EQUILIBRIUM FINITE ELEMENTS FOR THE SAFE LIMIT ANALYSIS OF PLATES

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SUMMARY

Equilibrium finite elements (EFE) overcome an inherent weakness of conventional conforming finite elements by providing strong forms of equilibrium irrespective of mesh refinement and thereby closing a significant opportunity for potentially costly FE malpractice. In predicting the plastic collapse of a structure conventional FE methods are slow (incremental analysis), awkward (many parameters to set) and produce collapse loads that with unconverged models may be considerably greater than the true value. In contrast, and by virtue of the lower-bound plasticity theorem, EFE provides predictions that are guaranteed to be less than the true value, i.e. safe approximations. This abstract presents some results from the EFE software tool and compares them with those produced by conventional conforming elements.

1: Introduction

Equilibrium is, of course, essential in structural design/assessment yet conventional conforming finite elements weaken this condition leaving the opportunity for finite element malpractice wide open. A classic example of this is the Sleipner Platform failure where stresses taken from an extremely crude conforming FE model underpredicted the true shear force by some 45% and contributed to the collapse of the structure [1].

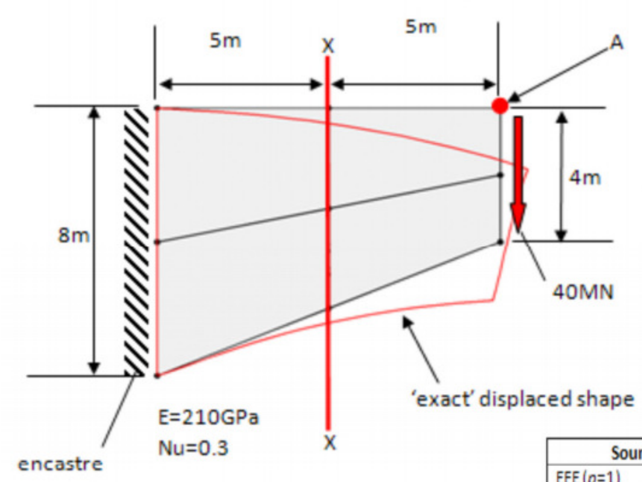
Beyond conventional conforming elements there is a range of other formulations of which equilibrium elements is one. These elements produce stresses that are always in equilibrium with the applied load irrespective of mesh refinement and thereby eliminate the aforementioned possibility of FE malpractice.

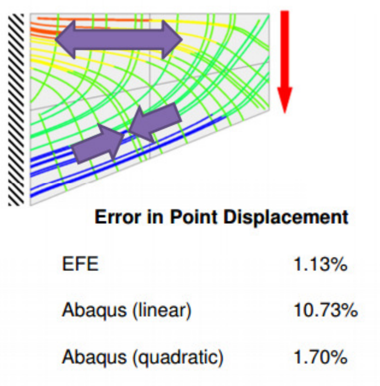
If one moves beyond elasticity into plasticity then equilibrium forms an essential ingredient in achieving safe lower-bound approximations to the collapse load - given a stress field that is in equilibrium with the applied load and that does not anywhere violate the yield criterion then the corresponding collapse load will be less than (or equal to) the true theoretical value.

Whilst limit analysis can be simulated using conventional FE software, [2], in an incremental fashion with a non-linear material definition, dedicated limit analysis software does this significantly more efficiently and, if formulated using equilibrium elements, leads to a conservative prediction of the collapse load.

Whilst EFEs have a long history, they have been more challenging from a theoretical and implementation perspective. It is for this reason that the majority of, if not all, commercial FE systems use conforming elements. Over the last decade or so there has been a revival of research into equilibrium elements leading to the practical realisation of robust equilibrium element formulations that can now be used to practical advantage in a commercial environment.

Ramsay and Maunder have contributed to this research into equilibrium elements and in 2004 set up a partnership (incorporating in 2009) to develop engineering software tools based on equilibrium finite elements (EFE) to assist the practising engineer.





Source	V (mm)	N (MN)	T (MN)	M (MNm)
EFE (ρ=1)	-3.58	0	40	200
Abaqus (linear)	-3.92	1.98	44.9	151.8
Abaqus (quadratic)	-3.48	0.007	39.2	215.2
'Exact'	-3.54	0	40	200

Figure 1: Deep tapered cantilever problem with results

2: A Plate Membrane Problem - Elastic Solutions from EFE

The problem is a deep tapered cantilever loaded with a uniform tangential traction at the free-end. A mesh of four quadrilateral elements is used and the results are presented in terms of the vertical displacement, V, at point A and the stress resultants along X-X. For the equilibrium element, linear statically admissible stress fields (p=1) were used whilst for the displacement element both four-noded and eight-noded elements were used.

