Revision Request for ASME B&PV Code

Angus Ramsay, PhD, FIMechE (Director at Ramsay Maunder Associates)

Jeremy Theler, ME (Director at SeamPlex)

Introduction

In [1], material data is provided for the practising engineer to be able to calculation the thermal expansion in a pressure vessel or similar component. The data is provided in tables listing thermal strain, ε_T, and both instantaneous, α', and average, α ,coefficients of linear thermal expansion at different temperatures, T. For example, Table TE-2 on p714 lists data for the *Thermal Expansion for Aluminium Alloys*. The particular data used will depend on how the engineer is going to calculate the thermal expansion. Often as not, nowadays this calculation will be performed through finite element analysis. The background to this Technical Note can be found in [2] and [3].

In many finite element (FE) systems, thermal expansion calculations are based on mean coefficients of linear thermal expansion (or for brevity coefficients of expansion), α, and these may be a function of temperature. The thermal strains used in the FE calculation are then determined from Eq (1), where $\Delta T = T_f - T_i$ is a uniform temperature change from the initial temperature, T_i to the final temperature T_f. Clearly, for accuracy, the mean coefficient of expansion should be appropriate for the temperature range being considered, i.e., $\alpha = \alpha|_{T_i}^{T_f}$.

Thermal Strain $$\varepsilon_T = \alpha \Delta T \qquad (1)$$

where $\Delta T = T_f - T_i$

Given the thermal strain as a continuous function of temperature then the instantaneous and mean expansion coefficient can be determined as illustrated in Figure 1 and defined in Eq (2) and Eq (3).

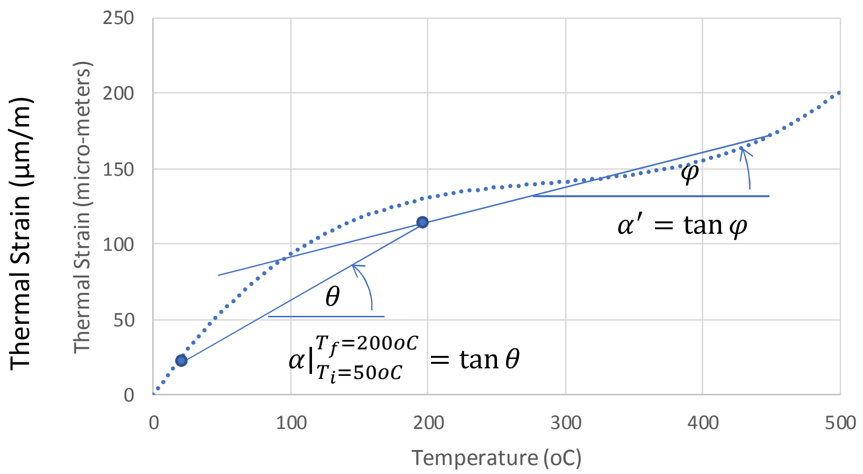


Figure 1: Coefficients of e: Temperature (°C)

Instantaneous Coefficient of Expansion $$\alpha' = \frac{\partial \varepsilon_T}{\partial T} \qquad (2)$$

The average coefficient of expansion is, from Figure 1, the gradient of the secant line cutting the thermal strain curve at the appropriate temperatures or the integral of the instantaneous value over the temperature range of interest divided by the temperature range as shown in Eq (3).

