LIVERPOOL POLYTECHNIC - FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL, MARINE

AND PRODUCTION ENGINEERING.

A FINITE ELEMENT APPROACH TO THE STATIC, LINEAR-ELASTIC BEHAVIOUR OF AXI-SYMMETRICALLY LOADED CORRUGATED BELLOWS EXPANSION JOINTS.

A dissertation submitted as part of the requirements for the Degree of Master of Engineering in Mechanical and Manufacturing Engineering of the Council for National Academic Awards.

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SUMMARY.

This project arose from an on-going partnership between Vickers Shipbuilding and Engineering Limited and Liverpool Polytechnic which has already resulted in a study of bellows design.

It was suggested to the present writer that an extension of the previous work in this field might usefully be undertaken by examining the validity of the finite element method as a prediction-tool for bellows design. The precise area chosen for this investigation was the prediction of static, linear-elastic, behaviour of corrugated bellows expansion joints subject to axi-symmetric loading.

A suitable specimen bellows was supplied by V.S.E.L. and this bellows provided the specimen data for all work in this report.

An initial literature search was carried out in order to assess the nature and extent of existing work in this field. In addition, correspondence with manufacturers of bellows was undertaken in order to discover the kind of standards which were being employed in the design of these products.

The literature search revealed that much theoretical work has been carried out in the field of bellows design, but it appeared that reported finite element work of the type proposed in this project was limited to the project carried out by J.J.Knight (1987) of Liverpool Polytechnic. The correspondence with bellows manufacturers revealed, interestingly, that the majority of bellows design is carried out on an empirical basis rather than by applying the theoretical principles revealed in the literature search. The preferred source of empirical data would appear to be the Standards of the Expansion Joint Manufacturers Association which uses as its primary source, work carried out by Atomics International (Report NAA-SR-4527) into the field of bellows design.

Since the present writer was only considering axi-symmetric loading, the finite element model used was of the axi-symmetric type in which the model consists of a generator plane. In addition to axi-symmetry, there was found to be symmetry between convolutions of a bellows and mirror-symmetry between the two halves of one convolution of the bellows. The resulting model was thus simply a generator plane consisting of half a convolution. This finite element model was run to produce data concerning:-

a) the linear axial stiffness,
b) the non-linear axial stiffness,
c) the stress distributions set up in a convolution due to axial displacement and internal pressure.

Experimental work was performed on the specimen bellows Four strain gauges were used, their locations being at the convolution root and the convolution crest in both the circumferential and the meridional directions. A test apparatus that could provide axial displacement and internal pressure was used. Recordings were made of the axial force and component strains as functions of both axial displacement and internal pressure.

Validation of the finite element model was thus dependent on the success or otherwise of the comparison of the finite element results with the experimentally observed results. The author found that the finite element method was suitable for predicting, in terms of magnitude, the bellows spring rates and stress distributions but that it was not capable of providing numerically accurate results for these parameters.

In the discussion to this report the author examined the reasons for this inability to predict behaviour accurately and he concludes that complex 'non-ideal' material and geometrical properties ie the anisotropy producing factors inherent in bellows due to their method of manufacture are the most likely cause for the inaccuracy.

As a result of the work carried out, the author was able to make the following conclusions:-

1. The finite element method has been found to be satisfactory as a technique for obtaining an order of magnitude prediction of axial stiffness and stress distributions in axi-symmetrically loaded bellows.

2. For an accurate value prediction of axial stiffness and stress distribution in axi-symmetrically loaded bellows, the finite element method has been found lacking. It is felt that the finite element methods inability to predict accurate values for these parameters does not lie in any inadequacy of the method but is due in the main to non-ideal material and geometric properties imparted to the material of the bellows during manufacture. Such variables cannot be accomodated in any simple finite element model with the technology of today. This does not, of course, preclude such a possibility in the future.

On completion of this work it was possible to make the following recommendations:-

Recommendations to V.S.E.L.

Based on the conclusions of this report the author would recommend that V.S.E.L. should adopt the finite element method as a useful predictor of behaviour in axi-symmetrically loaded bellows. However he would also wish to state clearly that this method can not be used alone in the design of bellows but that an empirical based method (eg the Standards of the Expansion Joint Manufacturers Association) must be used in conjuncture with any finite element analysis that may be employed.

Recommendations to Liverpool Polytechnic.

The student undertaking a project of this nature is at the mercy of a variety of factors which may render his work difficult or even impossible to complete to his satisfaction. In the case of this particular project, delays occured which, though small in themselves, made it impossible to proceed at an orderly pace and resulted in the frustration of 'out of sequence' working.

Perhaps the first step in any M.Eng. project should be an exercise in critical-path analysis performed jointly by the student and his supervisor. If this were done, liason with the industrial sponsor could become more effective and provision of materials and necessary technical assistance by the polytechnic could be properly planned and sequenced.

Recommendations for Further Work.

Although this project work appears to have reached its logical conclusion the author can see a number of routes in which further work could usefully be carried out in the field of bellows design. These are listed below.

1. A feasibility study could be made with a view to ascertaining whether the technology exists to include the 'non-ideal' material and geometrical properties imparted to the bellows material during manufacture in a finite element analysis. If this proved to be the case an attempt to apply the technology would be beneficial. If not, one could look into the creation of such a technology.

2. If finite element analysis is to be widely used in bellows design, it might well be beneficial to write a computer programme which, based on bellows specification data as input, gives as output a PAFEC data file.

3. Given the validity of the finite element method, a study into the optimisation of bellows design might produce interesting results. For example one could perhaps evaluate the relative merits of the different convolution geometries generally employed by manufacturers or, indeed, look at the possibility of a new convolution geometry.

4. The generation of a full bellows finite element model would provide a method by which the non-axi-symmetric loading regimes of lateral load and/or angular deflection could be tested and their respective characteristics noted.

NOMENCLATURE.

Quantity.	Symbol.
Young's Modulus.	E
Poisson's Ratio.	ν
Coefficient of Linear Expansion.	α
Effective Coefficient of Linear Expansion.	α'
Yield Stress in Tension.	σ_{yt}
Yield Stress in Compression.	σ_{yc}
Inner Radius of Bellows.	r_i
Outer Radius of Bellows.	r_o
Mean Radius of Bellows.	$\overline{R}$
Convolution Radius.	r
Thickness of Bellows Material.	t
Mean Radius of Pipe.	R
Number of Corrugations.	n
Internal Pressure.	p
Axial Force.	F
Axial Deflection.	x
Axial Pressure Thrust.	T
Axial Stiffness.	K
Working Temperature of Bellows.	T_w
Ambient Temperature.	T_o
Length of 'Straight' Portion of Convolution.	L
Prescribed Displacement.	Δ
Summation of Nodal Reactions.	R
Correction Constants (N.A.A. Standards (25))	C_f , C_d , C_p
Parameters Used by Haringx (18).	χ , λ_i , λ_o , μ_i , μ_o , ρ
Strain in the i Direction at Position j	ε_{ij}
Stress in the i Direction at Position j	σ_{ij}
Strain (Stress) Column Vector as Function of X.	$\overline{\varepsilon(x)}$ $\overline{\sigma(x)}$
Strain (Stress) Column Vector as Function of P.	$\overline{\varepsilon(p)}$ $\overline{\sigma(p)}$
Strain (Stress) Column Vector as Function of P & X.	$\overline{\varepsilon(p,x)}$ $\overline{\sigma(p,x)}$

Column Vector Order. eg. $\overline{\varepsilon(x)} = \{ \varepsilon_{\theta c}(x) , \varepsilon_{\theta R}(x) , \varepsilon_{mR}(x) , \varepsilon_{mc}(x) \}^T$

Stress/Strain Subscipts.

i- θ Circumferential (Hoop) Direction.
m Meridional Direction.

j- R Root of Convolution.
C Crest of Convolution.

Stiffness Subscripts.

EB Effective Stiffness of Unpressurised Bellows.
EBP Effective Stiffness of Pressurised Bellows.
C Combined Stiffness of Cuffs and Central Spool.
F Stiffness of Flexible Convolutions.

Additional Basic Relationships.

$$\overline{R} = (r_o + r_i)/2$$

$$L = (r_o - r_i - 2r)$$

$$\Delta T = T_w - T_o$$

Dimensions.

All dimensions in this report will be in S.I. Units.
In particular:-

Forces. Newtons (N)
Lengths. Millimetres (mm)
Pressures. Kilo-Pascals (KPa)
Stresses. Mega-Pascals (MPa)
Strains Micro-Strain ($\times 10^{-6}$)

Any Quantity not identified in this section will be defined at the appropriate point in the report.

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1. INTRODUCTION.

As the title of this report suggests, this project has been concerned with the validation of the Finite Element Method approach to axi-symmetrically loaded corrugated bellows expansion joints. The purpose of this section of the report is two-fold. Firstly, it is intended to introduce the reader of this report to the subject of expansion joints. Questions such as what are bellows expansion joints, why and where are they used, how do such joints vary and how do they fail in service will be looked at in this section. Secondly, it will be necessary to survey any previous work in the field of interest, to identify any established standards on the subject and to define the nature and scope of the work carried out during this project within the context of this framework of information.

1.1 What are Bellows Expansion Joints?

Bellows expansion joints are pipework members designed to accept and absorb displacements. Three distinct types of displacement may be identified. Axial movement, where displacement is in the direction of the pipe central axis. Lateral movement, where displacements are perpendicular to the pipe central axis, and Rotational movements where the two ends of the expansion joint are rotated with respect to each other about an axis perpendicular to the bellows central axis. These movements and the forces or moments associated with them are shown in figure 1.1 .
Note that rotational and lateral movements do not fall into the category of axi-symmetric loading and as such are not covered in any detail beyond this introductory section.

1.2 How can these Movements Occur?

A system of pipes will often be subject to cyclic forces. For example the exhaust system of a car is subject to vibrational effects transmitted from the engine and to thermally induced expansions and contractions. Thermally induced cyclic forces are, of course, of much longer wavelength and lower frequency than the characteristically small wavelengths and higher frequencies of vibrational effects. Depending upon the practical nature of the pipe system, in particular the extent of any constraints and the position of the expansion joint within that system, it is possible that any one of the above-mentioned movements or indeed more than one of these movements may occur at the expansion joint. Figure 1.2 depicts examples of practical systems where thermal expansion of the pipework will cause specific types of movements to occur at the expansion joint.

1.3 Why Use Expansion Joints? ? (2009)

In many pipe systems the magnitude of these cyclic forces may be negligible when compared to the strength of the pipework material. In these situations it is often possible to allow the pipework itself to absorb any force that occurs in the form of strain energy. However, where the magnitude of these forces is comparable or in excess of that which the pipework can withstand one must look for alternative methods by which to unload these forces.

Three distinct methods are available to the pipe system designer. The first method involves forming a force absorbing component in the actual material of the pipe itself. Two such techniques are employed. Bulging of pipes is a method by which flexible convolutions can be formed in the pipe material. The convoluted section of the pipe then absorbs the excess forces. Expansion loops may be formed in the pipe. This method allows the distribution of large forces evenly around the loop such that strains per unit length of pipe are kept within acceptable limits. Although the expansion loop technique is versatile in that one can simply increase the size of the loop to incorporate larger forces, the loop itself takes up a large amount of space and this may be undesirable in situations where space and indeed weight are at a premium (submarines or aircraft for example).

The second method is that of cold springing the pipe system. This method works by pre-straining the pipe in the ambient temperature position so that when the pipe is at its working temperature the thermally induced strains, which will be super-imposed on the pre-loaded strains, remain within the material's elastic limit.This method has two disadvantages. Firstly one is still limited to the amount of expansion that a pre-loaded pipe can accept. If the yield stress in compression is taken to be the same as that in tension then one is limited to a maximum strain of twice the yield strain. Secondly the range of stress the pipe material is subjected to will be twice as high as the case where the pipe system is not pre-loaded. Since the number of cycles to failure by fatigue is proportional to the stress range this method may well prove unsuitable where large numbers of cycles are to be expected.

The third method is the subject material of this project, namely bellows expansion joints. These joints take the same amount of room as an equivalent length pipe section and weigh, if anything, less than the equivalent length of pipe. The bellows joint unloads the pipe system of the majority of stress and can therefore be used in situations where many thermal or vibrational cycles are likely to occur in the pipe system's life span. Further, since the expansion joint can be designed to form the 'weak link' in the pipework, one can expect and hopefully predict, failure to occur at the expansion joint. This may be a consideration of some practical advantage in situations where, for reasons of safety perhaps, the joint could be located in an easily accessible position for routine scheduled replacements.

The ideas discussed in this section are shown in figure 1.3.

1.4 Limiting Temperature Range.

It is of interest to be able to estimate the limiting temperature range that a pipe system can withstand without the need to resort to induced flexibility. To this end the author proposes to analyse an idealised situation consisting of a straight pipe rigidly restrained in the axial direction. This is idealised because in reality one would find that the total axial restaint was, in-fact, only partial due to flexibility of the end connections.

Consider, then, a pipeline with total axial restraint. If the pipe is initially unstrained at some ambient temperature T_0, then it will be subject to a direct compressive stress σ_A when the temperature is raised to some working temperature T_W. The magnitude of this stress will be $\sigma_A = E\alpha\Delta T$.

This result reveals that for a given material with a given yield stress σ_{Yc} there will be a limiting value of ΔT (ΔT_L) such that $\Delta T_L = \sigma_{Yc}/E\alpha$. Any temperature range greater than this limiting value would result in plastic deformation of the pipe material.

The evaluation of the limiting temperature range is in theory somewhat complicated by the fact that Young's Modulus, the coefficient of linear expansion and the compressive yield stress for the material are all, to some degree, functions of temperature*. In practice, however, it is likely that empirical methods would be used to determine this property. Provided, therefore, this limiting temperature range is not exceeded in service then no expansion joint is necessary.

1.5 How do Bellows Vary?

The most visually apparent variation in bellows design is that of corrugation geometry. In the past, manufacturers of such joints have used a number of geometries

*Figure 1.41 .

and some of the more common ones are shown in figure 1.5 . More recently, however, there has been a tendency towards the U-shaped geometry for reasons of standardisation and ease of design and manufacture.

