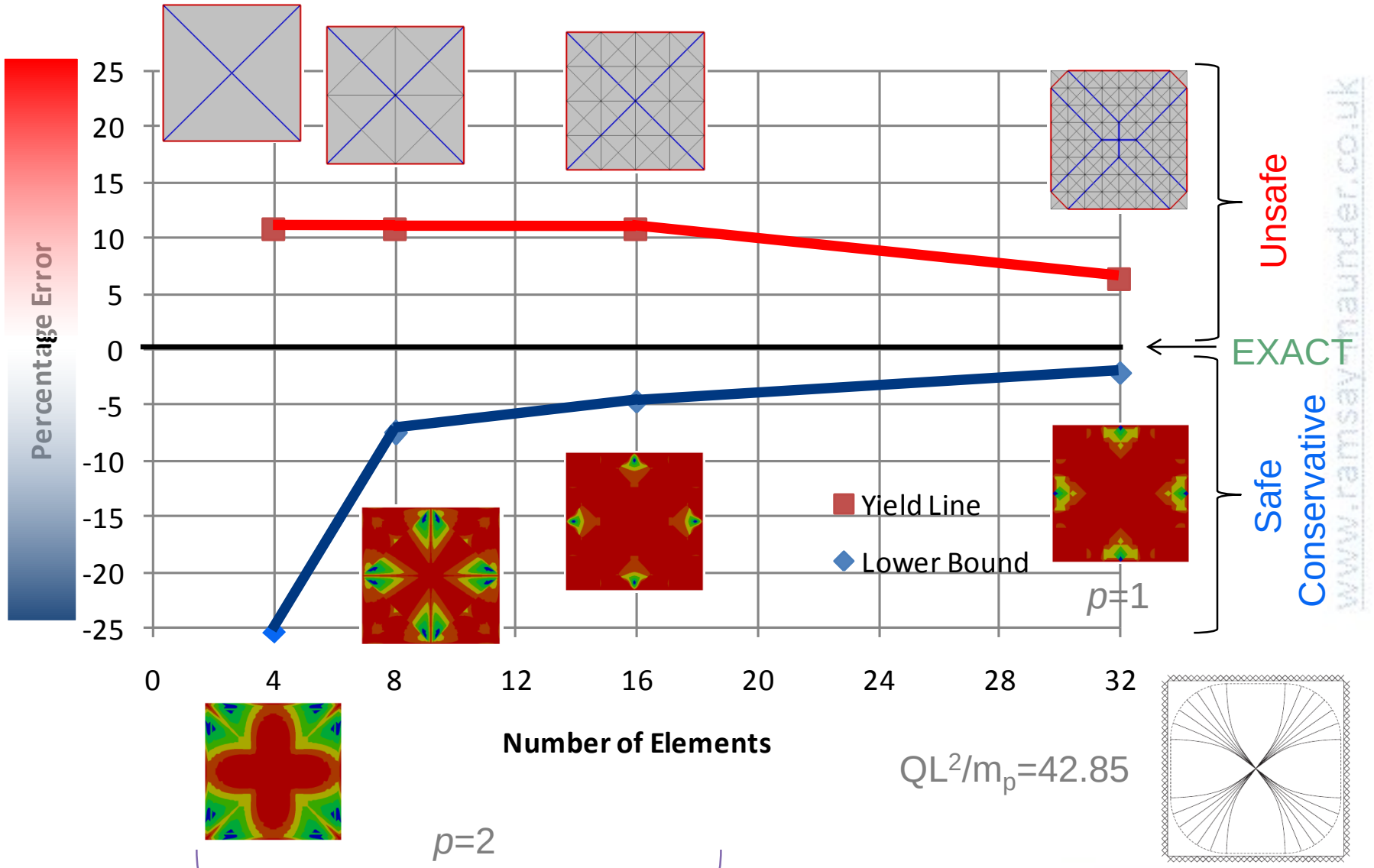

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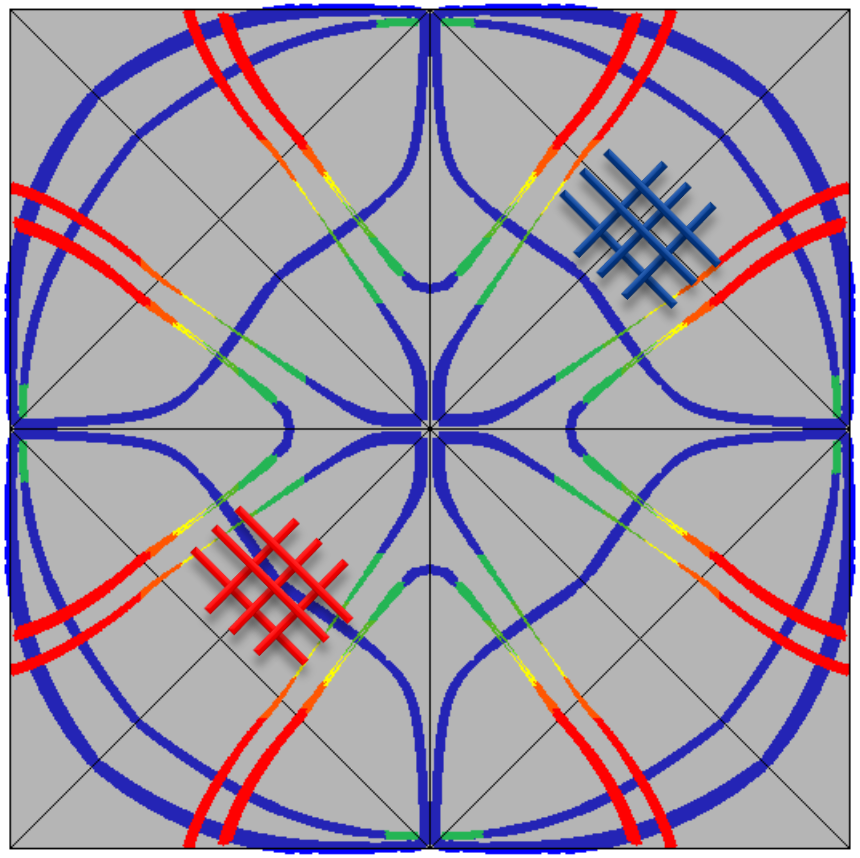
Principal Moment Trajectories (Lower Bound Solution)



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Any Questions

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EFE Equilibrium Finite Elements

for safe structural analysis and design optimisation

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