Average Coefficient of Expansion
$$\alpha = \frac{\int_{T_i}^{T_f} \alpha' dT}{T_f - T_i} = \frac{\varepsilon_T|_{T_f} - \varepsilon_T|_{T_i}}{T_f - T_i} \tag{3}$$

In general, thermal expansion data is defined relative to some datum or reference temperature where the thermal strain is assumed to be zero. This temperature might be defined as T_0. The practising engineer is may be interested in establishing the thermal strain due to a temperature rise from T_i to T_f. If $T_i \neq T_o$ the approach shown in Eq (4) is adopted.

$$\varepsilon_T|_{T_i}^{T_f} = \varepsilon_T|_{T_0}^{T_f} - \varepsilon_T|_{T_0}^{T_i} = \alpha|_{T_0}^{T_f}(T_f - T_0) - \alpha|_{T_0}^{T_i}(T_i - T_0) \tag{4}$$

Thermal Expansion Data in FE Software

As an example, the temperature dependent thermal expansion properties for a range of aluminium alloys, as published in ASME Boiler & Pressure Vessel Code, II, Part D, 2010 (p714), will be considered. These material properties are reproduced in Figure 2.

TABLE TE-2
THERMAL EXPANSION FOR ALUMINUM ALLOYS

Temperature, °C	Coefficients for Aluminum Alloys A	B	C
20	21.7	21.7	0
50	23.3	22.6	0.7
75	23.9	23.1	1.3
100	24.3	23.4	1.9
125	24.7	23.7	2.5
150	25.2	23.9	3.1
175	25.7	24.2	3.7
200	26.4	24.4	4.4
225	27.0	24.7	5.1
250	27.5	25.0	5.7
275	27.7	25.2	6.4
300	27.6	25.5	7.1
325	27.1	25.6	7.8

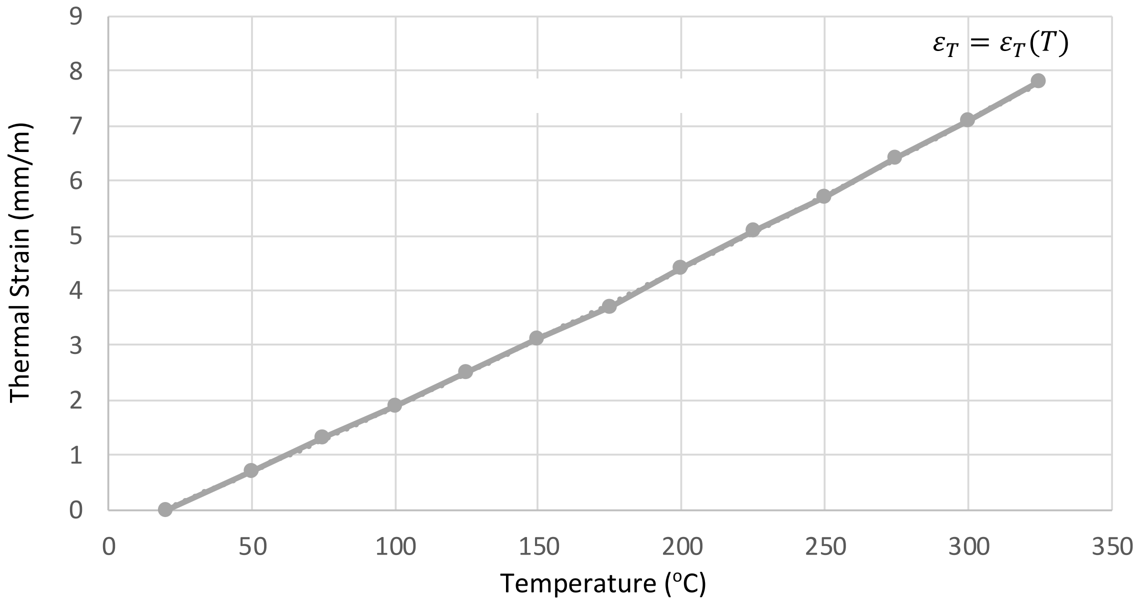
GENERAL NOTES:
(a) Aluminum alloys represented by these thermal expansion coefficients include:

A03560	A93003	A95254
A24430	A93004	A95454
A91060	A95052	A95456
A91100	A95083	A95652
A92014	A95086	A96061
A92024	A95154	A96063

