*Finite Element Specialists and Engineering Consultants*

On the Uniqueness of Limit Analysis Solutions

Whereas elastic solutions are unique, limit analysis solutions are only unique in the sense of the collapse load. This means that there can be many different solutions (yield line patterns or moment fields) each having the same collapse load.

This document shows some examples of multiple solutions in terms of yield line patterns and moment fields.

Yield Line Solutions

The problem examined has a solution both elastic (Kirchhoff formulation) and plastic that is a constant torsional moment field.
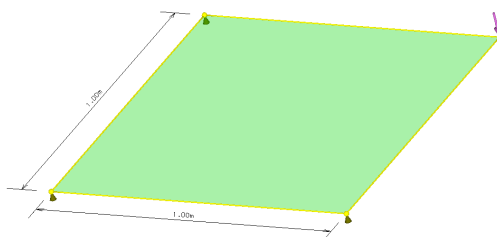

Figure 1: Square slab on three corner supports and with a corner load

If the applied corner load is q then the theoretically exact load factor for the Nielsen yield criterion is:

$$\lambda_c = \frac{2m_c}{q}$$

The principal moment system corresponding to the constant torsional moment field is represented by a set of orthogonal trajectories parallel to the diagonals of the slab and of equal and opposite magnitude. By selecting meshes with edges parallel to the diagonals a range of yield line solutions can be generated as shown in figure 2. Each has the same exact collapse load. If a mesh is chosen with edges that are *not* parallel to the diagonals then an approximate upper-bound to the exact collapse load is recovered.
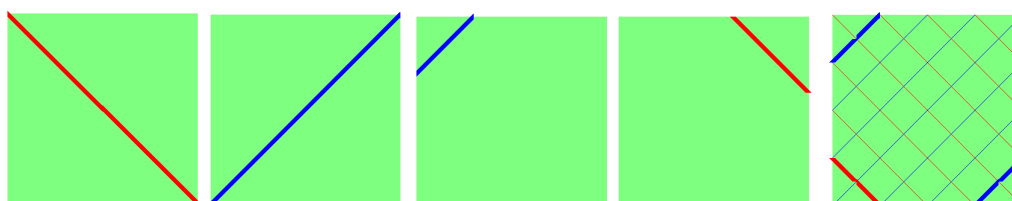
**Figure 2**: Yield line patterns all having the same collapse load

Lower Bound Solution

The problem examined is an isotropically reinforced simply supported square concrete slab with a uniformly distributed load.
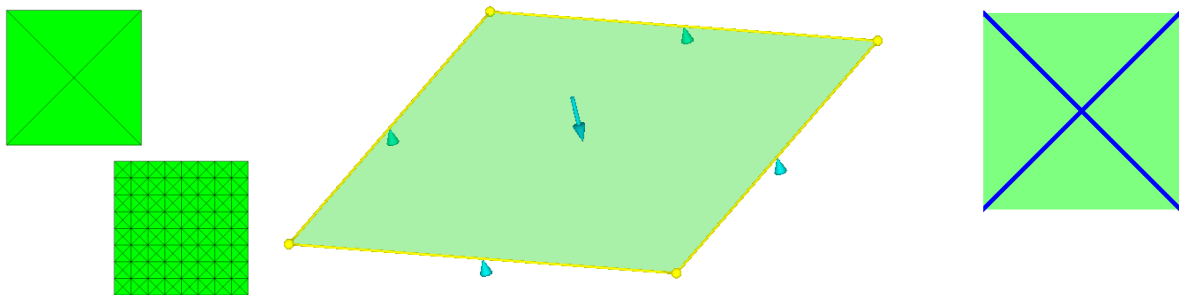
Figure 3: Simply supported square slab with a uniformly distributed load