A bellows, could in principle, consist of merely one convolution, the single omega-type being a good example. In general, however, a series of n convolutions adjacent to each other makes up one expansion joint. Such a close-coupled series is called a single bellows expansion joint. The flexibility of an expansion joint is proportional to the number of convolutions. Single bellows expansion joints are limited in their ability to absorb lateral displacements and it is for this reason that expansion joint manufacturers market the double or universal expansion joint which consists of two single expansion joints mounted on a common central spool piece. Figure 1.51 illustrates the distinction between single and universal expansion joints. The degree of lateral displacement that a universal expansion joint can absorb is then directly proportional to the length of the central spool piece.

1.6 Associated Components.

In certain applications it is necessary to strengthen or restrict movements of bellows. In cases where bellows are subject to high internal pressures it may prove necessary to strengthen the convolution root. Reinforcing rings carry out this function and their effectiveness may be enhanced by pre-tensioning the rings. Equalising rings may be used to ensure that all convolutions deflect to the same degree, though analysis suggests that this is only necessary where thickness varies from convolution to convolution because of inaccurate manufacturing techniques. A more important role for these rings could be envisaged were they to be manufactured such that they act to take the load if the bellows were deflected into a failure degree. In this case the rings would prevent further deflection by rendering the bellows rigid.

Internal sleeves may be used where the fluid transfered by the bellows is likely seriously to errode the thin convolution (typically liquid/solid two-phase flow) or where flow induced vibration is not desired: Flow in the transition region may enter the bellows as laminar and, due to the corrugated surface of the bellows, leave as turbulent. This may be undesirable in specific situations.

Hinges, tie-rods and gimbals may be used in certain applications where it is desirable to limit a set of bellows to move in a pre-determined manner. A bellows with none of the aforementioned attachments may move axially, laterally or angularly. Hinges are used to restain any axial or lateral movement thereby allowing only angular rotation. Tie-rods remove the bellows ability to move in the axial direction yet allow lateral and angular movements. Gimbals (double hinges) allow angular rotation in any plane but remove axial and lateral displacement possibilities. There may be a number of reasons why a pipe designer would wish to remove only certain degrees of freedom. It is common, however, to use such devices where axial thrust due to internal pressure in the bellows is high because these devices take the axial thrust themselves therefore not transmitting it to the surrounding pipework. The various devices discussed in this section are shown in figure 1.6 .

1.7 Modes of Failure.

Bellows, like any engineering component, are susceptible to failure through excess strain and fatigue. Failure through excess strain can be avoided by keeping strains within appropriate limits. Failure by fatigue may similarly be alleviated by keeping the stress range the bellows are subjected to within appropriate limits. Bellows may also fail through surface effects. Since they are, by definition, thin walled vessels, any damage to the surface may severely reduce the load-bearing capacity or the life of an expansion joint. Damage to the surface may be caused by mishandling, corrosion via the working fluid or chemical attack from the environment. In this latter field the damaging effects of certain paints and lagging materials are well known and documented. B.S.6129 (28) covers this aspect of failure well and it recommends handling and packing procedures along with paints and laggings which should be avoided.

Like any thin walled vessel, where the length to diameter ratio is high, bellows may be subject to the instability known as squirm. Figure 1.7 shows this phenomenon The understanding and prediction of this mode of failure are well known and well documented by Haringx (18) who presents a method of analysis by which the onset of this mode of failure can be predicted.

1.8 Background to this Project.

The background and history of this particular project lies in a relationship established between Vicker's Shipbuilding and Engineering Limited (V.S.E.L) and Liverpool Polytechnic and, as a result of this relationship, an interest was developed in the theory of bellows expansion joint design. Specifically V.S.E.L., as expansion joint users, were suffering the early failure of such joints in the exhaust systems of submarine diesel generators. As a preliminary, a literature survey was performed and this now follows.

1.9 Literature Survey.

A review of current literature on the theory of stresses and stiffness of bellows expansion joints was conducted. Many papers exist on this subject and, in addition, several standards on expansion joints were uncovered.

The study of bellows stiffness has been undertaken by a number of workers. Calladine (11) presents a analysis based on an approximate energy method whereby he deals with S-shaped bellows and chooses to keep the angle α (figure 1.5) as a variable.

Berliner and Vikhman (10) present an empirical formula based on the theory of bending of annular discs. They provide a series of test results and show that their formula fits, with reasonable accuracy, their results.

Wilson (23) conducted a computerised survey of bellows literature and his paper reviews available axial stiffness formulae up until 1983. A study of this paper reveals that many different formulations exist. The basis for these formulations has frequently been a simplified approach where the bellows is considered as a beam, a disc or, in the more elaborate analyses, a shell.

The assumptions inherent in many of these formulations in practice mean that the suggested formula is only applicable to a particular bellows geometry and, more often than not, it is only applicable for limited ranges of form factor. (A variety of form factors is used in bellows work: these are, in general, non-dimensional geometric parameters.).

All the evidence, then, points to the fact that no generalised theory applicable to bellows stiffness has been developed. Further, it is evident that application of these existing formulae gives very different results when applied to the same set of bellows. Wilson (23) carried out an experimental programme on a set of polyethylene bellows and then applied a number of formulae to these same bellows. His results show that although all formulae arrive at a stiffness that is in the correct order of magnitude, considerable divergence in actual results exists (maximum divergence in Wilson's results was 32%).

The stress in bellows which is set up by axi-symmetric loading conditions (ie axial load and/or internal pressure) has been studied by a number of writers in the field. Again, simplified theories have been applied. Feely and Goryl (13) apply an analysis based on beam theory to the crimped plate geometry (figure 1.5) and they provide a number of design formulae presented with some test results which appear to validate their theory.

In the main, however, we find that the mathematical processes used in analysing stresses in bellows follows the same chronological pattern as those for stresses in piping elbows. The first analysis of piping elbows represented the distortion of the elbow as a truncated Fourier series and was applicable to large values of the form factor Rt/r^2. Clark (14) in a previous paper on piping elbows attacked the 'shell theory' equations and obtained a solution asymptotic to the exact solution for small values of the form function.*

Turner and Ford (22) set up the strain energy integral in terms of the unknown stress. They then represented the stress as a five term truncated Fourier series and integrated the strain energy integral. The resulting equation was minimised (the principle is of minimising the functional where, in the case of mechanics of materials, the functional is strain energy) and solutions were then found for the unknown coefficients in the Fourier series.

This approach sounds straightforward until one realises that even with a five term truncation of the Fourier series, the strain energy integral contains of the order of 1400 terms in its integrand. For certain limits of the integral many of these terms will disappear but in the general case where these terms do not disappear this represents an awesome algebraic task. Their analytical work is backed up by an experimental programme carried out on bellows of S-shaped geometry.

Laupa and Weil (20) approach the problem in a similar manner, again with a five term Fourier series solution. They provide an elegant analysis for convoluted bellows of U or S-shaped (α=90) for the cases of axial and internal pressure loading.

Dahl (12) also approached the problem with a Fourier series solution, however, in this work a four term truncation is applied. This work is applicable to toroidal bellows.

The second approach to the problem of stresses is that shown by Clark (14) who sets up the exact complex differential equations and then provides a method of asymptotic integration by which a solution asymptotic

* Figure 1.9 shows the range of validity for solutions.

to the exact solution at small values of the form funcion is achieved. This method, although only applicable to small values of Rt/r^2, can be used for larger values of this form factor by incorporating a correction factor.

It becomes clear that, like the bellows stiffness formulae, there exists no generalised solution for the stress distribution in a set of bellows. It also becomes evident that those particular solutions that do exist for certain geometries are intractable for the everyday application. We are, thus, faced with a situation where in a generalised solution is unlikely to be available until more advanced computational techniques become available.

When faced with this kind of dilemma, where a solution is requires but the exact process is intractable, engineers often resort to a simple model and correct by experimentally determined correction factors. The Atomics International Report NAA-SR-4527 (25) and the Standards of the Expansion Joint Manufacturers Association (24) provide a technique where simple equations are used and correction is taken from a graphical plot. An extract from the latter is shown, for reference, in the appendix to this report.

1.10 Review of Finite Element Work.

Several writers in the field of bellows have applied the finite element method to the problem of stresses in bellows. Hamanda et al (16) provide 'design diagrams and formulae for U-shaped bellows' however the author was unable to examine this paper: the paper was not received after requisition. Snedden (21) in his Ph.D dissertation uses the finite element method in his analysis of the non-linear 'elasto-plastic' behaviour of laterally loaded bellows. Snedden's work is beyond the scope of the author's work and is, therefore, not discussed in any detail.

A study was performed by J.J. Knight (19) of Liverpool Polytechnic and submitted as his M.Eng dissertation in 1987. This study was entitled 'The Factors Affecting the Performance of Marine Bellows.' Knight introduces his work by stating his intention to determine a better understanding of the factors affecting the bellows in question. In the event, presumably for reasons of time and resources, his study concerned itself with one factor only viz the effect of internal pressure and lateral displacement on the strain in the bellows material. This work included an experimental programme where his apparatus could laterally displace one of the end flanges relative to the other and he arranged a number of strain gauges around one half convolution thereby giving a picture of strain for that convolution. His finite element models include a completed model of half a convolution suitable for axi-symmetric loading only and an incomplete attempt at a 'whole-bellows' model capable of modelling the effects of unsymmetric loading.

From his experimental and finite element programme of work Knight was able to conclude that the finite element model predicted with some accuracy the stresses on the bellows external surface for the case of internal pressure. He was also able to show the strain distribution caused by lateral displacement, around one half of one of the convolutions. The usefulness of this last is highly questionable since he, by necessity (due to the finite length of strain gauges), had to arrange his gauges in a circumferential arc. As a result he was picking up a unrepresentative picture of strain. Since strain for unsymmetric loading is a function of the circumferential position and since the principal planes only lie in the meridional and cirumferential directions for the case of axi-symmetric loading I would submit that for a practical solution to the real problem he was investigating he should have positioned all his gauges at a constant value of θ .

Summarising Knights work then, it is clear that a germ of a solution in terms of finite element analysis exists for the case of axi-symmetric loading. The task of the present writer has therefore been to extend and verify his axi-symmetric work using a different bellows expansion joint and a more refined finite element model. Should broad agreement be found between the results of the two projects then it would appear that it is valid to regard the axi-symmetric finite element model as a suitable design tool for the prediction of actual behaviour on bellows which are subject to axi-symmetric loading.

By way of extension the present writer has looked at the case of axial deflection in addition to internal pressure loading. Further, since there is no evidence in Knight's work of an exhaustive literature survey, the present writer has taken the opportunity to review the published materials in this field (section 1.9) and to consult with specialist firms engaged in bellows design and manufacture (appendix 4). It is hoped that the data thus assembled will be of assistance to future workers in this field.

2. EXPERIMENTAL PROGRAMME.

2.1 Introduction.

An experimental programme was carried out on a specimen set of bellows. This section of the report deals with this experimental programme in detail.

2.2 Design of Test Rig.

In the early stages of this project it was thought that a test rig should be designed and manufactured to suit the bellows specimen available. To this end the author designed a test rig with the capability of providing axial load, pressure loading and lateral deflections in two directions. The manufacturing drawings (appendix 3) were supplied to the Polytechnic by the author. In the event, through an apparent lack of commitment from V.S.E.L. and as a result of the inability of the Polytechnic to fund such a venture, the manufacture of the test rig was cancelled.

Faced with this setback, the author identified two pieces of existing Polytechnic testing equipment that might prove suitable for the axial loading of the bellows specimen.

The Denison testing machine (model T.42.62) shown in plate 2 is a 20 ton capacity machine. Preliminary tests on this machine, however, revealed that although the axial displacement readability was ±0.005", the axial force readability was only ±12.5N. Since the bellows stiffness was thought to be in the region of 25-50N/mm and the author was not expecting to displace the bellows by more than 15mm this level of readability was clearly unacceptable.

The Instron testing machine (model 8032) shown in plate 3, however, provided a much higher degree of readability accuracy both in terms of displacement ± 0.024mm and applied force ± 2.4N . It was decided, therefore, to carry out all experimental work on the Instron. It should be noted that the Instron and Denison testing machines are only suitable for axially displacing the bellows. The possibility of investigating lateral movement was, therefore, ruled out at this juncture.

The only requirement in terms of additional material was for the provision of two blank flanges for blanking off the open ends of the bellows. Flanges of the correct type were not available, however some 1/2" plate steel was found and the blanks were burnt from this. These burnt blanks were machined and drilled for the appropriate P.C.D. by the polytechnic workshops. For accurate indexing in the Instron, the blanks were recessed by the author to fit over the Instron's loading platterns.

The final requirement of being able to pressurise the bellows was met by machining a flow port in one of the blank flanges. A suitable system of pipes and pump was assembled and it was then fitted ready for testing. It is worth noting here that the pressure gauge used was of the Bourdon type with a range 0-200KPa gauge pressure. The gauge had a readability of ±2.5KPa. The gauge was calibrated using a Budenburg (serial Nº 7192/279) dead-weight tester and it was concluded that the accuracy of the gauge was well within the limits of readability. Pressures taken from the gauge are identified as nominal pressures in this report.

2.3 Bellows Test Specimen.

The bellows specimen supplied by V.S.E.L. were of the universal expansion joint type, consisting of two single bellows expansion joints mounted on a common central spool. V.S.E.L. were unable to provide any precise information regarding dimensions or material properties. This problem was greater than may at first appear. One can only get an accurate picture of the dimensions and an accurate assesment of the material properties by destroying the bellows. This could clearly not take place until the experimental programme had been completed The approach adopted, therefore, was to continue work on the theoretical side with the best approximations of dimensions and material properties available, to push the experimental work on and, when complete, to destroy the bellows, obtain exact dimensions and more realistic material properties with which to update the theoretical work. This approach is outlined in the network analysis shown in appendix 6. Fully dimensioned diagrams of the expansion joint may be found in figure 2.3

2.4 Strain Gauge Installation.

2.41 Identification of Principal Planes.

Before one can identify suitable positions for strain gauges, one must have a approximate idea of the likely stress distribution. If one does not know the directions of the principal planes then one must always use three gauges (a rosette) in order to fix the three principal planes in space and to obtain values for two of these principal strains. If, however, one knows the orientation of the principal planes beforehand then it is only necessary to use two gauges (aligned with the principal directions) in order to obtain the same information. In the case of the bellows expansion joint in question we have the latter situation.