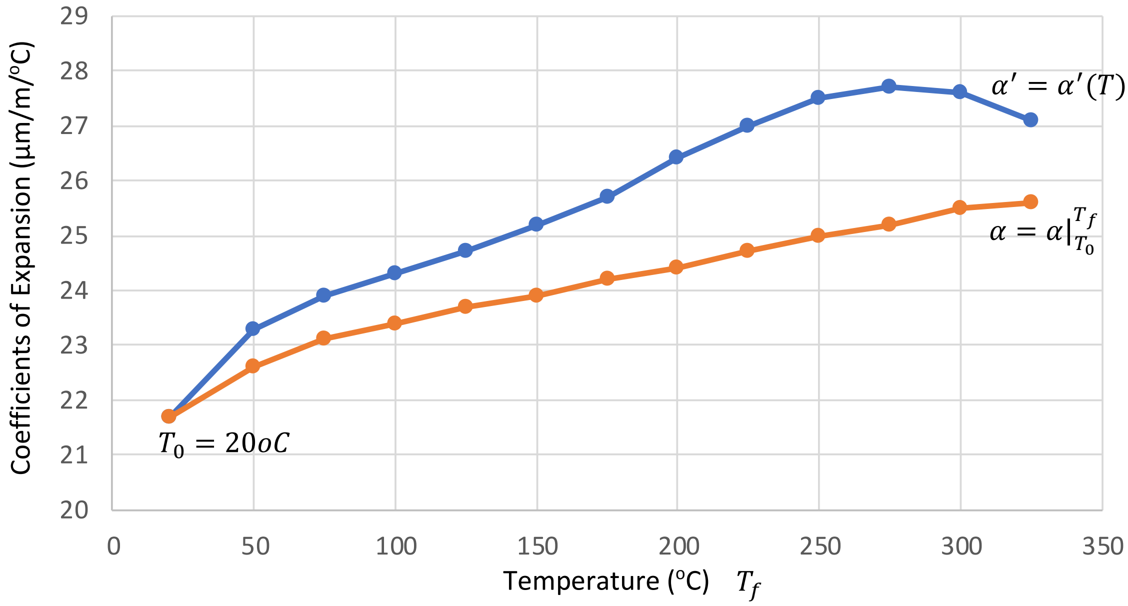
(b) Coefficient A is the instantaneous coefficient of thermal expansion × 10^{-6} (mm/mm/°C). Coefficient B is the mean coefficient of thermal expansion × 10^{-6} (mm/mm/°C) in going from 20°C to indicated temperature. Coefficient C is the linear thermal expansion (mm/m) in going from 20°C to indicated temperature.

Figure 2: Temperature dependent thermal expansions properties for aluminium alloys

As stated in note (b), the instantaneous and mean coefficients of expansion are presented in the columns denoted A and B, respectively, and the thermal strain in the column denoted C. It is unclear from the notes how these values were derived. However, it is assumed that the thermal strains are measured data and the coefficients of expansion have been derived from this data using Eq (2) and Eq (3). It is possible, likely, that the thermal strains have been processed, for example by curve fitting, to produce a (C^1) continuous representation of thermal strain as a function of temperature – this being useful in ensuring that the evaluation of Eq (2) leads to unique values at temperatures where the thermal strain is measured. The data from ASME is plotted in Figure 3.



(a) Thermal strain as a function of temperature



(b) Coefficients of expansion as functions of temperature

Figure 3: Temperature dependent thermal expansion data from ASME

Let us consider the FE system ANSYS which allows the input of thermal expansion data as either thermal strain or instantaneous or mean coefficients of expansion. Whichever form the data is provided, ANSYS converts it into mean coefficients of expansion and uses Eq (1) to then calculate the thermal strains.

Assuming that the ASME data is consistent, i.e., that the quantities are related by Eq (2) and Eq (3), then it is a simple matter to check the results from ANSYS. As noted previously, ANSYS maps instantaneous thermal coefficients into mean coefficients, i.e., column A into column B presumably using Eq (3). If the data is provided in the form of thermal strains then it will map this into mean coefficients, i.e., column C into column B presumably using Eq (2) followed by Eq (3).