The exact solution to this problem has yield line pattern comprising sagging yield lines on the two diagonals. The lower-bound solution provides the exact collapse load with a 1x1x4 mesh of p=1 elements.
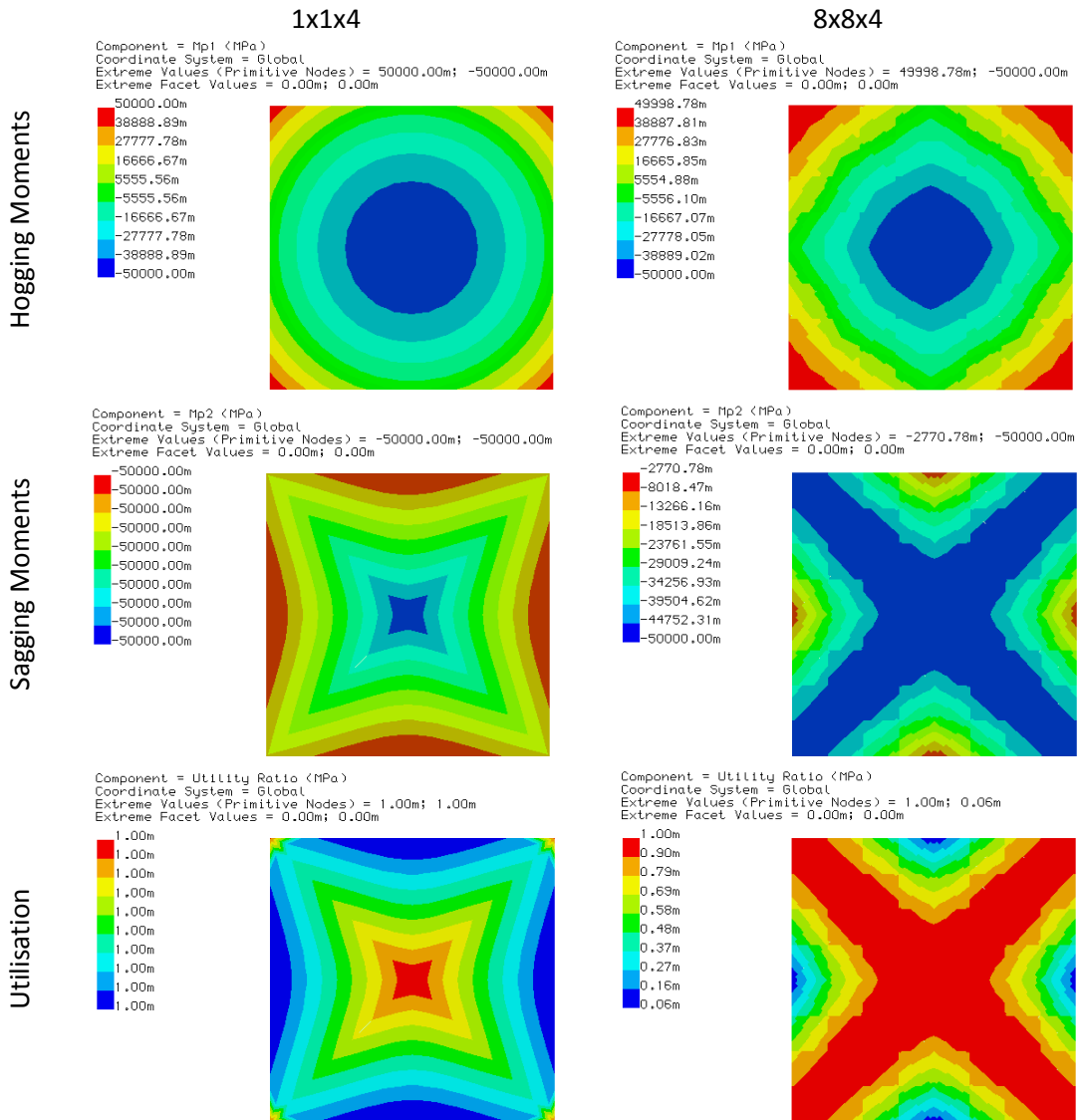
Figure 4: Principal moments and utilisation for two meshes having the same collapse load