Equilibrium Finite Elements in the Education of Engineers

Angus Ramsay & Edward Maunder

Ramsay Maunder Associates

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RMA – Partnership 2004-2009, Limited Company 2009-Present.

Ramsay Maunder ASSOCIATES ***Finite Element Specialists and Engineering Consultants***

EFE Equilibrium Finite Elements

for safe structural analysis and design optimisation

Edward Maunder

- My background
  - IC talk by Prof B M Fraeijs de Veubeke on equilibrium
  - Design experience
    - Tower 42 RC design - details of reinforcement from CFE output – a precursor of Sleipner?
  - Teaching/research at Exeter University
    - CFE with warnings about "lack of equilibrium"
    - Sleipner (1992)
    - Equilibrium research with John Robinson, Angus, and IST Lisbon
  - RMA
  - The BOOK

The Book!

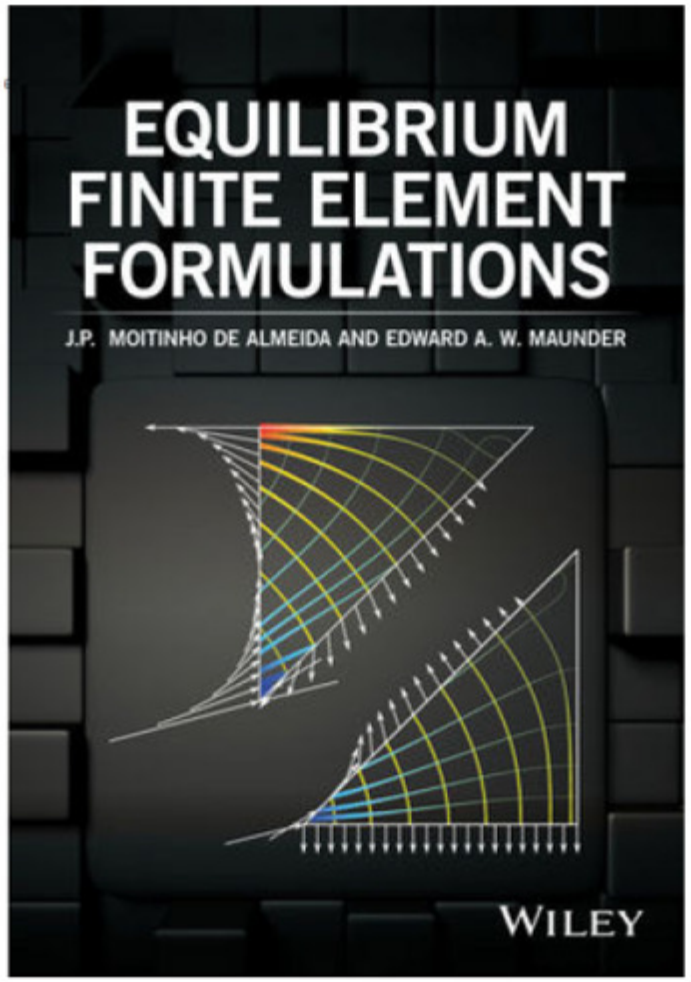
Implementation and references

A theoretical exposition for academics and engineers.

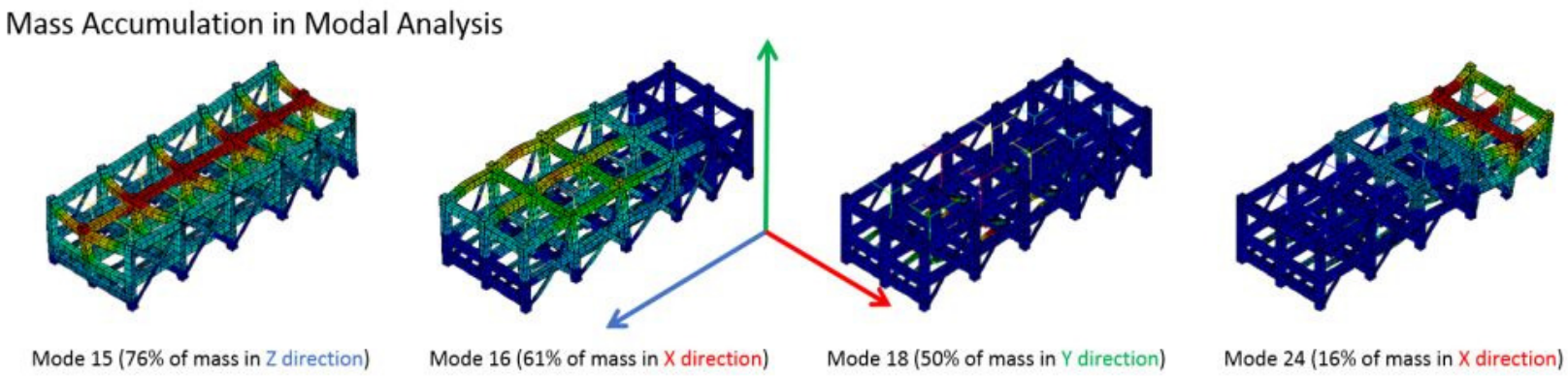
Wiley 2017

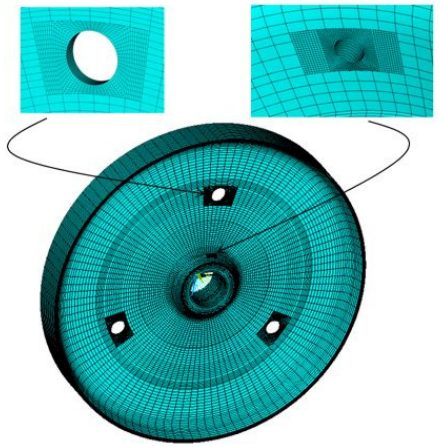
To try “academic” software, contact:

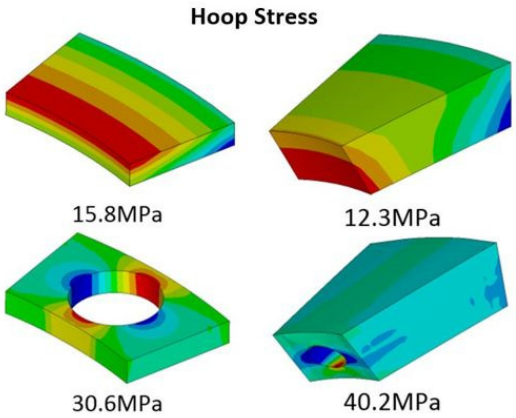
moitinho@civil.ist.utl.pt

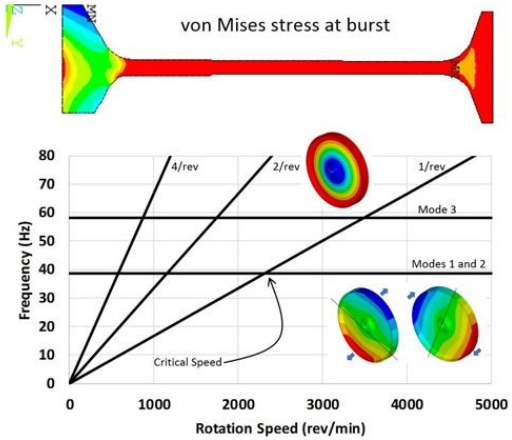


Recent Projects at RMA

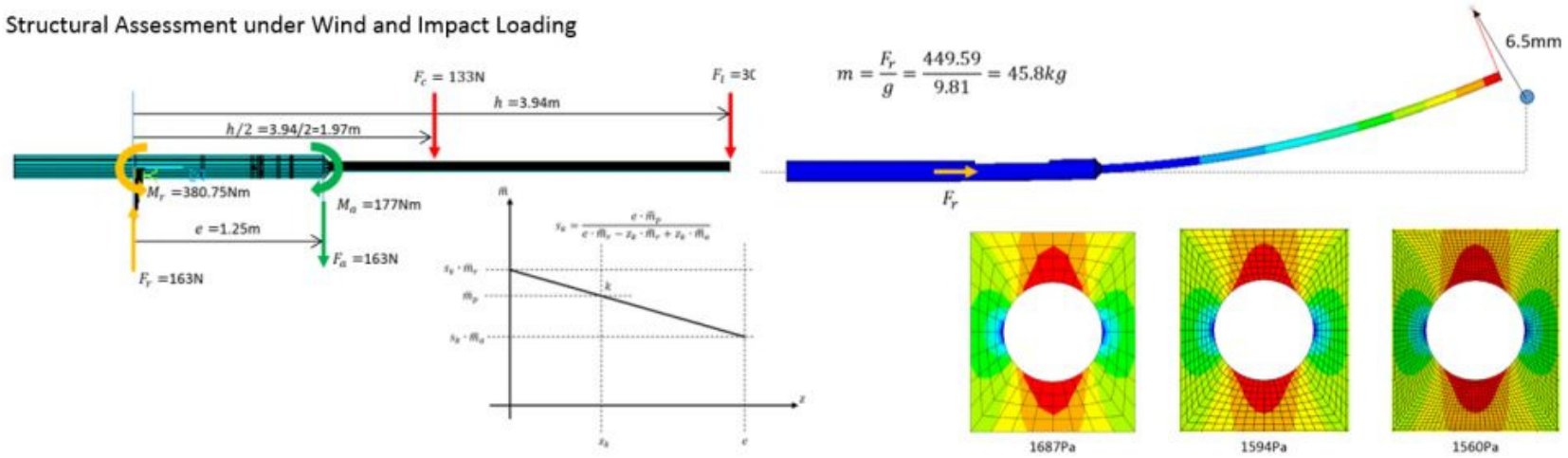




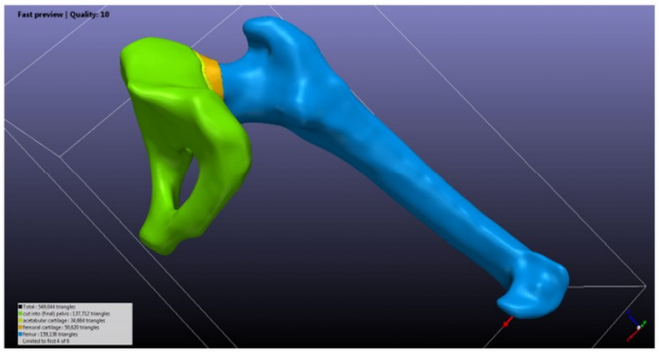




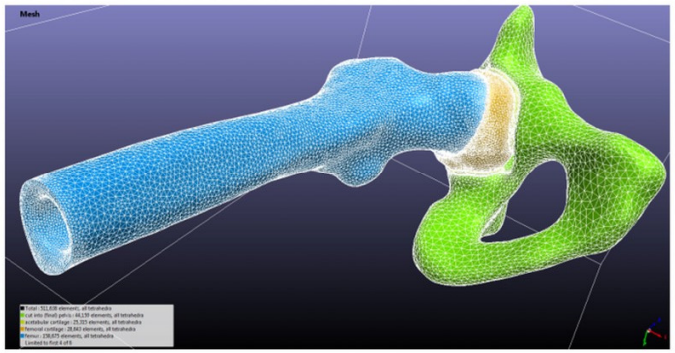
Recent Projects at RMA



Study of Femur/Implant Mechanics

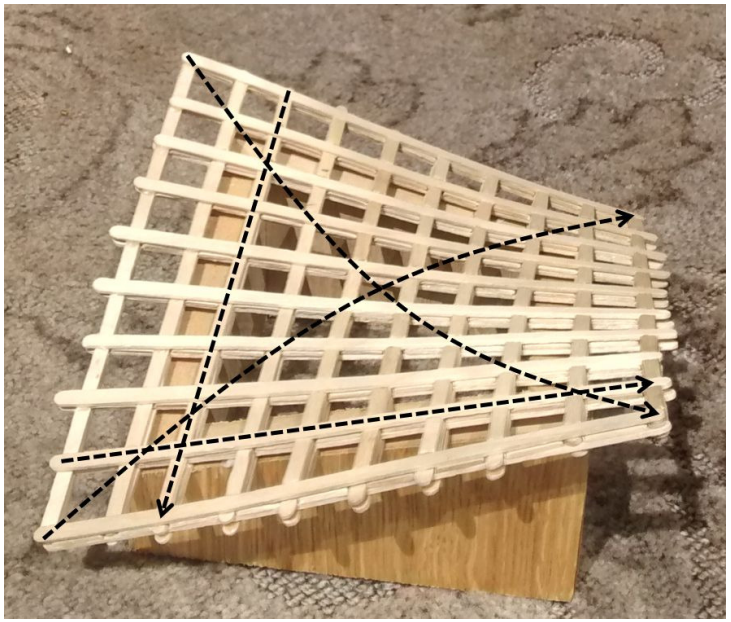


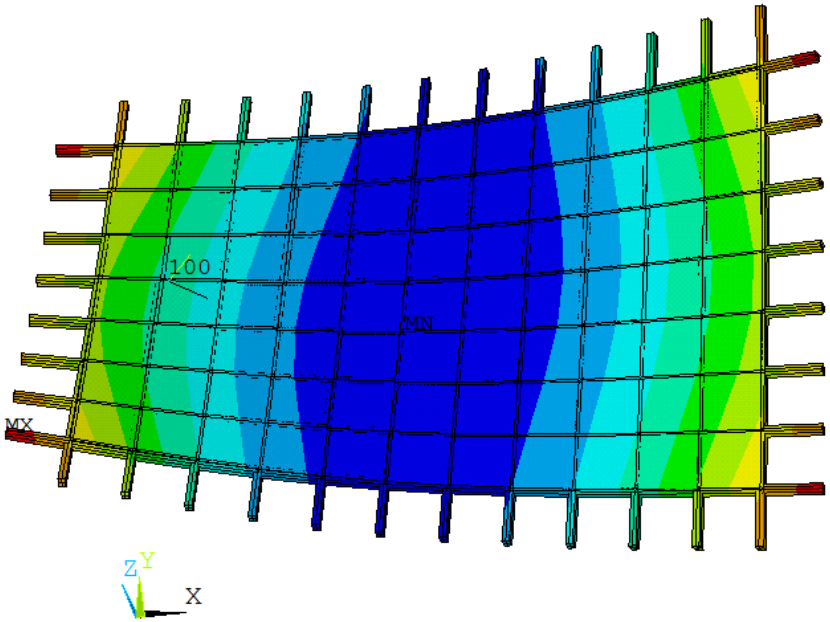
Solid model of Pelvis & Femur



FE model ready for adding Implant