The reason that we know the direction of the principal planes follows from the type of loading that is being studied. Since the loading studied in this project is axi-symmetric then if the material of the bellows is isotropic and homogeneous it follows that deflections and therefore strains will be axi-symmetric. The bellows will remain a body of revolution before and after application of load. From this one can conclude that the three principal stress vectors at any point in the bellows point in the meridional, circumferential and transverse directions

It should be pointed out here that this thinking is only applicable to the case of axi-symmetric loading. In the case of unsymmetric loading one would not be able to identify the principal planes intuitively and three strain gauges would therefore be required at each point.

The principal directions identified above turn out to be circumferential (θ), meridional (M) and transverse (T). Unlike a plane cylinder where it is possible to identify principal directions at any point in the body with a global set of axes, it is necessary to identify principal directions at any point in the bellows by a set of local axes. The axis sets used in this report are shown in figure 2.411. If one desires, of course, it would always be possible to relate the configuration of the local axes at any point to the global axis, however, there is no gain for our purposes by doing this.

From this 'common-sense' approach to identifying the principal planes it is clear that one is quite legitimate in placing strain gauges in the circumferential and meridional directions at any point on the bellows. The third principal stress (transverse) cannot be measured by strain gauges and here one must assess the practical significance of this principal stress. Clearly, for an internally pressurised bellows the transverse stress at the inner surface will, at all points, be equal to the pressure and there will be zero transverse stress at the outer, atmospheric, side of the bellows. Does the magnitude of this transverse stress require consideration in terms of bellows failure? Taking the analogous case of internally pressurised <u>thin</u> cylinders ($t/d<1/20$) the answer is clearly 'no'. However, how good is this analogy?. This question may only be answered by investigating the relative magnitudes and directions of the circumferential and meridional stresses in relation to the transverse stress.

The benefit of analysing an axi-symmetrically loaded structure like the bellows is that behaviour does not vary in the circumferential direction and each convolution can be considered as identical in behaviour to any other. This allows relative freedom in positioning of strain gauges. One can obtain an accurate picture of strain around the meridion of a convolution by staggering gauges at different meridional positions around the circumference of any convolution. Again this is a difficult idea to get over in words and for this reason figure 2.412 is supplied.

Having identified the important directions of the '2-D' surface principal stress system and noted that one is relatively free to position gauges on the bellows, one must now look towards the selection of suitable strain gauges.

2.42 Strain Gauge Selection.

In any situation where one wishes to analyse strains in a body one must select a gauge size that is negligible in terms of width and length when compared to the rate of change of strain with distance at a point of interest on the body. This is common-sense, since, the strain gauge will in effect give an average result of strain for the area it covers. Unfortunately, the only strain gauges that were available to the author did not fit this 'ideal' situation. For the bellows in question, a suitable gauge length and width would have been less than 1mm square, however, the only gauges available were 7mm long by 2mm wide. Accepting this situation it was concluded that one could only obtain a picture of strain that would stand up to any scrutiny at the positions at the root and crest of a convolution. Gauges positioned at intermediate points between these limits would have been averaging the strain over too many radii to stand up to any scrutiny whatsoever. Strain gauges were, therefore, positioned at the root and crest of a convolution, two at the crest, two at the root measuring strain in the circumferential and meridional direction respectively.

The table below identifies the strain gauge type with position.

Gauge Position	Length (mm)	Width (mm)	Gauge Factor	Resistance
$\varepsilon_{\theta c}$	7	2	2.055+1.0%	120 +0.3%
$\varepsilon_{\theta R}$	7	2	2.055+1.0%	120 +0.3%
ε_{mR} *	1	1	2.045+0.5%	120 +0.3%
ε_{mc}	7	2	2.055+1.0%	120 +0.3%

* Strain gauge left over from previous project.

The gauges were installed using the usual technique of scrupulous surface cleaning, quick drying epoxy-resin adhesive and soldered connections. A set of dummy gauges was found (attached to some similar material) and the gauges were connected to a Brüel and Kjaer Strain indicator box (type 1526).

The strains thus measured were converted to stresses by the 2-dimensional form of Hooke's law as shown below.

$$\sigma_{\theta j} = \frac{E}{1-\nu^2} (\varepsilon_{\theta j} + \nu\varepsilon_{mj})$$

$$\sigma_{mj} = \frac{E}{1-\nu^2} (\varepsilon_{mj} + \nu\varepsilon_{\theta j})$$

The following sections document the series of tests that took place.

2.5 Test Series.

2.51 Axial Stiffness.

The bellows were set up in the Instron testing machine and the load and displacement readouts were adjusted to give a null reading. Having previously determined that the material yielded at approximately 15mm axial displacement, a limit of 10mm total displacement was set. Incremental displacements of 2mm were then applied up to the 10mm maximum and back to zero axial displacement. The axial load and axial displacement were recorded after each increment. This test was repeated a number of times and it was concluded that the results (table 1) were repeatable and that no plastic deformation had occurred.

2.52 Strain as a Function of Axial Displacement.

The strains were recorded during the axial stiffness tests. These strains exhibited repeatability and are shown in table 1.

2.53 Strain as a Function of Internal Pressure.

The bellows were set up with zero axial displacement and the force and displacement readouts were zeroed. At this position of zero axial displacement pressure was applied incrementally to a maximum of 200KPa. The strains and axial load were recorded at each 50KPa increment. The test was repeated a number of times and the results showed repeatability. The results from this test are shown in table 6.

2.54 Strain as a Function of Axial Displacement and Internal Pressure.

A series of test as described in section 2.51 (axial stiffness) were carried out at 50KPa increments from zero to 200KPa. At each increment the axial force, strains and axial displacement were recorded. The results are shown in tables 2 through 5.

2.55 Material Test.

After all experimental work (above) had been completed and the results checked for their integrity, the bellows specimen was cut open and two tensile test specimens were taken from the cuff in the circumferential direction The tensile test pieces were manufactured to the preferred dimensions* stated in B.S. 18 (26). The test pieces were tested on a Hounsfield tensile testing machine and a value for Young's Modulus in the order of 200GPa was obtained.

This material test will be discussed in more detail in the discussion section of this report.

2.6 Interpretation of Results.

All experimental tests were carried out in the linear-elastic area of material behaviour and from that point of view one can expect linear results. The occurrence of non-linear behaviour due to large displacements,

*Figure 2.55

although expected, was not observed in the results. Given, then, this predominantly linear behaviour the results were processed to yield a 'best straight line' equation based on the 'sum of least squares' approach. As with any reduction of data some characteristic of the original data will be lost in the process. In this instance the best straight line equations have a small value for their intercept. This is clearly not the case in reality however, the benefit gained from performing this 'smoothing' operation outweigh this small lose of accuracy.

This analysis results in two sets of five linear equations, the first set being the axial force and four strains as functions of axial displacement, the second set being the axial force and four strains as functions of internal pressure. By applying the principle of superposition and, therefore by implication ruling out non-linearities due to large displacements, the two sets of equations can be added vectorially to give one set of five linear equations for the axial force and the four strains as functions of both axial displacement and internal pressure. The aforementioned analysis is summarised below.

$$F(X)=62.9X+14.5 \quad \text{and} \quad F(P)=19.7P+16.1$$

By the Principle of Superposition $F(X,P)=F(X)+F(P)$

hence $F(X,P)=62.9X+19.7P+30.6$

$$\overline{\varepsilon(x)} = \begin{bmatrix} 57.8 & 0.00 & 3.70 & 0.00 \\ -53.7 & 0.00 & -5.50 & 0.00 \\ -36.5 & 0.00 & -3.90 & 0.00 \\ 29.5 & 0.00 & 0.30 & 0.00 \end{bmatrix} \times 10^{-6} \begin{Bmatrix} X \\ P \\ 1 \\ 0 \end{Bmatrix}$$

$$\overline{\varepsilon(p)} = \begin{bmatrix} 0.00 & -0.494 & -0.38 & 0.00 \\ 0.00 & -0.139 & -1.18 & 0.00 \\ 0.00 & -1.849 & -4.47 & 0.00 \\ 0.00 & -1.402 & -2.24 & 0.00 \end{bmatrix} \times 10^{-6} \begin{Bmatrix} X \\ P \\ 1 \\ 0 \end{Bmatrix}$$

And, again, by the principle of super-position

$$\overline{\varepsilon(p,x)} = \begin{bmatrix} 57.8 & -0.494 & 3.32 & 0.00 \\ -53.7 & -0.139 & -6.68 & 0.00 \\ -36.5 & -1.849 & -8.37 & 0.00 \\ 29.5 & -1.402 & -1.94 & 0.00 \end{bmatrix} \times 10^{-6} \begin{Bmatrix} X \\ P \\ 1 \\ 0 \end{Bmatrix}$$

Applying the 2-dimensional form of Hooke's law

$$\overline{\sigma(p,x)} = \frac{E}{1-\nu^2} \begin{bmatrix} (57.8+29.5\nu) & -(0.494+1.402\nu) & (3.32-1.94\nu) & 0 \\ -(53.7+36.5\nu) & -(0.139+1.849\nu) & -(6.68+8.37\nu) & 0 \\ -(36.5+53.7\nu) & -(1.849+0.139\nu) & -(8.37+6.68\nu) & 0 \\ (29.5+57.8\nu) & -(1.402+0.494\nu) & -(1.94-3.32\nu) & 0 \end{bmatrix} \times 10^{-6} \begin{Bmatrix} X \\ P \\ 1 \\ 0 \end{Bmatrix}$$

The results from all experimental work are shown in the graph section of this report. The best straight line is plotted with the experimental points in each case.

Of particular interest are the partial derivatives of the stress and force equations with respect to axial displacement and internal pressure. These are tabulated below for reference.

$$\frac{\partial F}{\partial x} = 62.9 \text{ N/mm}$$

$$\frac{\partial F}{\partial p} = 19.7 \text{ N/KPa}$$

$$\frac{\partial \sigma_{\theta c}}{\partial x} = 14.8 \text{ MPa/mm}$$

$$\frac{\partial \sigma_{\theta R}}{\partial x} = -14.3 \text{ MPa/mm}$$

$$\frac{\partial \sigma_{mR}}{\partial x} = -11.5 \text{ MPa/mm}$$

$$\frac{\partial \sigma_{mc}}{\partial x} = 10.2 \text{ MPa/mm}$$

$$\frac{\partial \sigma_{\theta c}}{\partial p} = -0.198 \text{ MPa/KPa}$$

$$\frac{\partial \sigma_{\theta R}}{\partial p} = -0.146 \text{ MPa/KPa}$$

$$\frac{\partial \sigma_{mR}}{\partial p} = -0.423 \text{ MPa/KPa}$$

$$\frac{\partial \sigma_{mc}}{\partial p} = -0.345 \text{ MPa/KPa}$$

3 THEORETICAL WORK.

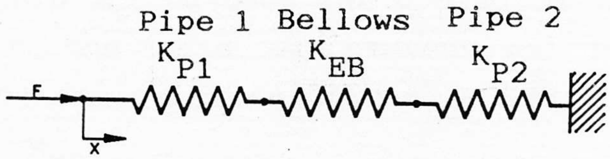
3.1 Introduction.

By way of introduction to this theoretical section of work, the author intends to show that in a pipeline incorporating flexible elements,such as a bellows expansion joint, the pipes may often be considered rigid for the purpose of force analysis. This is shown in the first section 3.2 .

Sections 3.3 and 3.4 are intended as introductory analyses by which to highlight the basic relationships for axial stiffness.

3.2 Axial Force in Pipeline.

If one wishes analytically to consider a straight pipe section incorporating a bellows expansion joint, it is clear that such a system can be modelled mathematically as a mechanical system consisting of three linear stiffness elements (springs) in series. The governing relationship can then be shown to be:-



$$F = K_E x \quad , \quad K_E = \frac{K_{P1}K_{EB}K_{P2}}{K_{P1}K_{EB}+K_{P1}K_{P2}+K_{EB}K_{P2}}$$

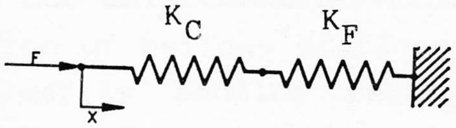
where $x = \Delta T(L_{P1}\alpha_1 + L_B\alpha' + L_{P2}\alpha_2)$

Evaluation of the stiffness and the axial displacement would enable the axial force and, hence, the direct stress in the pipe to be calculated.

This stiffness equation can be simplified by making the generally valid assumption that the stiffness of the pipework is much greater than that of the bellows expansion joint. In this case $K_E \to K_{EB}$. This assumption introduces an error in the calculated stiffness that can be shown to be less than 1% provided the identity $K_P \geqslant 200\ K_{EB}$ is satisfied.

3.3 Bellows Axial Stiffness.

A universal expansion joint consists of flexible convolutions and 'stiff' elements such as the central spool and end cuffs. This mechanical system can be modelled as two linear stiffness elements in series and the governing equations are then:-

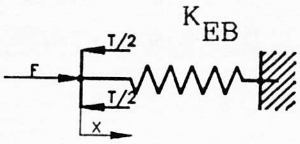


$$F = K_{EB}x \quad , \quad K_{EB} = \frac{K_C K_F}{K_C + K_F}$$

Again, by assuming that the stiffness of the cuffs and central spool is much greater than that of the flexible convolutions, this relationship can be reduced to $K_{EB} \rightarrow K_F$. The magnitude of error induced in the calculated stiffness by applying this assumption is less than 1% provided the expression $K_C \geqslant 100\ K_F$ is satisfied.

3.4 Pressurised Bellows Axial Stiffness.

When the bellows are subjected to an internal pressure a thrust will be set up in the axial direction due to the pressure acting over the projected area of the convolution. In the case of pressurised bellows expansion joints, the following relationship can be seen to apply:-



$$F = K_{EBP}x \quad , \quad K_{EBP} = K_{EB} + \frac{T}{x}$$

3.5 Review of Axial Stiffness Formulae.

The foregoing analyses have shown that for the basic case of a straight section of pipe incorporating a bellows expansion joint, the direct stress set up in the pipework can be determined by quantifying the three parameters K_{EB}, T and X. The evaluation of the pressure thrust is a simple matter: multiplication of the internal pressure by the projected area will quantify T. The evaluation of the axial displacement is a touch more complicated since it requires a knowledge of the functional relationship between the respective coefficients of linear expansion and temperature. This is not covered in this report; suffice it to say that 'worse-case' methods are normally used in practical situations.