In Figure 4, the mean coefficients of expansion are plotted as defined in ASME and as derived by ANSYS from the thermal strain and from the instantaneous coefficient of expansion.

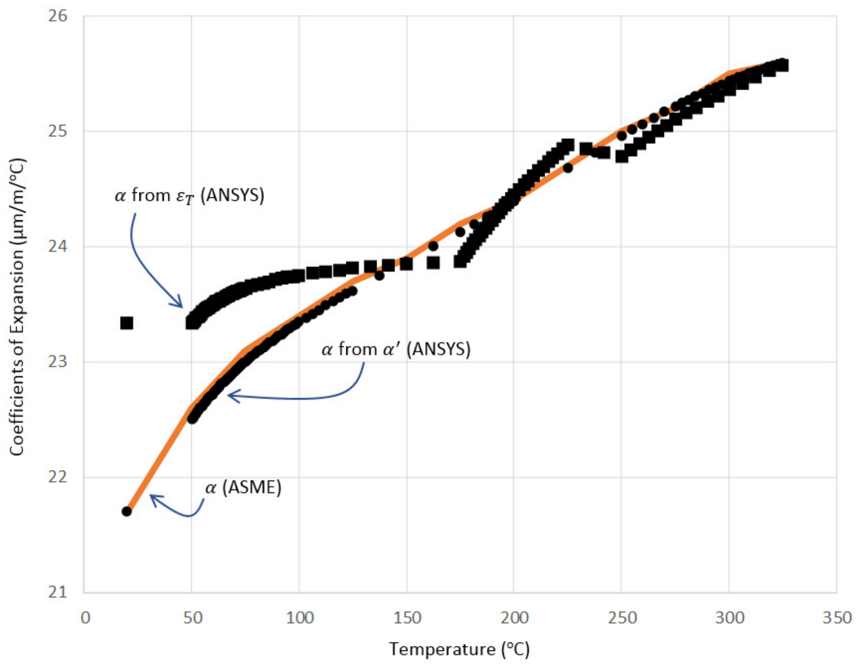


Figure 4: Mean coefficient of expansion from ANSYS

In Figure 4, the mean coefficient of expansion from ASME is plotted as a solid orange line. The solid black circles are results from ANSYS produced by inputting the instantaneous coefficients of expansion from ASME. The results appear reasonable although the distribution of data points is rather uneven.

The solid black squares in Figure 4 represent the mean coefficients of expansion calculated by ANSYS from the ASME thermal strains. The same uneven distribution of data points as noted above is observed for these results. However, of more concern is the lack of agreement between the ASME data and that calculated by ANSYS.

One might be forgiven, when seeing the results of Figure 4 to question the veracity of the mean coefficients of expansion from ANSYS. It turns out, however, that the mean coefficients of expansion produced by ANSYS from the original ASME thermal strain data is correct, at least at the original temperatures specified in the ASME data, in that these mean coefficients of expansion lead back to the original ASME thermal strains. At 50°C the thermal strain from ASME is 0.7mm/m. ANSYS converts this into a mean coefficient of expansion as per Eq (5).

$$\alpha|_{20oC}^{50oC} = \frac{0.7 - 0}{50 - 20} = 23.333\mu m/m/oC \tag{5}$$

This is different from the ASME value which is 22.6 µm/m/°C. However, if the ANSYS coefficient is used to calculate the thermal strain then, clearly, the correct value is recovered.