Recent Projects at RMA



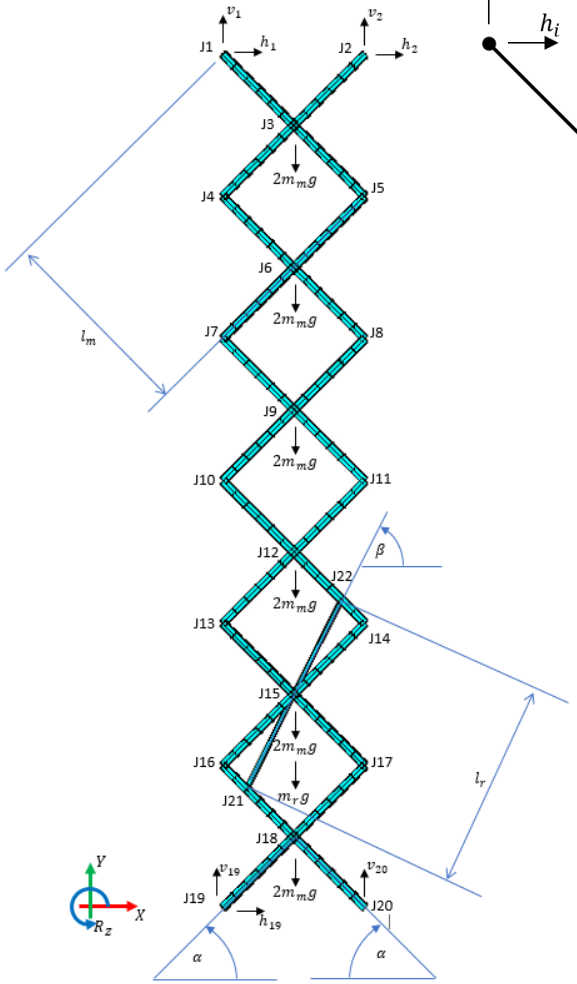


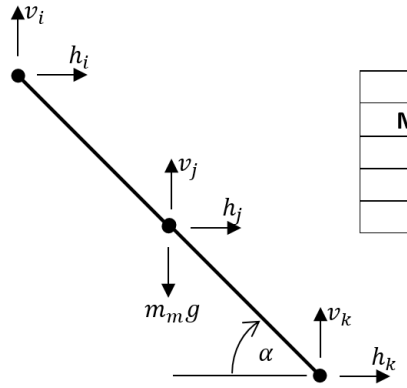
Macro/Microscopic forms of Equilibrium

- Sir Christopher Wren (1660s)
  - The design must be regulated by the art of **staticks**, or invention of the centers of gravity, and the duly poising of all parts to **equiponderate**; without which, a fine design will fail and prove abortive. Hence I conclude, that all designs must, in the first place, be brought to this test, or rejected.