The evaluation of K_{EB} for a particular set of bellows is our main concern. An intuitive approach to the evaluation of bellows stiffness suggests that, like any other primarily bending phenomenon, the stiffness will be a function of Young's Modulus, the second moment of area and the cube of the material thickness. In addition it has already been noted that the stiffness is also inversely proportional to the number of convolutions. A review of bellows literature revealed a number of formulae for evaluating the axial stiffness. The formulae that appeared to be relevant to the bellows specimen studied in this report are listed below.

Kellogg (3)

$$K_{EB} = \frac{\sqrt{2}\, E(r_i+r_o)t^3}{3n(r_o-r_i)^{\frac{5}{2}}\, C^{\frac{1}{2}}}$$

Turner and Ford (22).

$$K_{EB} = \frac{\pi E(r_i+r_o)t^3}{2(1-\nu^2)n\{6\pi C^3+24C^2(r_o-r_i-2C)+3\pi C(r_o-r_i-2C)^2+(r_o-r_i-2C)^3\}}$$

Berliner and Vikhman (10).

$$K_{EB} = \frac{2Et^3}{n\left(\frac{L^2}{3} + \frac{\pi Lr}{2} + \frac{\pi r^3}{4L} + 2r^2\right)}$$

The term 'C' is equivalent to r, the convolution radius, for U-shaped bellows.

Atomics International Report (25).

$$K_{EB} = \frac{0.431\ \bar{R}\ E\ t^3}{n(r_o-\bar{R})^3\ C_f}$$

Haringx (18).

$$K_{EB} = \frac{2\pi(1-\rho^2)Et^3}{3n\chi r_o^{\,2}(1-\nu^2)(1-\rho^2+2\rho Ln\rho)(1-\rho^2-2\rho ln\rho)}$$

where

$$\chi = \frac{1 + 2C\{(1-\rho^2+2\rho^2\ Ln\rho)^2\ \lambda_o + \rho(1-\rho^2+2Ln\rho)^2\ \lambda_i\}}{r_o(1-\rho^2)\{(1-\rho^2)^2 - 4\rho^2(Ln\rho)^2\}}$$

$$\lambda = \frac{Sinh\ 2\mu C + Sin\ 2\mu C}{2\mu C(cosh\ 2\mu C + cos\ 2\mu C)}$$

$$\mu = \sqrt[4]{\frac{3(1-\nu^2)}{r^2t^2}}$$

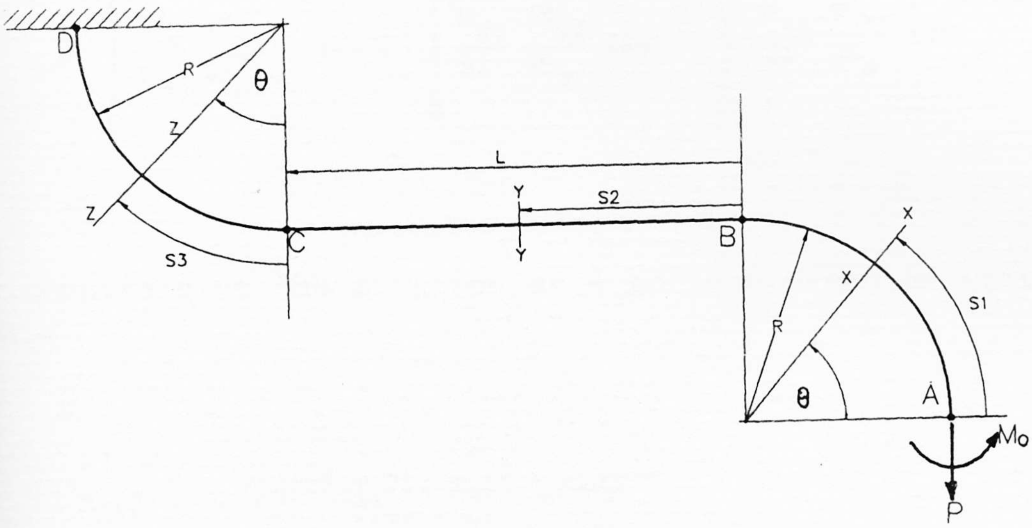
$$\rho = r_i/r_o$$

3.6 Application of Castigliano's Theorem to Bellows Axial Stiffness.

The author also applied Castigliano's First Theorem in order to assess the likely quality of results obtained by this method. Clearly such a treatment involves the making of assumptions, but these assumptions are of the same order of magnitude as those made by the other analyses recorded in the literature search. The assumptions applied in this analysis are that the 'straightened' out' convolution behaves in a similar manner to the original convolution, and that the strain energy associated with shear forces, torsion and end effects are negligible in comparison to the strain energy of bending.

A synopsis of the Castigliano analysis is shown below, and the results of this analysis are discussed in the discussion section of this report.

Model



The moment equations can be written:-

$$M_{xx} = Pr(1 - \cos\theta) - M_o$$

$$M_{yy} = P(r + S_2) - M_o$$

$$M_{zz} = P(r + L + r\sin\theta) - M_o$$

The partial derivatives can be written:-

$$\frac{\partial M_{xx}}{\partial P} = r(1 - \cos\theta) \quad , \quad \frac{\partial M_{xx}}{\partial M_o} = -1$$

$$\frac{\partial M_{yy}}{\partial P} = r + S_2 \quad , \quad \frac{\partial M_{yy}}{\partial M_o} = -1$$

$$\frac{\partial M_{zz}}{\partial P} = r + L + r\sin\theta \quad , \quad \frac{\partial M_{zz}}{\partial M_o} = -1$$

The angular deflection at point A can be written

$$\theta = \int_0^{\pi/2} \frac{M_{xx}}{EI} \frac{\partial M_{xx}}{\partial M_o} r d\theta + \int_0^L \frac{M_{yy}}{EI} \frac{\partial M_{yy}}{\partial M_o} dS_2 + \int_0^{\pi/2} \frac{M_{zz}}{EI} \frac{\partial M_{zz}}{\partial M_o} r d\theta$$

By constraining the rotation at A to zero, M_o can be written:-

$$M_o(P) = \frac{P(2\pi r^2 + 2Lr + L^2 + \pi L r)}{2(\pi r + L)}$$

Now the equation for the vertical deflection at A can be set up

$$\Delta = \int_0^{\pi/2} \frac{M_{xx}}{EI} \frac{\partial M_{xx}}{\partial P} r d\theta + \int_0^L \frac{M_{yy}}{EI} \frac{\partial M_{yy}}{\partial P} dS_2 + \int_0^{\pi/2} \frac{M_{zz}}{EI} \frac{\partial M_{zz}}{\partial P} r d\theta$$

By letting:-

$$I = \frac{2\pi \bar{R} t^3}{12} \quad ,$$

the stiffness of the whole bellows can therefore be written:-

$$K_{EB} = \frac{\pi \bar{R} t^3 E(\pi r + L)}{n\{6\pi^2 r^4 + 30\pi L r^3 + (24+3\pi^2)L^2 r^2 + 4\pi L^3 r + L^4\}} \quad .$$

A summary of the results of these foregoing axial stiffness formulae applied to the bellows specimen is given below.

Method	K_{EB}(N/mm)	Deviation
Kellogg	26.6	-58%
Turner & Ford	28.1	-55%
Berliner & Vikhman	14.4	-77%
Atomics International	63.0	+0.2%
Haringx	33.1	-47%
Castigliano	25.9	-59%
Experimental Observation	62.9	/
Finite Element	60.0	-5%

3.7 Non-Linearity.

By non-linearity one understands that the relationship between cause and effect is not a linear one. In the context of expansion joints subjected to axi-symmetrical loading, two such non-linear relationships become important:-

(a) The non-linear relationship between stress and strain is of interest if the bellows is to be plastically deformed. This so called 'elasto-plastic' behaviour is beyond the scope of this work, it is, however, looked at in some detail by Snedden (21).

(b) The second non-linearity is that occurring in the elastic region of material behaviour, due to large displacements of the loaded body. When a body is subject to large displacements one may often find that a non-linear relationship between load and displacement exists. In effect the stiffness ceases to be a constant but, instead, becomes a function of the displacement. This form of non-linearity is considered in this text and to distinguish it from any other type of non-linearity it will be referred to as non-linear stiffness throughout the remainder of this report.

Referring again to the formulae for axial stiffness it can be seen that each of these is independent of axial displacement. If the bellows were subject to large displacements (typically, displacements are characterised as being large when they exceed half the thickness of the material) then these equations would not suffice as they stand. If, however, one could accurately predict the changes in geometrical dimensions of the bellows as it displaced, these equations could be used iteratively to generate the non-linear solution. Since the formulae recorded seem to diverge greatly from the experimentally observed stiffness value and since, as has been shown, prediction of geometrical changes with displacement is approximate at best, this approach is not considered. In the following section a finite element method of analysis will be shown that can be used to predict the nature and extent of the non-linear stiffness likely to be exhibited in expansion joints of the type under consideration.

3.8 Finite Element Approach to Stiffness.

When a body is a 'body of revolution' subjected to an axi-symmetric system of loading, the process of finite element modelling may be greatly simplified. Although the body is three-dimensional the geometry and loading are not functions of the circumferential (θ) direction and , as a result, the problem can be analysed using two-dimensional techniques only.

Further model simplifications can be applied with advantage if the assumption is made that each convolution is identical to any other in behaviour and that each convolution has a plane (r,θ) of symmetry through the root. The resulting model is shown in appendix 2.

The type of element chosen to model the bellows under consideration in this study was the PAFEC 36210, eight noded isoparametric curvilinear quadrilateral element. PAFEC describes this element as 'a flat element which carries only loads in its own plane.'.

Such a model must be loaded and restrained realistically with respect to the situation encountered in reality otherwise one cannot expect predictions resulting from the model to be of any practical use. Restraints* were applied in the axial (X) direction along the crest edge and the load was applied by prescribing equal displacements to all nodes along the root edge. Using prescribed displacements, as opposed to point loads, ensured that the edge remained parallel to a radial (r) line and hence forced the condition of compatability with a hypothetical neighbouring convolution.

The stiffness of the whole bellows was then determined by summing the nodal reactions along the root edge and applying the following relationship.

$$K_{EB} = \frac{R}{2n\Delta}$$

3.9 Finite Element Approach to Non-Linear Stiffness.

The finite element methods and in particular the finite element software generally available are predominantely devices by which to solve large sets of linear equations. These are, therefore, not able to cope directly with non-linear equations. This, however, does not preclude the solution of non-linear equations by a step-wise iterative approach. The method involves incrementing the total load to be applied in suitably chosen small steps. After each step a reformulation of the element and global stiffness matrix takes place based upon information from the previous step. In PAFEC software one must include the statement LARGE.DISPLACEMENTS in the control module and include a new module called INCREMENTAL in the data file. For all the finite element data files see appendix 2.

3.10 Review of Stress Formulae.

It was stated in the introductory section of this report, that no generally applicable equation can be formulated for the stress distribution in a set of bellows. This is, of course, understandable since bellows

*See figure 3.81

vary enormously both in terms of geometry and in the types of loading to which they are subject. It was discovered through the literature survey that two main approaches have been employed in finding or approximating stress distributions in axi-symmetrically loaded expansion joints. Which approach has been employed in a particular case depends on the value of certain form functions and a validity range for these form functions was shown.

Both approaches require large amounts of tedious algebraic manipulation. Such manipulation is ideal for a computer but fraught with the possibility of error if performed manually. Further, both approaches are based on simplifying assumptions which are bound by their very nature to make the resulting distribution questionable to some degree.

With this in mind and, since we are attempting to validate finite element methods by direct experimental observation, it seems a fruitless exercise to carry out such an analysis on the bellows specimen. Such analysis, therefore, is not carried out in this report. This, of course, does not preclude a comparison being made between the finite element results generated in this report and those of the many workers in this field of mechanics.

Acknowledging the intractable nature of a generalised thoery for bellows expansion joint stress distribution, engineers in the United States have formulated a set of basic equations based upon empirical correction factors by which the designer may predict the maximum stress in a expansion joint. These equations, and their respective graphical correction factor plots, are shown for reference in the appendix to this report.

3.11 Finite Element Approach to Stress Distribution.

I have discussed the modelling of the bellows in finite elements in a previous section and, therefore, will not reiterate what has already been noted there.

The element type used in this model gives not only outputs for the two principal stresses in the plane of the element, but also the out of plane circumferential (hoop) stress for axi-symmetric models. Given these three stresses one has available a complete picture of stress distribution in a convolution. It is noted in the experimental programme section of this report that the three principal directions associated with a axi-symmetrically loaded convolution are in fact meridional, circumferential and transverse. This implies conveniently, that no additional manipulation of the two in-plane stresses is necessary after the output from PAFEC is received.

With any finite element model one must spend time in refining the mesh. Where the rate of change of stress with distance is large one must clearly have a grid consisting of smaller elements than at a point where this derivative is small. In the case of the convolution model we have no point loads, (although pressure application in PAFEC is via point loads), and, hence, no localised loading effects need to be considered with respect to mesh refinement. The choice of mesh density was made on the basis of experimental results from Turner and Ford (22), where it was clearly shown that the rate of change of stress with distance in both the transverse and meridional directions was large.

In finite element modelling one can estimate the accuracy of one's results by investigating the discrepancy in magnitude of stress at a particular node as evaluated from all connecting elements. Alternatively one can consider the maximum residual stress normal to an unloaded surface as being approximately equal to the difference between one's results and a ideal 'totally refined' model with zero residual stresses in this treatment.

This latter method was used. Residual stresses normal to the unloaded external surface were, at all nodes, at least two orders of magnitude below the principal stress values recorded. This accuracy was considered acceptable.

It should be noted here that the accuracy talked about above is the accuracy between one's own imperfect model and a hypothetical 'totally refined' model. If it is in addition inaccurate in terms of geometry and/or loading then the generated results will have a considerable inherent inaccuracy: an inaccuracy sufficient to render them of no validity.

4. DISCUSSION.

Let us begin this discussion with a summary of the work that has been carried out. The primary objective of this work was to attempt to validate the finite element method as a design tool for predicting the static, linear-elastic, behaviour of axi-symmetrically loaded corrugated bellows expansion joints. To this end experimental work was carried out on a specimen bellows and a finite element model was set up and run.

In addition to the abovementioned work, a number of equations for the axial stiffness of a bellows expansion joint, already formulated by workers in the field, were evaluated for the bellows specimen data. This apparent diversion from the main objective was provided to highlight the need for an accurate and general method by which to determine data concerning the strength of bellows. I propose to deal first with the results from these equations. For ease of reference these results are shown below.