If, on the other hand, the ASME coefficient is used to calculate the thermal strain the result shown in Eq (6) is obtained.

$$\varepsilon_T|_{20oC}^{50oC} = 22.6 \cdot 30 = 0.678 \; \mu\text{m/m} \tag{6}$$

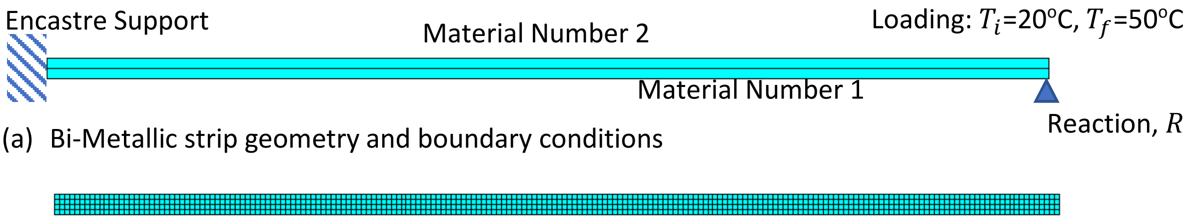
The percentage error in thermal strain calculated from the ASME mean coefficient of expansion is given in Eq (7).

$$e|_{20oC}^{50oC} = \frac{0.7 - 0.678}{0.7} = 3.14\% \tag{7}$$

Clearly, the error as calculated above depends on the ASME thermal strain being exactly 0.7. In practise, as the thermal strain data is only given to a single significant figure, then one must assume that the actual value could lie in the range 0.65 to 0.75. If these values are used in Eq (7) then the error ranges from -4.3% to +9.6%, i.e., a range of about 14%.

Demonstration of Potential Uncertainty

This error or uncertainty is not insignificant. Indeed, in most linear FEM computations, relative uncertainties in the input data is directly transferred as relative uncertainty in the output results. However, one can find cases where the important input parameter is the difference of uncertain data resulting in a significantly increased uncertainty in the results. In particular, if the model of interest has components of different materials interacting with each other and both materials have a degree of uncertainty in the thermal expansion coefficients. An example of this is the bi-metallic strip problem shown in Figure 5. The strip is 1m long and comprises strips of two materials with 0.01m thickness which are fully bonded together. The left-hand end of the strip is fully restrained and the right-hand end is propped, as shown. The strips are meshed with square elements (higher-order or eight-noded) of dimension 0.005m.



(a) Bi-Metallic strip geometry and boundary conditions

(b) Mesh

Figure 5: Bi-Metallic Strip Problem

The elastic properties for the two materials are assumed to be identical with an elastic modulus of 210GPa and Poisson's ratio of 0.3 and a plane stress constitutive relationship is adopted. The mean coefficients of expansion are derived from the thermal strain listed in [1] and tabulated in Table 1.

Material Number	Designation	Tabulated	Min	Max
1	9Cr-1Mo Steel	0.3	0.25	0.35
2	Aluminium Alloy	0.7	0.65	0.75

Table 1: Thermal strains (µm/m) at 50°C with a reference temperature of 20°C

With the minimum and maximum values of the mean coefficient of expansion for the two materials there are four possible combinations and these are listed in Table 2 together with the reaction force from the FE analyses.

Analysis Number	1	2	3	4
	+, +	+, -	-, -	-, +
α_1 (µm/m/°C)	11.6’	11.6’	8.3’	8.3’
α_2 (µm/m/°C)	25.0	21.6’	21.6’	25.0
R (kN)	6.31	4.73	6.31	7.89

Table 2: Mean coefficients of expansion and reaction force from FE model

The mean reaction force for the four scenarios of expansion coefficients is 6.31kN which means that the reaction force has an uncertainty of $\pm 25\%$! Note that although different reaction forces would be obtained if a plane strain constitutive relationship had been adopted, the uncertainty would be identical to the plane stress case used in this example.

It should be noted, of course, that for a linear model the response is proportional to the loading. In this case the loading is the difference between the thermally induced strains and one could have predicted *a priori* that the uncertainty would be $\pm(2 \cdot 0.05)/(0.7 - 0.3)$= $\pm$25%.

Closure

There are two distinct issues here:

Issue Number 1: The data provided in [1] is not consistent in that the thermal strain derived from the tabulated mean coefficient of expansion is not equal to the tabulated thermal strain – see Eq (6). The reason for this might be that the choice of interpolation function used for the thermal strain is such that it does not go through the tabulated thermal strains.