Cauchy 1823 – Differential Equation of Equilibrium of a Continuum

Current Project at RMA





	Joints		
Member	i	j	k
3	4	6	8
5	7	9	11
9	13	15	17

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ -S_\alpha & -C_\alpha & 0 & 0 & S_\alpha & C_\alpha \end{bmatrix} \begin{Bmatrix} h_i \\ v_i \\ h_j \\ v_j \\ h_k \\ v_k \end{Bmatrix} = m_m g \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

Member	h3	v3	h4	v4	h5	v5	h6	v6	h7	v7	h8	v8	h9	v9	h10	v10	h11	v11	h12	v12	h13	v13	h14	v14	h15	v15	h16	v16	h17	v17	h18	v18	h19	v19	v20	h21	v21	h22	v2
1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0.71	0.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	-0.71	0.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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3	0	0	-0.71	-0.71	0	0	0	0	0	0	0.71	0.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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4	0	0	0	0	0.71	-0.71	0	0	-0.71	0.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.71	-0.71	0	0	-0.71	0.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.71	-0.71	0	0	0	0	0	0	0.71	-0.47	-0.47	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	-1	0	1	0	0	0	0	0	0
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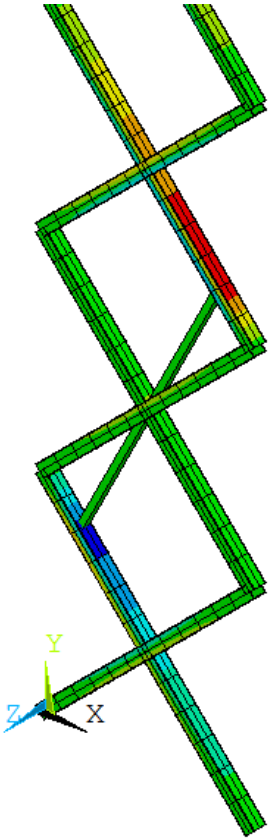
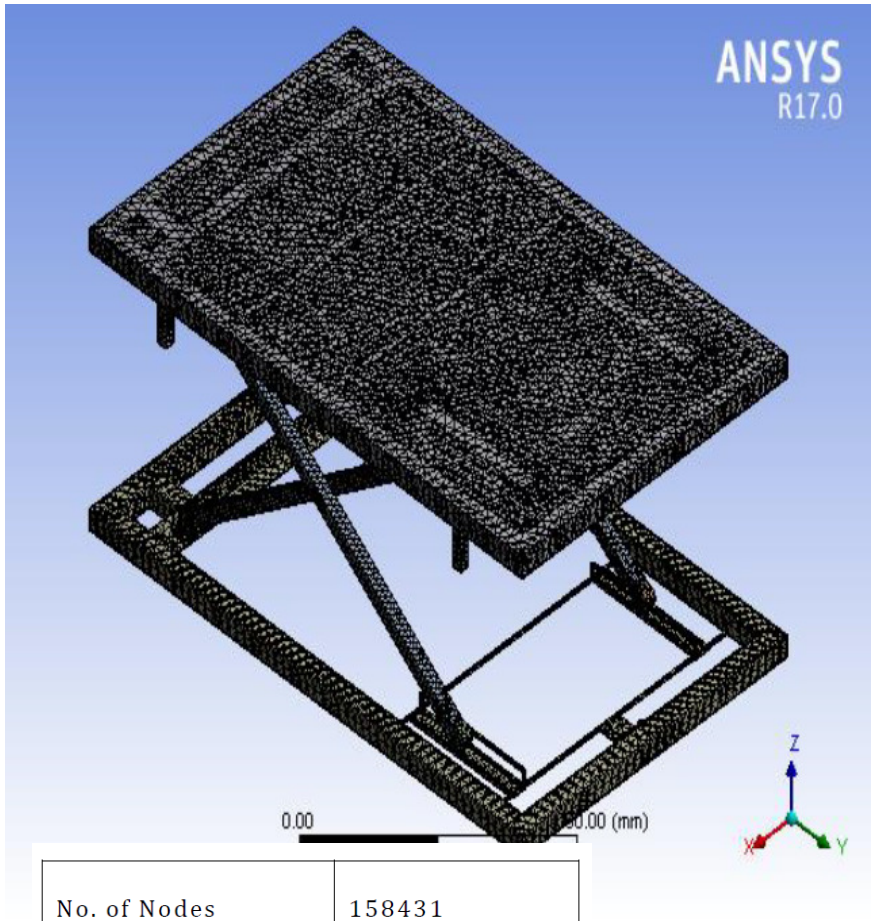
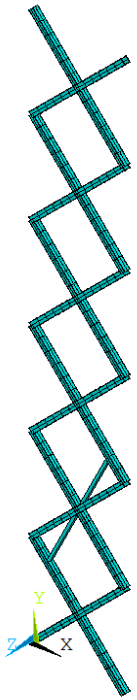
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.71	-0.71	0	0	0.71	-0.71	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	-1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.89	0.45	0.89	-0.4



FE Malpractice?



No. of Nodes	158431
No. of Elements	78141



No. of Nodes	444
No. of Elements	145

Codes of Practice

- SLS Conditions – limits on maximum deflection and possibly stress to keep within the fatigue limit of the material.
- ULS Conditions – limits on load to avoid collapse (elastic and/or plastic). Often undertaken member by member.
- Utilisation = Demand/Capacity (Partial Factors on both Demand and Capacity). Should be less than unity.
- Some structures governed by SLS others by ULS BUT generally the engineer should consider both conditions.
- Structural demand comes from analysis in terms of stress resultants.
- CFE can provide good and continuous displacements but generally rather poor stresses unless sufficient mesh refinement is conducted. But how much is sufficient?