Method	K_{EB}(N/mm)	Deviation
Kellogg	26.6	-58%
Turner & Ford	28.1	-55%
Berliner & Vikhman	14.4	-77%
Atomics International	63.0	+0.2%
Haringx	33.1	-47%
Castigliano	25.9	-59%
Experimental Observation	62.9	/
Finite Element	60.0	-5%

In addition to applying these formulae, the author applied Castigliano's first Theorem to the bellows. This was again done purely out of interest and because the fact that the implicit assumptions used in this model were much the same as those used in many of the other formulations. As the reader can see, the result from this analysis was comparable with those of the other analyses. If, for the time being, we ignore the N.A.A. formulation, we can make the statement that all formulations provide stiffness values that are comparable, in terms of magnitude, with the experimentally observed value. Given that this is so, not one of these formulations provides a result that is usefully near the actual value. All formulations provide answers smaller than the observed value. This deviation is probably due to the premise inherent in most formulations that the bellows can be considered as a straight corrugated beam whereas, in reality, additional axial stiffness is gained from the bellows being a corrugated shell of revolution. The clear conclusion that can be drawn from the above observations is that the assumptions applied in the formulation of these equations are not, on the whole, valid in practice. When faced with this sort of problem, namely the need for an accurate solution but the availability of only an inadequate theoretical analysis, engineers often resort to the use of empirically corrected formulations. The N.A.A formulation is such an example and, as can be seen from the results, it provides an answer of good accuracy. The N.A.A standards are, understandably, widely employed by both bellows manufacturers and bellows users in this country (evidently the U.K. has no such standard) and on this basis I would recommend that V.S.E.L. should research and evaluate this standard should they wish, in the future, to carry out design in this field.

The work carried out in this project is an attempt to step back again from empiricism into the theoretical domain of engineering mechanics. Many of the references quoted in this report show work that was carried out before the use of computers and in particular before the finite element method became generally accepted in engineering. This 'step back' is therefore both desirable and appropriate in a theoretical science.

Having thus highlighted the importance of this work it is now time critically to analyse the experimental and finite element work that contributes the major part of this report.

Before any comparison can be made or conclusions can be drawn on the results one must possess a full understanding of the nature and extent of any experimental errors and the validity of any assumptions on which the finite element model is based. The following sections deal with this aspect of the work.

4.1 Analysis of Experimental Errors.

In this section I wish to identify and describe three potentially significant sources of experimental error, and then to discuss the nature and extent of these errors The three sources of error can be identified as:-

1. Incorrect position/orientation of strain gauges,
2. Amplification of bending stress due to adhesive thickness,
3. Size of strain gauge relative to strain gradients.

1. Incorrect Position/Orientation of Strain Gauges.

In many strain gauge installations the misalignment and/or mis-orientation of gauges is inexcusable. Traditional methods of marking out with a scribe are easily appied in the majority of cases. With the bellows, however, marking out with a scribe was clearly unsuitable because of the damage it would cause to such a thin walled vessel and the fact that the bellows, by its very nature, is a flexible body. Strain gauge installation was further hampered by the shape of the bellows surface. The surface both at the root and at the crest of convolutions curves in two mutually perpendicular directions. For these reasons the author is not able to guarantee the exact positioning of the strain gauges. However, a visual inspection of the installation shows that there is no great deviation in either their position or their orientation. See plate 4.

2. Amplification of Bending Stress Due to Adhesive Thickness.

In general, the strains at a body's surface are the vector sum of bending strains and direct strains. Strain gauge bridge circuits normally read the total of these two strains. With the measurement of bending strains we have a source of error. Because the gauge is located on a layer of adhesive, the measured bending strain will be amplified over the actual strain present at the surface. The extent of this source of error may be investigated by working in the stress domain and applying the simple bending theory.

$$\frac{\sigma_{BA}}{\sigma_{BM}} = \frac{t}{t+2\delta t}$$

σ_{BA} = Actual Bending Stress

σ_{BM} = Measured Bending Stress

δt = Adhesive Thickness

t = Bellows Thickness

From this relationship we note that where the material thickness is large in comparison to the adhesive thickness, the presence of the adhesive represents an insignificant source of error. However, with the bellows in question, where the adhesive thickness is comparable to the material thickness the adhesive film most certainly represents a significant source of error.

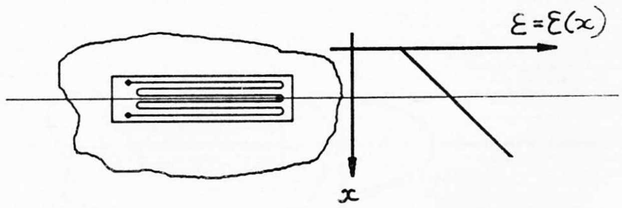
Strain gauge manufacturers recommend a adhesive thickness of 0.0001". Taking, therefore, a worst case of say 0.0005" the bending strain measured by the strain gauge would be amplified in the order of 5% over the actual bending strain.

To estimate the error in recorded strain one would clearly have to know the relative magnitudes of the bending strains and the direct strains at a point of interest ie $\sigma_D/\sigma_B = ?$. These are, of course, not known so we can only conclude that, for the worst case possible where the ratio $\sigma_D/\sigma_B = 0$, the maximum error due to bending strain amplification would be of the order of +5%.

3. Size of Strain Gauges Relative to Strain Gradients.

This is a source of error that is again inexcusable in theory since one should always use strain gauges that suit the task in hand. Since this theoretical ideal was not possible for reasons already mentioned it is necessary to discuss this source of error.

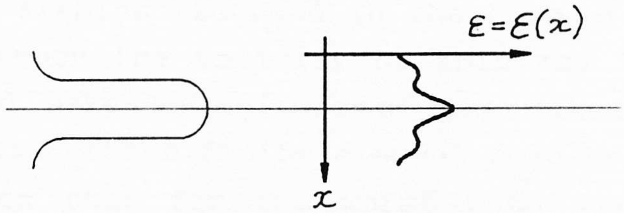
The reason for error here is that the strain gauge is not reading strain over an area of constant strain but over an area of changing strain. As a result the strain gauge is ,in effect, giving a value that represents an average strain in the direction of interest over the area of the gauge. One can envisage strain fields where this average strain reading will equal the actual strain at the centre line of the gauge. An example of this type of strain field is given below.



Typical Strain Distribution where Actual Strain at Gauge Centre is Equal to the Measured Strain.

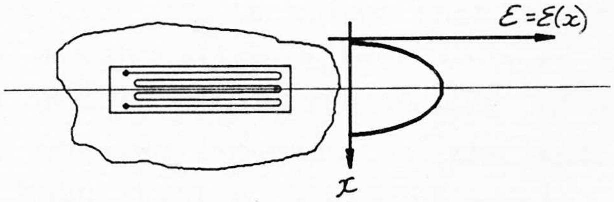
Figure 4.11

In the case of the bellows, however, this is not the case. One of the assumptions associated with axi-symmetric loading of bellows is that the strain field in the bellows will be a cyclic field with a wavelength equal to one convolution. Further, by symmetry, we may assume that the strain field in half a convolution will be a 'mirror image' of that in the other half of the convolution. It follows from this that positions at the convolution crest and root will have strains (meridional and circumferential) that are local maxima or local minima. This idea is shown below.



Strain Field Showing Symmetry and Points of Local Extrema.

Figure 4.13



Typical Bellows Strain Distribution where Actual Strain at the Gauge Centre is Not Equal to the Measured Strain.

Figure 4.12

This time the average strain will not equal the actual strain at the centre line of the gauge. If we were to make a 'triangular approximation' of the strain field one could estimate the likely extent of this source of error. From this qualitative analysis we can conclude that for points at the root and at the crest of a convolution the measured strain will always be less in magnitude than the actual strain at the point in question. Further, since the meridional strain gauges have their length orientated with the direction of changing strain whereas the circumferential strain gauges have their length orientated with a direction of constant strain it is likely that the strains recorded in the meridional direction will suffer more from this source of error than the strains recorded in the circumferential direction. Although the validity of this may be questionable, one could make an esimate of the likely error through an analysis of the finite element results. Such an analysis yields that for the worst case viz strain in the meridional direction with the largest local stress gradient, the error could be of the order of -8%

Applying the above reasoning to the experimental errors one can envisage the possibility of:-

a) an unquantifiable error. (cause 1)
b) a +ve error (cause 2)
c) a -ve error (cause 3)

Although the exact value of these errors is unobtainable in practice it is clear that there will, to some extent, be a cancelling effect because of the opposite signs of b and c above. The extent of cancellation is, of course, unknown however, if the author were forced to make a quantitative judgement on the errors present in the experimental results he would say ±5%.

4.2 Analysis of Finite Element Work.

In this section of the discussion the author wishes critically to analyse the finite element method and in particular the application of this method to the field of bellows design.

Two principles must be stated at the outset since they are inherent in the finite element technique.

1. The results generated by a finite element model can only be as accurate as the model used.
2. If one can totally refine the mesh of a particular model one will end up with accurate results for that model.

This first statement deals with the assumptions made in modelling and the reality of any loads or constraints. We have already established (section 3) that the selected loads and constraints are realistic. It remains, therefore, to look at the assumptions implicit in modelling. These are as follows:-

1. That the material thickness is constant around the convolution.
2. That the material is, at all points, homogeneous and isotropic.
3. That before any application of load the material of the bellows is unloaded.

These assumptions, though necessary, are all invalid to some extent and the reason for this lies in the method of manufacture of bellows.

The 'raw material' for a set of bellows consists of a sheet of material which, due to its manufacture (ie rolling), will exhibit a certain degree of anisotropy. The sheet material is then again rolled to form the bellows shell. Again we can see that the rolling process is likely to impart directional properties to the material. Although their effects are not known quantitatively

It is reasonable to assume that the extent of anisotropy in the final bellows shell will be dependent on whether or not the bellows shell is rolled in the same direction of rolling of the original sheet material. In addition to this anisotropy, the rolling process to form the bellows shell is also likely to result in a non-uniform material thickness around each convolution.

After rolling, the bellows shell will be welded along a meridional seam. As with all welding processes, there will exist an area around the weld, called the heat-affected zone, which exhibits different material properties from the unaffected material. We should note here that heat treatment would tend to reduce this effect. However, according to my sources, bellows manufacturers tend not to heat treat the bellows after welding because they wish to retain the beneficial material properties imparted to the material on rolling. The heat-affected zone is therefore likely to remain a significant factor. It is also clear that, depending on how the joint to be welded is forced together, there may exist a certain amount of pre-load. The author noticed, when cutting the bellows up for material samples, that there was a certain amount of pre-tension in the circumferential direction of the convolutions.

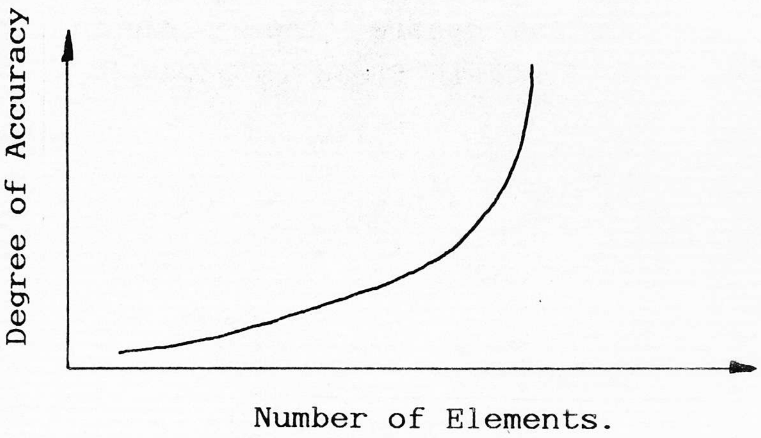
In addition to these processes, it will be likely during manufacture, especially during the rolling process that some surface damage may have occured. With such a thin walled vessel as a bellows this may also affect the ultimate strength.

This discussion on the manufacture of bellows has highlighted the fact that in practice many of the assumptions on which any finite element model can be based are in reality invalid. Unfortunately we are unable to quantify these effects and even were such quantification possible, we could not easily include it in the model.

As an aside, it is interesting to note that as computational engineering becomes more advanced it may ultimately be possible to simulate the manufacturing process such that all these factors could be taken as input data to a finite element analysis.

In conclusion, then, we are forced to accept a certain degree of error by using a finite element model that is based upon simplifying assumptions that are, to some extent, invalid. This point shows the clear advantage of an empirical approach where all these 'unquantifiable' sources of error can be included in the empirical correction factor. However, with the finite element method we are able to obtain a picture of stress distribution (not available with current empirical methods) and, even accepting erroneous effects, we should still be able to accept the general pattern of stress distribution as truly representing the 'real life' situation.

The second statement deals with the fact that within the assumptions on which the model is based the particular model used may not be perfectly refined. As a consequence, the results generated by a particular model may be less than correct when compared to those generated by the same model but with a higher degree of refinement. This can be shown diagramatically although the exact functional relationship is, clearly, not known.



Because in reality any body possesses an infinite number of degrees of freedom, any finite element analysis, where by definition there are assumed a finite number of degrees of freedom, will result in an answer that is approximate. The art of finite element analysis is to balance model accuracy, which is related to the number of elements used, and computational cost. One must realise that finely meshing regions where the stress distibution gradients are small is a waste of computational effort whereas coarsly meshing regions where the stress gradients are high will introduce appreciable inaccuracies.

The model chosen was thus a balance of these factors. Methods exist for approximating accuracy of models and these have been outlined in the theoretical section of this report. The reader can see, by inspection of the finite element graphical results (graphs 7 and 8), that the results provide smooth curves and this is an indication of the accuracy of mesh refinement.

4.3 Commentary on the Results.

Having looked at the integrity of the experimental and the finite element results it is now necessary to make comparisons and to draw conclusions from these results.

Looking first at the axial stiffness results, we see that the finite element result compares extremely well with the experimentally observed result. The experimental results were taken for a maximum axial displacement of 12mm and, as can be seen from the graph, to all intense and purpose they are linear.