Issue Number 2: The data offered by ASME is of rather low precision (one significant figure). This lack of precision, particularly for lower temperatures, can lead to a significant degree of uncertainty in quantities of engineering interest calculated from the ASME thermal expansion data – see above.

Recommendations to ASME

In the introduction to the section of [1] containing the thermal expansion data, the reader is informed that this data should be considered to have an uncertainty of $\pm$10% - see Figure 6.

SUBPART 2
PHYSICAL PROPERTIES TABLES

INTRODUCTION

Subpart 2 of Section II, Part D provides, to the extent possible, physical properties for most of the alloys used in Code construction. Included in this Subpart are tables of thermal expansion (instantaneous, mean, and linear), thermal conductivity and thermal diffusivity, and modulus of elasticity. These values are all listed as a function of temperature from 20°C to as high as 900°C. Subpart 2 also contains tables of density and Poisson’s ratio for ferrous and nonferrous alloys.

All of the properties provided in Subpart 2 are considered typical. They are neither average nor minimum. Thermal–physical properties such as thermal expansion, thermal conductivity, and thermal diffusivity are affected more by alloy content than by crystal structure or heat treatment. Due to the permitted range for elements comprising alloys (specification ranges of chemical compositions), the thermal–physical properties described in Tables TE-1 through TE-5 and Table TCD should be considered to have an associated uncertainty of ±10%.

Moduli of elasticity and Poisson’s ratio are also typical values, but the values of modulus of elasticity, shown as a function of temperature in Tables TM-1 through TM-5, tend to be closer to average values since their temperature dependency is factored against an “average” room-temperature value.

For those alloys for which physical properties are not yet addressed in Subpart 2, the user of the Code may use other authoritative sources for the needed information. In those instances, the user is encouraged to bring the information to the attention of the ASME Boiler and Pressure Vessel Committee so it might be added to Subpart 2. Information should be directed to:

Secretary
ASME Boiler and Pressure Vessel Committee
Three Park Avenue
New York, NY 10016-5990
Telephone: (212) 591-8533
Fax: (212) 591-8501

Figure 6: Introduction to the ASME Physical Properties Tables

The reason for this uncertainty appears to be that the tabulated data encompasses a range of alloys within a single table. Nonetheless, there are cases where the lack of precision in the tabulated data exceeds this assumed uncertainty. For example, the thermal strain at 50^{o}C for 9Cr-1Mo Steel is 0.3 µm/m. With the acknowledged uncertainty, this value could lie in the range 0.27 to 0.33 but the lack of precision would lead the reader to understand that the uncertainty should lie in the range 0.25 to 0.35. This inconsistency clearly leaves the reader uncertain as to the true uncertainty in the data.

This issue could be resolved by tabulating the data to an additional significant figures, i.e. to 2 significant figures in the case of thermal strain data.

The apparent inconsistency in the thermal expansion data, e.g., between the mean coefficients of expansion and the thermal strain needs to be explained in order that the reader can make a judgement to which is more accurate.

This issue could be resolved by declaring in detail how the data is derived, presumably with the coefficients of expansion being derived from the thermal strain.

References

[1] ASME, ***Boiler & Pressure Vessel Code 2010***, II, Part D, Properties (Metric), Materials.

[2] Angus Ramsay, ***ANSYS/ASME: Potential Issue with Thermal Expansion Calculation***, RMA Technical Note (July 2017):

http://www.ramsay-maunder.co.uk/downloads/ANSYS%20Potential%20Issue%20with%20Thermal%20Expansion%20Calculations.pdf

[3] Jeremy Theler, ***On the Evaluation of Thermal Expansion Coefficients***, SeamPlex Technical Note (July 2017):

https://www.seamplex.com/docs/SP-WA-17-TN-F38B-A.pdf