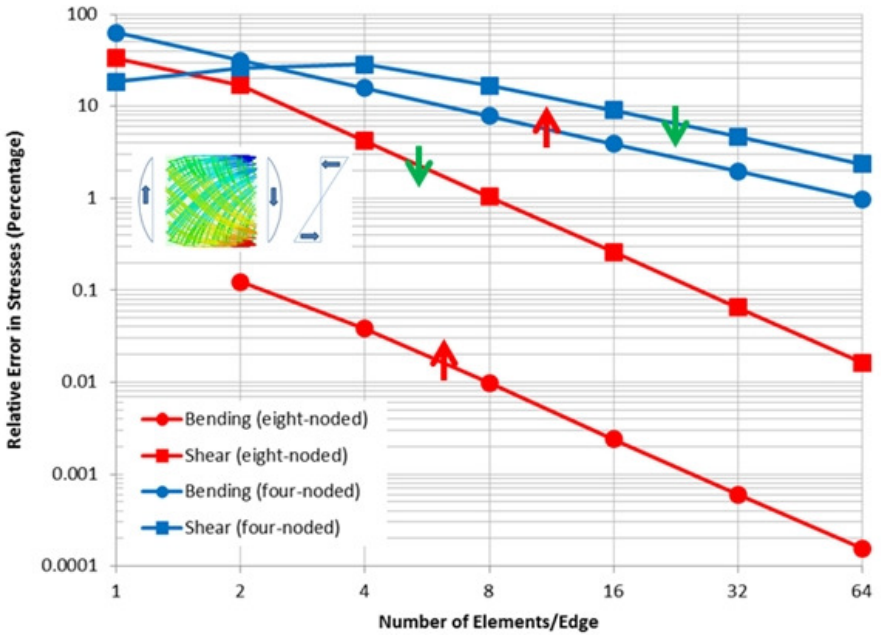
CFE Elements – the Status Quo

Low Fidelity:

Order	Displacement	Stress
Lower	Linear	Constant
Higher	Quadratic	Linear

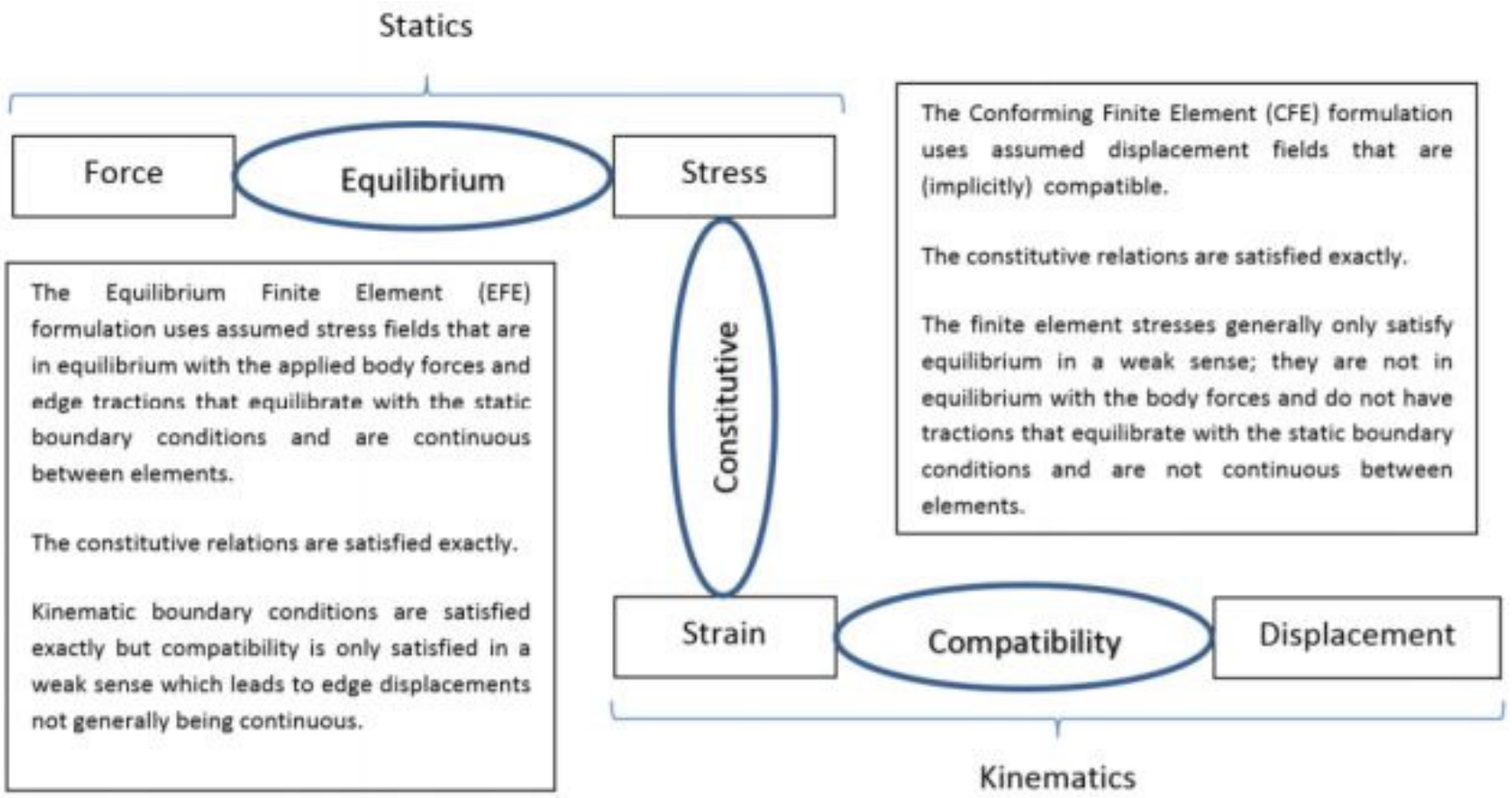
Can you think of many practical problems where linear stresses represent the exact solution?!