The finite element results (graph 6) predict 12mm axial displacement to be on the limit of linear behaviour One might on this basis conclude that the finite element model is an accurate method by which to predict axial stiffness. However, as with any theoretical method, one can only make such a conclusion if the theory holds true for all cases. The author therefore tested the axi-symmetric finite element model against a set of experimental results observed by Turner and Ford (22) In this work the writers quote geometric and material properties for a number of bellows and an experimentally observed axial stiffness is recorded for each bellows. This model is shown in appendix 8 and, as can be seen the experimental and finite element results do not compare anything like as well as those exhibited in this report for the specimen bellows. The results of this model are 30% too small.

There may be a reason for this discrepancy. In the manufacture of Turner and Ford's bellows the shells were much thicker and the manufacturing process was significantly different from that used for the bellows specimen used in this report. It would appear that the bellows used by Turner and Ford were manufactured from half-convolution shells welded together in the circumferential direction.(The bellows specimen used in this project were welded in the meridional direction only.)

The resulting heat-affected zones would have much more effect on the stiffness than those in the bellows specimen of this report. If one accepts that the material in the heat-affected zone is stiffer than the surrounding material then this would explain the higher stiffness that was observed than was predicted by the finite element model. This check on the axi-symmetric finite element model shows clearly that the stiffness of a bellows is highly affected by the method of manufacture and, more importantly emphasises the fact that a finite element model that does not take into account the method of manufacture is liable to give erroneous results. The author feels, therefore, that although stiffness can be approximately predicted an accurate prediction of this parameter through finite element analysis is not possible since there are changes in the material of a bellows due to the manufacturing processes used that can not readily be included in the finite element model.

Application of the N.A.A. empirical stiffness formula to Turner and Ford's bellows results in a figure that is in close agreement with the finite element result. This again reinforces the authors suspicion that manufacturing methods greatly affect a bellows' stiffness. The empirical model will, by its very nature, only produce accurate results if the empirical correction factor is appropriate to the method of manufacture of a particular bellows. In this case it appears that the empirical correction factors of the N.A.A. suit the bellows studied in this report but not those studied by Turner and Ford.

One final point that may be made regarding bellows stiffness is the extent of the non-linear stiffness encountered with large axial displacements. The finite element model, although not backed up by experimental observation, show that with large axial displacements the stiffness can not be considered constant. Indeed, the stiffness almost halved as the displacement exceeded 32mm. This non-linear prediction confirms the 'rule of thumb' statement, made earlier in the report, that the stiffness of a body remains predominantly linear for displacements up to half the material thickness of that body.

Turning our attention now to the results in terms of stresses and stress distributions, the author would like to start this part of the discussion with the statement that the finite element results give a stress distribution around a convolution whereas the experimentally observed results only show point stress values. The reason for this has already been noted and is, therefore, not reiterated here. One must, though, be careful not to read too much into the stress distributions resulting from the finite element analysis for the simple reasin that we do not have experimental observations with which to support any conclusions that may be drawn. Despite this, it is perfectly legitimate to make a qualitative analysis of these stress distributions if one can compare ones results with, for example, the experimental results of some other worker in the same field. To this end the author has included some experimental results observed by Turner and Ford (22) in appendix 8.

The reader is referred to graphs 7 and 8 for this part of the discussion. With the exception of one experimental point (graph 8 circumferential stress at the convolution root) there appears to be a constant underestimation of the magnitude of the discrepancy between the finite element results and the experimentally observed behaviour. This constant difference is understandable given the errors in the experimental design and given the possible invalidity of assumptions in the finite element model. What is less understandable is the reason why one experimental point should not exhibit this same error. It was noted during the pressurised bellows test that the strain gauge at this point ($\varepsilon_{\theta R}$) was not returning to zero when the bellows was unloaded. This was at first dismissed as creep however, as the graphical results (graph 3) show there may, in the event, have been something more than creep taking place at this strain gauge location. The author suspects, with hindsight, that during the tests undertaken with the bellows pressurised, the root circumferential strain gauge was loosened. He would therefore suggest that the results for internal pressure taken at this position be treated with a certain degree of circumspection.

Let us now make some comments on the stress distributions found in the bellows, noting again that there is no experimental data with which to back these distributions up. Inspection of the results, graphs 7 and 8, shows that the inner meridional stress distribution closely approximates to a mirror image, about the abscissa, that of the outer meridional stress distribution. This observation is true for both axial load and internal pressure loadings. This kind of stress distribution resembles closely that due to bending ie stress at inner surface equals the negative of that at the outer surface. Further, since the symmetry lies about the abscissa one might reasonably assume that the neutral axis for a convolution is at a depth of half the material thickness.

Unlike the meridional stresses where the stress distributions appear to be predominantly due to bending, the circumferential stress distributions do not lend themselves so easily to such a simple analogy. Referring now to Turner and Ford's experimentally observed stress distributions, we can see that for the loading case of axial displacement (Turner and Ford did not include a study of pressurised bellows) the general shape of the curves and the characteristics thus displayed compare well with those found in this report (graph 7). We may, therefore, conclude on this basis, and this basis alone, that the finite element method is a capable tool for the prediction of stress distributions in axi-symmetrically loaded bellows.

The reader may have noticed that graph 9 is basically a repetition of graph 8. However, graph 9 shows the values of circumferential stress that would be obtained if the thin-cylinder theory had been used. The thin-cylinder theory will, obviously, produce a linear result if the actual bellows radius at a point is used or, alternatively, a constant result if the mean radius of the bellows is used for all points on the meridion. One can draw little in the way of conclusion from this analysis since the systems of loading (although both axi-symmetric) are incomparable to each other. However, it is possible to say that the thin-cylinder theory produces results that are of the correct order of magnitude when compared to those obtained by finite element analysis. It should be noted that this conclusion is only applicable to the bellows in question since it has not been tried and tested with the results of other workers.

Whilst on the subject of internally pressurised bellows it would seem to be appropriate to say a few words about the phenomenon known as squirm.

The phenomenon known as squirm, although not the main concern of this project, is an important mode by which a bellows may fail in service. Haringx (18) provides a good theoretical basis for this topic and the reader who is interested in this aspect of bellows failure is referred to his work. A qualitative description of this important phenomenon is given here for reference.

One can compare squirm in thin cylinders or bellows to the buckling of struts. Squirm in a bellows is initiated by a small lateral deflection of the unsupported part of the bellows. This lateral deflection causes one side of the bellows to stretch and the other side to compress. The internal pressure then has a greater area of application on the stretched side than on the compressed side and the lateral deflection therefore increases. This phenomenon is known as squirm. From Haringx's work it can be seen that the value of the ratio length of unsupported bellows/ mean radius is an important parameter in designing to avoid the possibility of squirm. This ratio is analogous to the slenderness ratio used in the theory of buckling of struts.

[illegible].4 FURTHER WORK.

The main part of this discussion has now been completed we have critically analysed the experimental and finite element results and thus provided a firm footing on which the conclusions may be laid. To bring this discussion to a logical conclusion the author would like to suggest two possible approaches in which the finite element results from an axi-symmetric analysis of bellows could be usefully employed.

Firstly, accepting the finite element method as a suitable predictor of actual behaviour, one could look at the possibility of optimising bellows design. Such an optimisation process could take several forms, one could, for example, investigate the relative merits of different convolution geometries in terms of stiffness or, perhaps, in terms of component stress values. In such a study one would need to generate finite element models of all the different convolution geometries and this process could be aided if a suitable computer programme that would take bellows specification data as input and output a finite element data file were available. The first step to be taken in writing such a programme would be to represent mathematically each convolution geometry that is to be studied. For the simpler geometries, those consisting of piecewise linear segments, this process is relatively simple. For boundary curves that are non-linear eg parabolic or elliptic, however, this process may be quite complicated in terms of mathematics. The complication arises because of the constraint, implicit when modelling a constant thickness bellows, that the two boundary curves must be displaced, relative to each other, by a constant normal distance. The author spent some time investigating this problem of representing the boundary curves of a non-linear convolution geometry in a mathematical format and the results of this investigation are shown in appendix [illegible].

The second of the possible routes for further work is, in reality, the next logical step to be taken after the finite element analysis, namely the prediction of elastic failure in bellows. In the discussion that follows, the author would wish to remind the reader that in practice bellows are often loaded into the plastic region of material behaviour, Sneddon (21) devoted three years research into the so called 'elasto-plastic' behaviour of laterally loaded bellows. As a result it should be realised that elastic failure may well not be the only criterion of failure used in bellows design.

The method I wish to describe considers the circumferential and meridional stresses only although the same procedure could be applied to the transverse stress distribution if it were known.

If one accepts the finite element analysis as an acceptable predictor of stress distributions in axi-symmetrically loaded bellows and if, also, one accepts that the principle of superposition holds for the loading cases of axial displacement and internal pressure then the following method may be found useful.

For each of the four surface stress distributions predicted by the finite element analysis one can employ the method of Numerical Harmonic Analysis. This is a numerical method whereby a continuous mathematical description (Fourier Series) may be obtained for a set of discrete points. If this method were applied one would obtain two sets of four equations. The first set being the surface stresses as a function of meridional position and axial displacement and the second set being the surface stresses as a function of meridional position and internal pressure. Adding these results would yield a set of four 4-dimensional equations for the surface stresses as a function of meridional position, axial displacement and internal pressure.

Applying calculus to the resulting equations one could find local extrema within a given working range of axial displacement and internal pressure. The method of numerical harmonic analysis is applied to the outer surface circumferential stress in appendix 9.

Should the finite element method be adopted as a tool for the design of bellows then it may well prove advantageous to computerise this aforementioned method.

5. CONCLUSIONS.

In the light of the work carried out in this report, the author is able to offer the following conclusions:-

1. The finite element method has been found to be satisfactory as a technique for obtaining an order of magnitude prediction of axial stiffness and stress distributions in axi-symmetrically loaded bellows.

2. For an accurate value prediction of axial stiffness and stress distribution in axi-symmetrically loaded bellows, the finite element method has been found lacking. It is felt that the finite element methods inability to predict accurate values for these parameters does not lie in any inadequacy of the method but is due in the main to non-ideal material and geometric properties imparted to the material of the bellows during manufacture. Such variables cannot be accomodated in any simple finite element model with the technology of today. This does not, of course, preclude such a possibility in the future.

In the light of these observations it can be seen that the finite element method does have a useful place in the design of bellows at least in terms of predictions For accurate value predictions it is clear that the finite element method is not, on its own, sufficient. It appears likely to the author that through the use of empirical data obtained from experimental work on a series of bellows, all manufactured in the same way, accurate predictions of behaviour might be made which could greatly assist the design of 'families' of bellows if the finite element method be used to interpolate and extrapolate.

6. RECOMMENDATIONS.

On completion of this work it was possible to make the following recommendations:-

Recommendations to V.S.E.L.

Based on the conclusions of this report the author would recommend that V.S.E.L. should adopt the finite element method as a useful predictor of behaviour in axi-symmetrically loaded bellows. However he would also wish to state clearly that this method can not be used alone in the design of bellows but that an empirical based method (eg the Standards of the Expansion Joint Manufacturers Association) must be used in conjuncture with any finite element analysis that may be employed.

Recommendations to Liverpool Polytechnic.

The student undertaking a project of this nature is at the mercy of a variety of factors which may render his work difficult or even impossible to complete to his satisfaction. In the case of this particular project, delays occured which, though small in themselves, made it impossible to proceed at an orderly pace and resulted in the frustration of 'out of sequence' working.

Perhaps the first step in any M.Eng. project should be an exercise in critical-path analysis performed jointly by the student and his supervisor. If this were done, liason with the industrial sponsor could become more effective and provision of materials and necessary technical assistance by the polytechnic could be properly planned and sequenced.

Recommendations for Further Work.

Although this project work appears to have reached its logical conclusion the author can see a number of routes in which further work could usefully be carried out in the field of bellows design. These are listed below.

1. A feasibility study could be made with a view to ascertaining whether the technology exists to include the 'non-ideal' material and geometrical properties imparted to the bellows material during manufacture in a finite element analysis. If this proved to be the case an attempt to apply the technology would be beneficial. If not, one could look into the creation of such a technology.

2. If finite element analysis is to be widely used in bellows design, it might well be beneficial to write a computer programme which, based on bellows specification data as input, gives as output a PAFEC data file.

3. Given the validity of the finite element method, a study into the optimisation of bellows design might produce interesting results. For example one could perhaps evaluate the relative merits of the different convolution geometries generally employed by manufacturers or, indeed, look at the possibility of a new convolution geometry.

4. The generation of a full bellows finite element model would provide a method by which the non-axi-symmetric loading regimes of lateral load and/or angular deflection could be tested and their respective characteristics noted.

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FIGURES.

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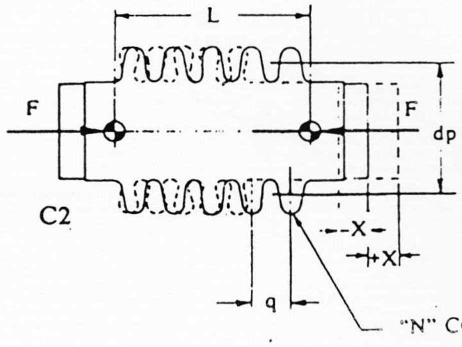
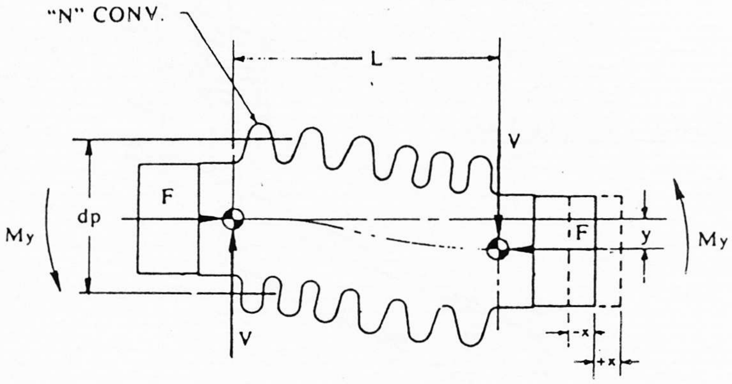
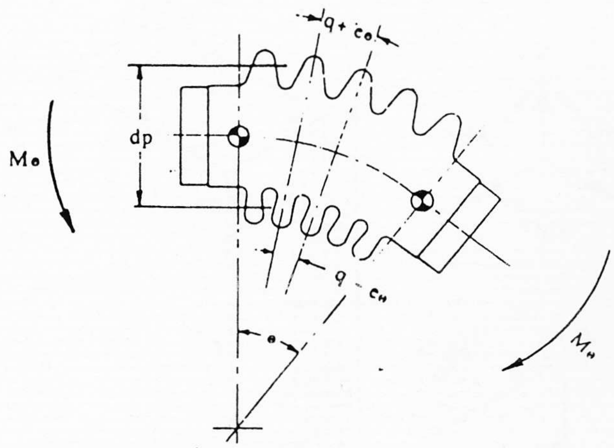


Axial Movement



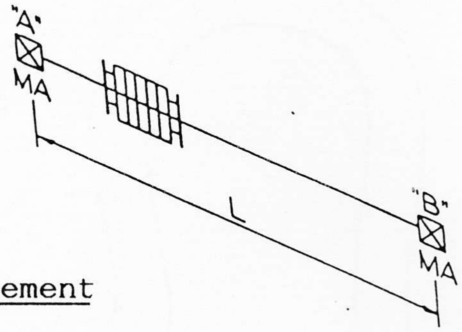
Lateral Movement.