The stress resultants N (normal), T (tangential) and M (moment) on X-X are calculated by integrating the FE stresses along the section. Conforming elements do not generally, particularly for coarse meshes, satisfy equilibrium and the discrepancies in the stress resultants are not insignificant! For the equilibrium model, and even though the FE stresses are not the exact values, the stress resultants form an equilibrium set.

3: A Plate Bending Problem - Limit Solutions from EFE

In the second NAFEMS Benchmark Challenge, [3], a plate bending problem (simply supported with a UDL) was presented and readers were asked to predict the collapse load using published results and simulated limit analysis.

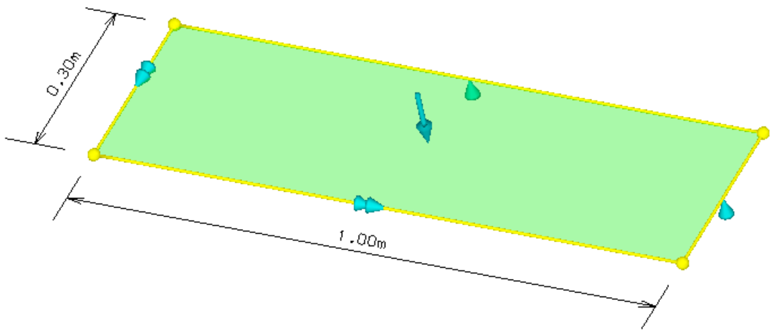


Figure 3: Symmetric quarter of the Challenge Plate

This challenge produced some interesting responses from Benchmark readers and uncovered some interesting issues with commercial FE systems (CS) and published results. Some of these results are seen in the following figure and it is interesting to record that the solution times for some of the responses were in excess of 5 minutes, c.f. 1 second for EFE!

The published results, Roark and the Steel Designers' Manual, are rather conservative. The commercial FE systems (CS) on the otherhand produce unsafe results which converge with mesh refinement to the correct value. The results for EFE are seen to lie in the safe region (predicted collapse loads below the exact value). Even for the coarsest mesh the collapse load for EFE is close to the true value.

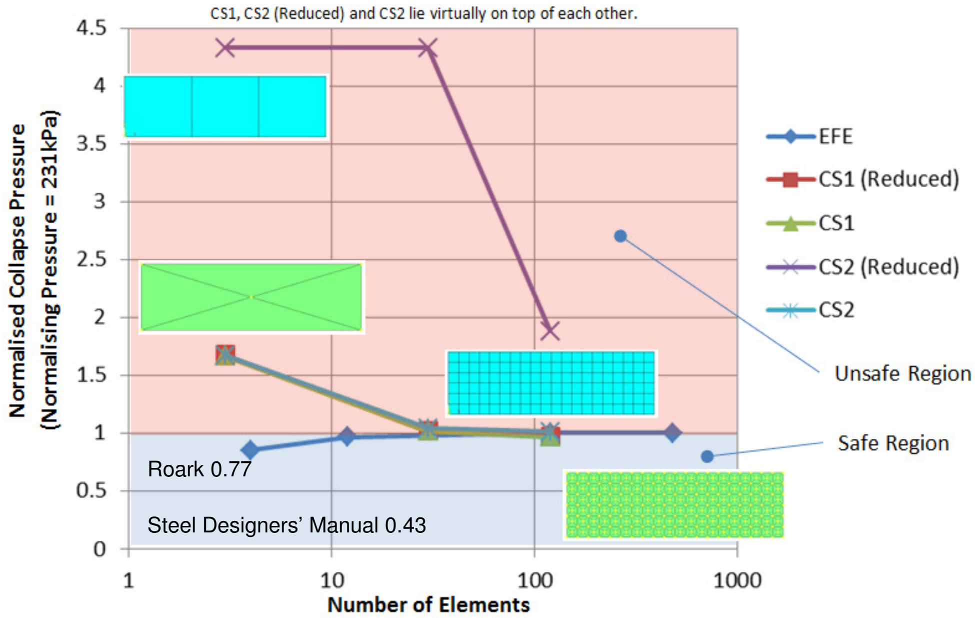


Figure 4: Results for the Challenge Plate

4: Closure

The presentation for which this abstract has been prepared will provide further explanation of equilibrium finite elements and how they might be used to advantage by practising engineers.

REFERENCES

[1] *Sleipner Failure* - http://en.wikipedia.org/wiki/Sleipner_A

[2] Ramsay, A.C.A. & Maunder, E.A.W., 2014. *Simulated Limit Analysis of Plates using Incremental Plasticity Finite Elements* (http://www.ramsay-maunder.co.uk/r--d/resources/)

[3] Ramsay, A.C.A. & Maunder, E.A.W., 2014. *NAFEMS Benchmark Challenge No 2 – Assessment of a Simply Supported Plate with Uniformly Distributed Load* http://www.ramsay-maunder.co.uk/downloads/Benchmark%20Challenge%20002.pdf