Hence the Basic Mesh (that mesh required to capture the geometry alone) is not normally sufficient for accurate strength predictions.



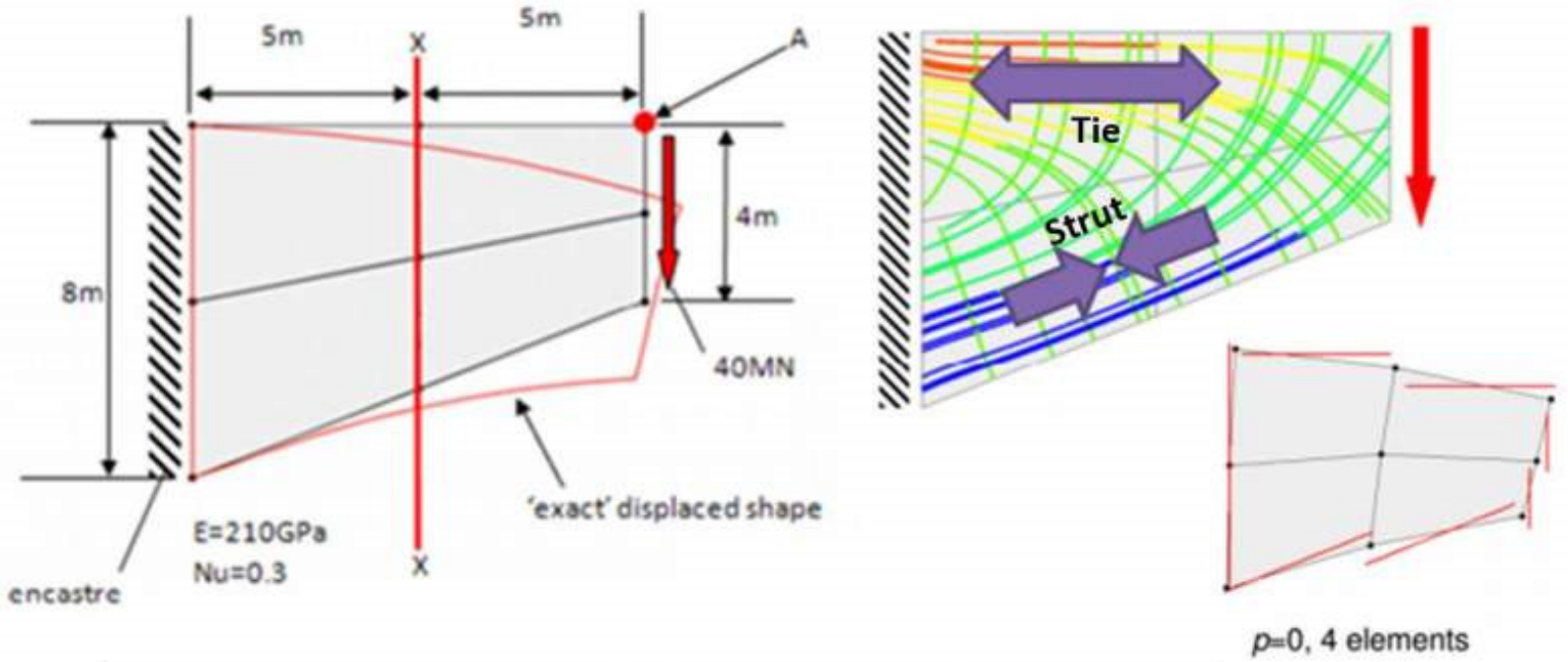
The results from CFE models can leave the practising engineer in a predicament!

Nature of FE Approximation for Pure FEs (CFE and EFE)



http://www.ramsay-maunder.co.uk/knowledge-base/publications/what-is-equilibrium-finite-element-analysis/