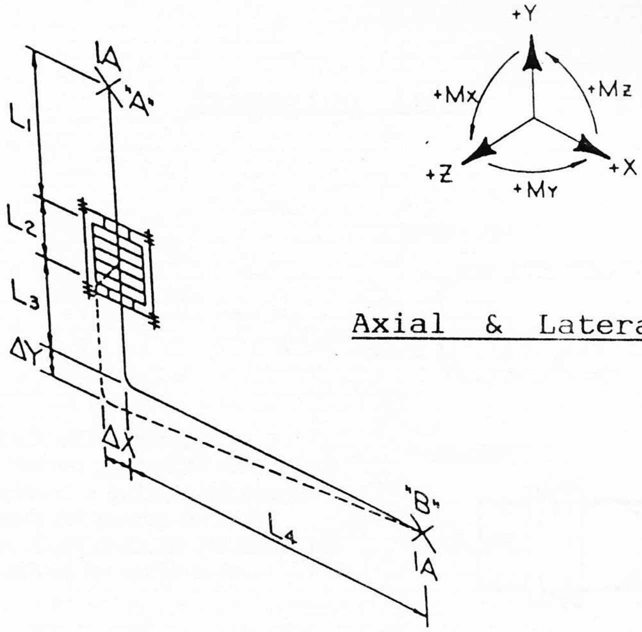
Rotational Movement

Types of Movement in Bellows.

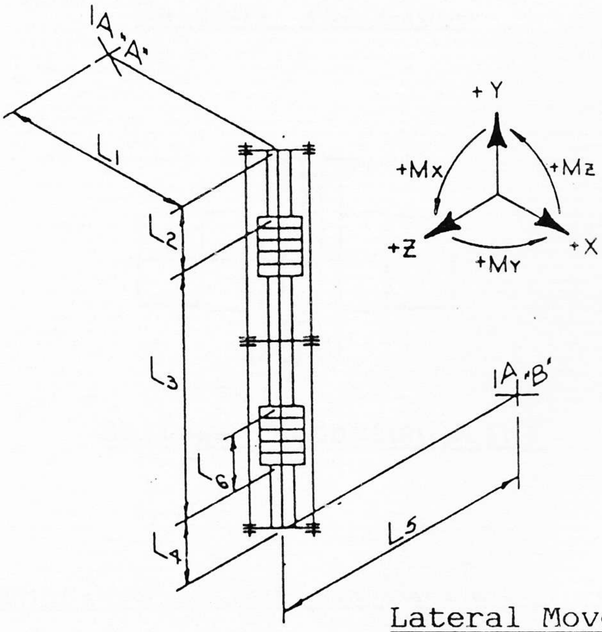
Figure 1.1



Axial Movement



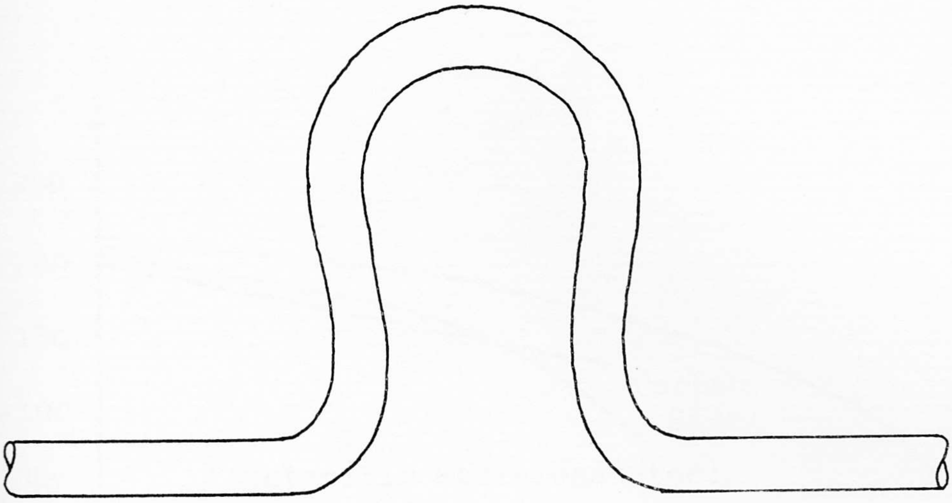
Axial & Lateral Movement



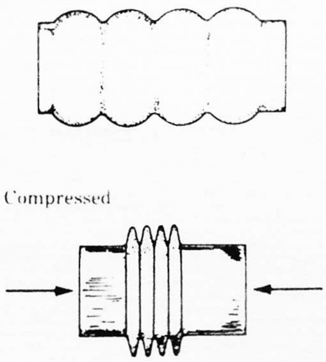
Lateral Movement in Two Planes

Practical Pipe Systems Causing Specific Movements Due to Thermal Expansion.

Figure 1.2

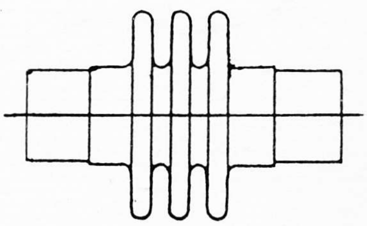


Expansion Loop



Sequence of processes for manufacturing metal bellows. A straight tube is first bulged by internal pressure. It is then compressed longitudinally to collapse the bulged sections and thus form bellows. Such products are useful for flexible connectors, such as for gas lines for appliances.

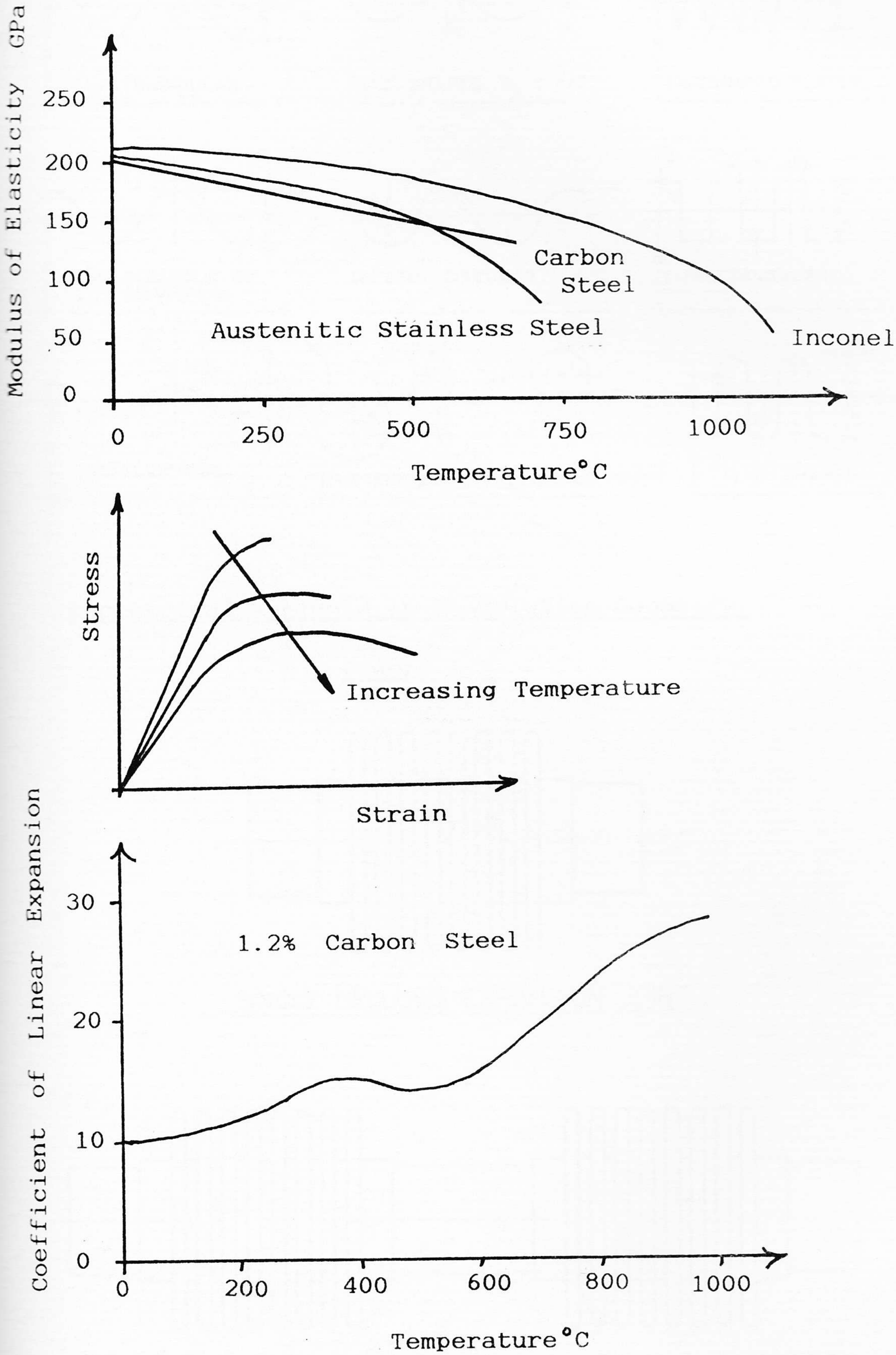
Bulging Technique



Bellows Expansion Joint

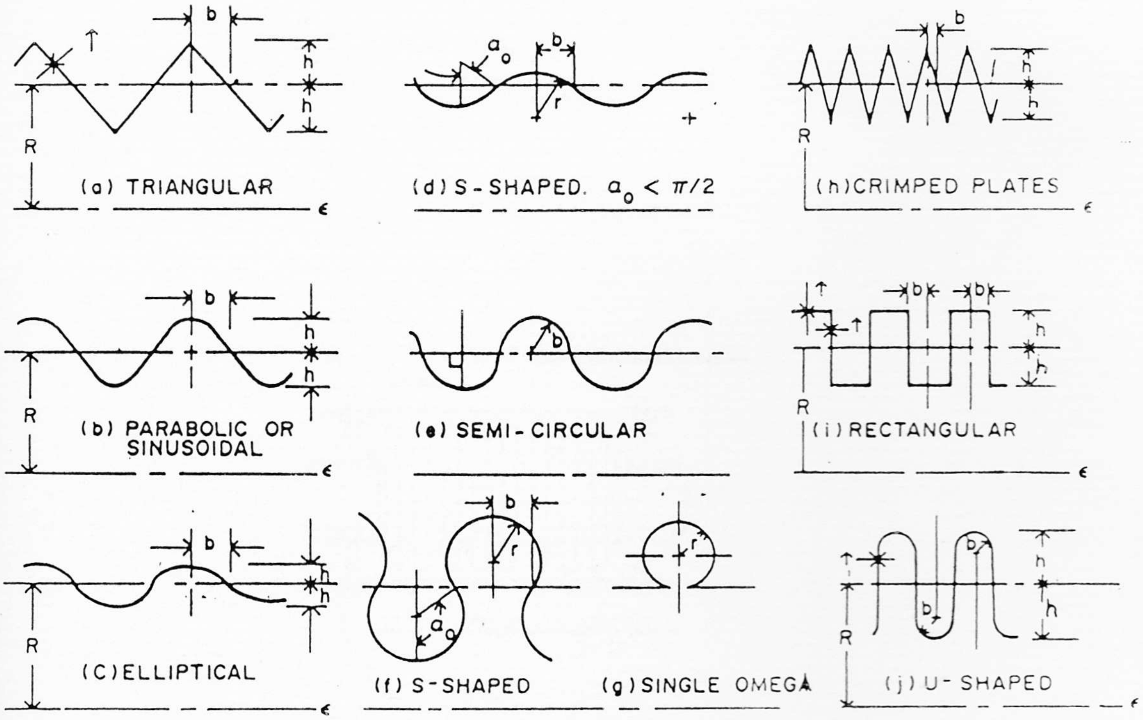
Alternatives to Bellows Expansion Joints.

Figure 1.3



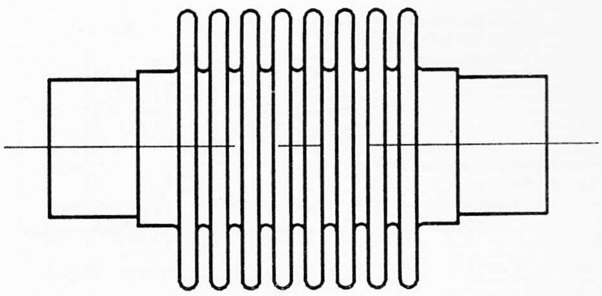
Material Properties as a Function of Temperature.

Figure 1.41

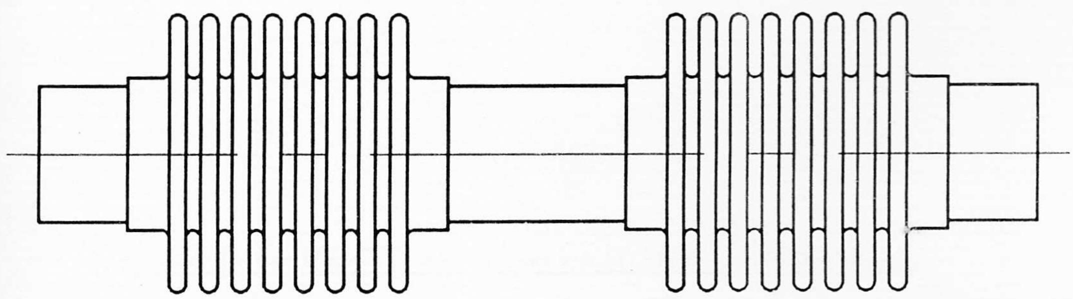


Geometrical Varients in Convolution Geometry.