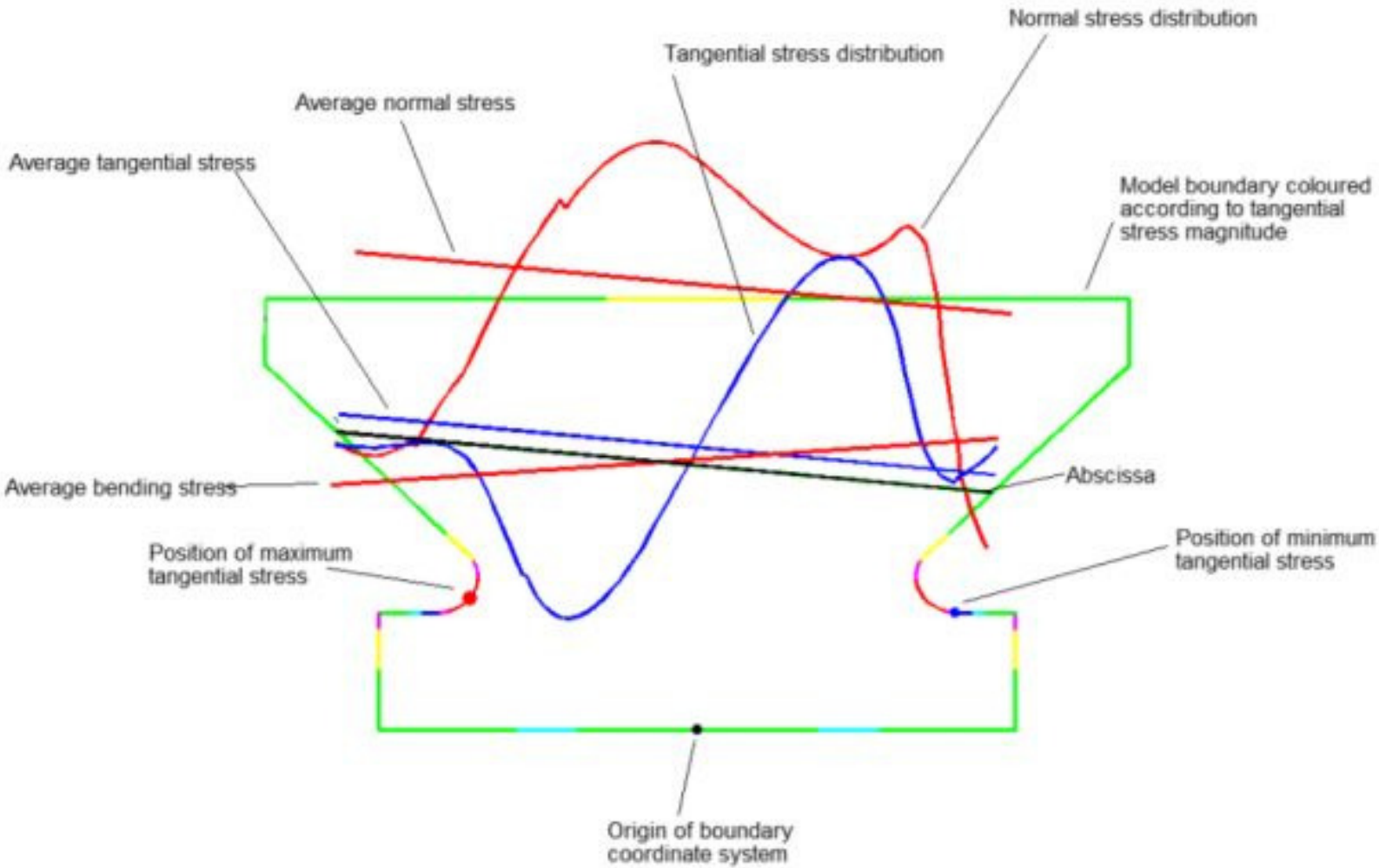
Nature of FE Approximation for Pure FEs (CFE and EFE)



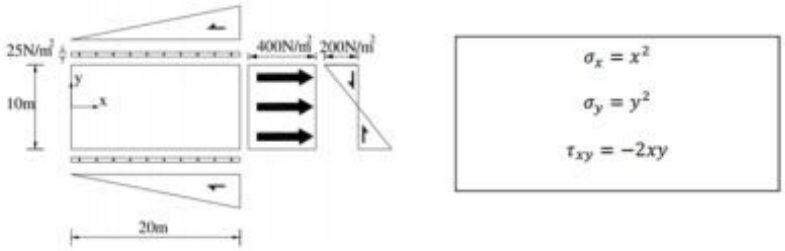
	Displacement (mm)	Normal (MN)		Tangential (MN)		Moment (MNm)	
		Left	Right	Left	Right	Left	Right
EFE (p=1)	-3.58	0	0	40	40	200	200
CFE (four-noded)	-2.56	27.1	19.7	14.5	80.0	200.2	62.8
CFE (eight-noded)	-3.48	0.0	0.0	39.5	38.9	205.2	220.7
'Exact'	-3.54	0	0	40	40	200	200

http://www.ramsay-maunder.co.uk/knowledge-base/publications/what-is-equilibrium-finite-element-analysis/

Equilibrating Tractions on a Model Section



EFEs – Particular and Hyperstatic Stress Fields

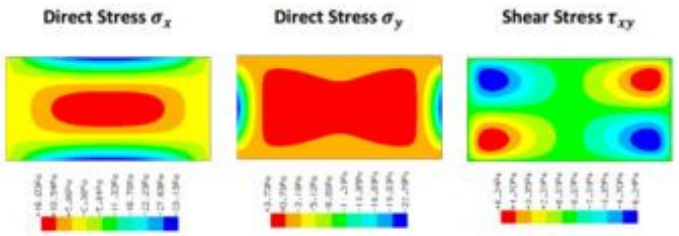


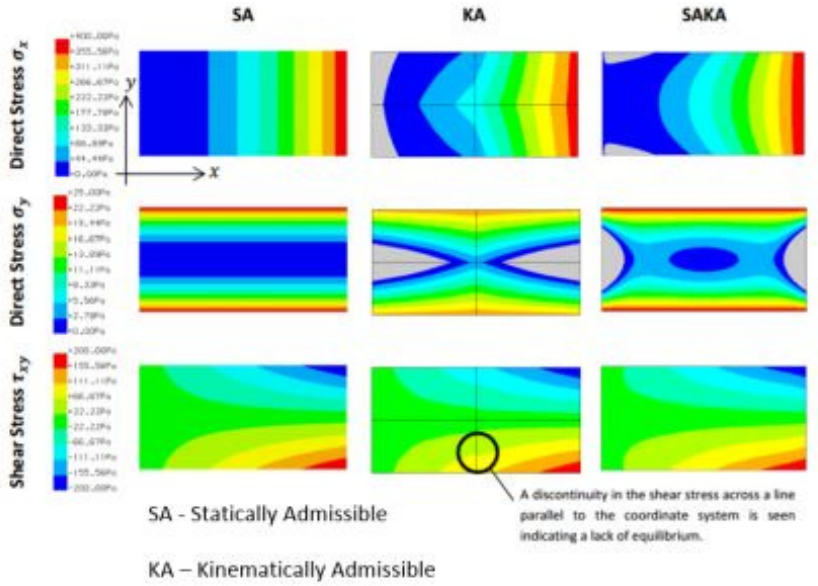
$$\sigma_x = x^2$$

$$\sigma_y = y^2$$

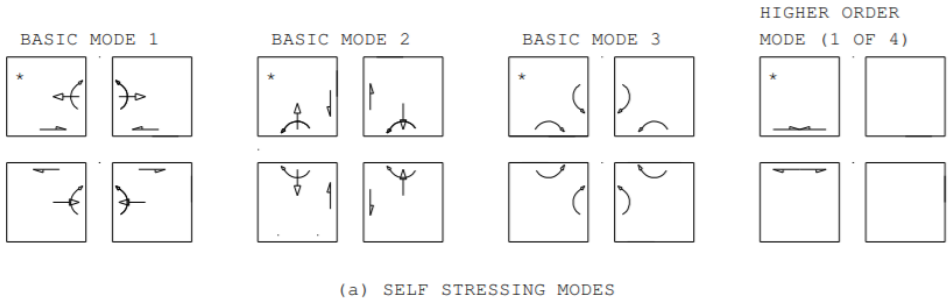
$$\tau_{xy} = -2xy$$

Hyperstatic (SAKA-SA)



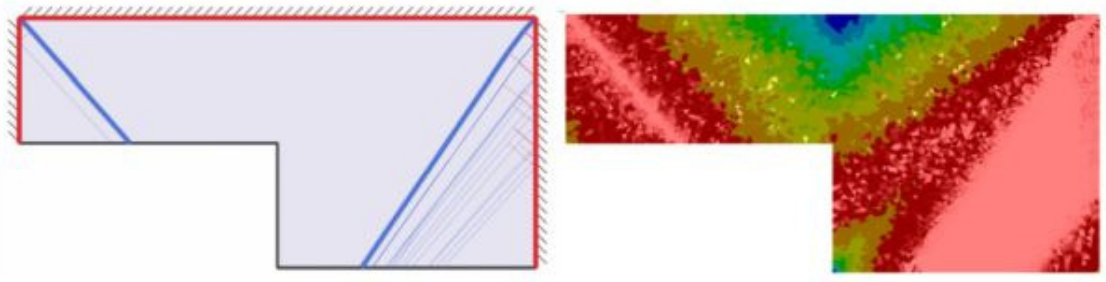


http://www.ramsay-maunder.co.uk/knowledge-base/glossary/hyperstatic-stress-fields/



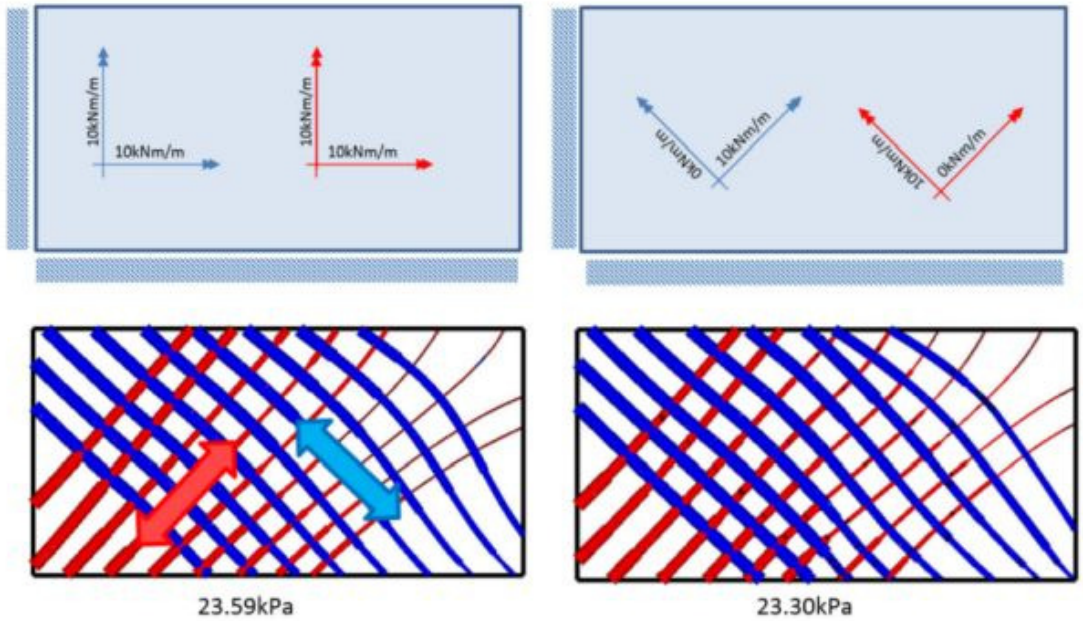
http://www.ramsay-maunder.co.uk/knowledge-base/thesesdissertations/doctoral-thesis/

EFEs – Limit Analysis/Design



(a) CFE ($\lambda = 4.21$) (b) EFE ($\lambda = 4.20$)

http://www.ramsay-maunder.co.uk/knowledge-base/publications/yield-line-analysis-of-reinforced-concrete-slabs-is-the-10-rule-safe/



http://www.ramsay-maunder.co.uk/knowledge-base/publications/equilibrium-finite-elements-for-rc-design/

Closure

We (RMA et al) believe that EFEs have a useful role to play in practical engineering analysis:

Essential for understanding how a structure works (load path, trajectories)

Essential for safe design (avoidance of, for example, Sleipner)

Essential for error estimation in CFE models (V&V with the current status quo)

Acknowledgement

The authors are grateful to NAFEMS for providing another opportunity to present our ideas on EFEs in the context of educating engineers. A ‘Why do?’ NAFEMS book is in preparation and is likely to be published later in the year or early next year.