Figure 1.5

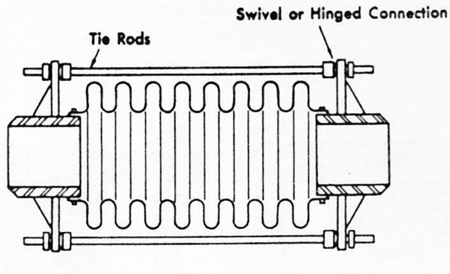


SINGLE BELLOWS EXPANSION JOINT.

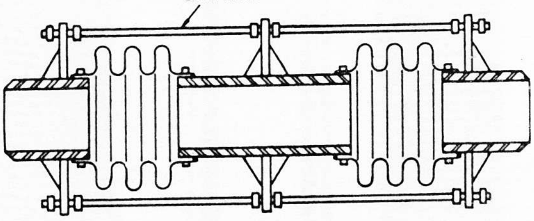


UNIVERSAL EXPANSION JOINT CONSISTING OF TWO SINGLE EXPANSION JOINTS CONNECTED BY A COMMON CENTRAL SPOOL.

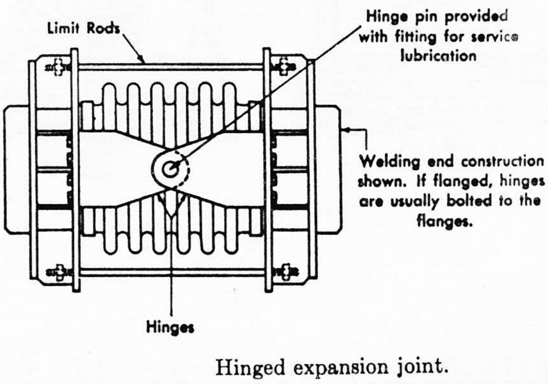
Figure 1.51



Tied expansion joint.



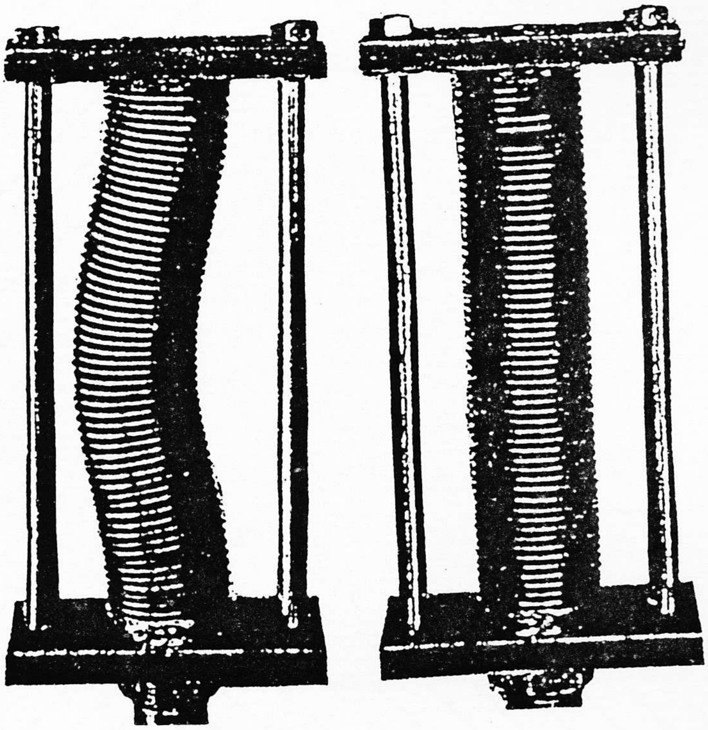
Universal type expansion joint.



Hinged expansion joint.

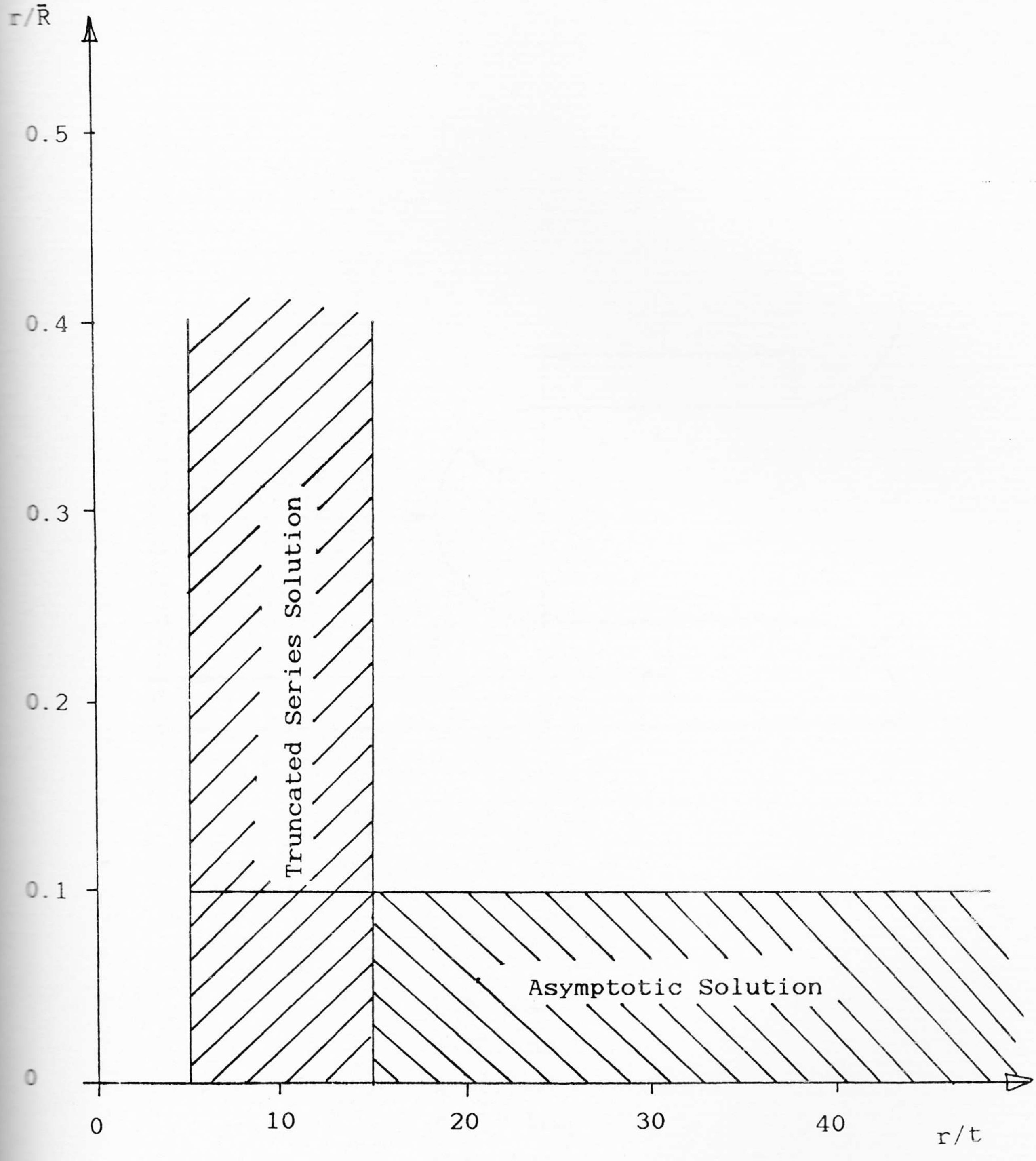
Associated Bellows Restraining Components.

Figure 1.6



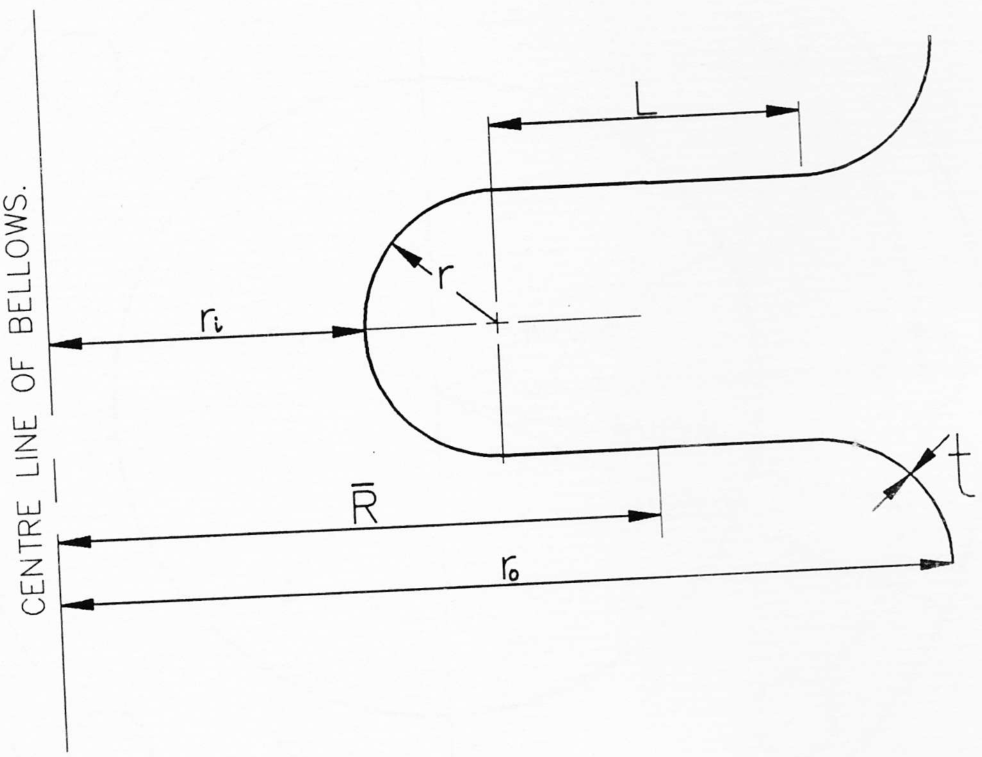
The Phenomenon Known as Squirm.

Figure 1.7



Range of Validity of Simplified Solutions.

Figure 1.9



$t = 0.45mm$

$r_i = 70.5mm$

$r_o = 88.5mm$

$r = 5.825mm$

$L = 6.35mm$

$\bar{R} = 79.5mm$

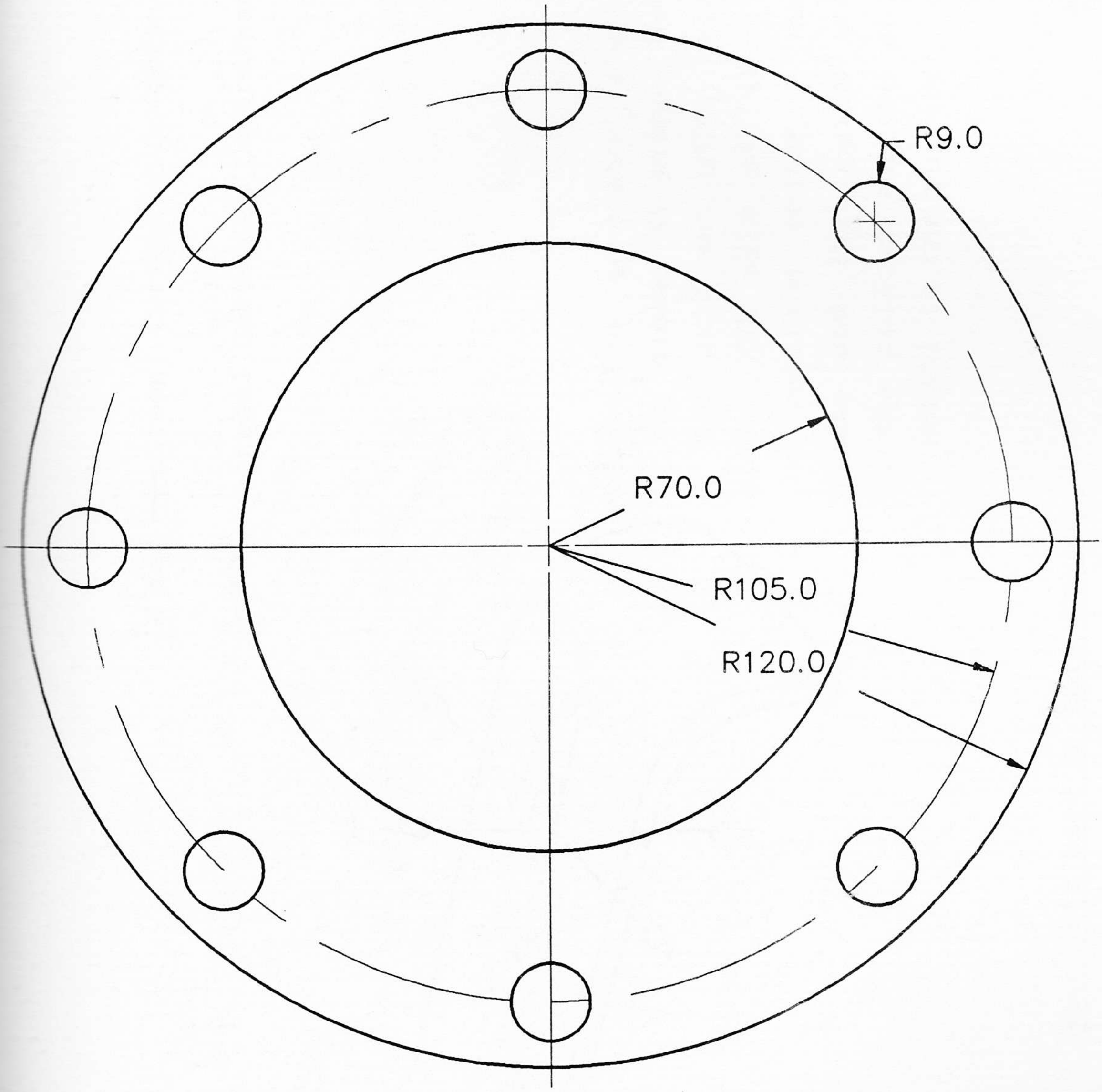
$E = 207GPa$

$\nu = 0.278$

$n = 16$

Full Specification of Bellows in Terms of Geometrical and Material Properties.

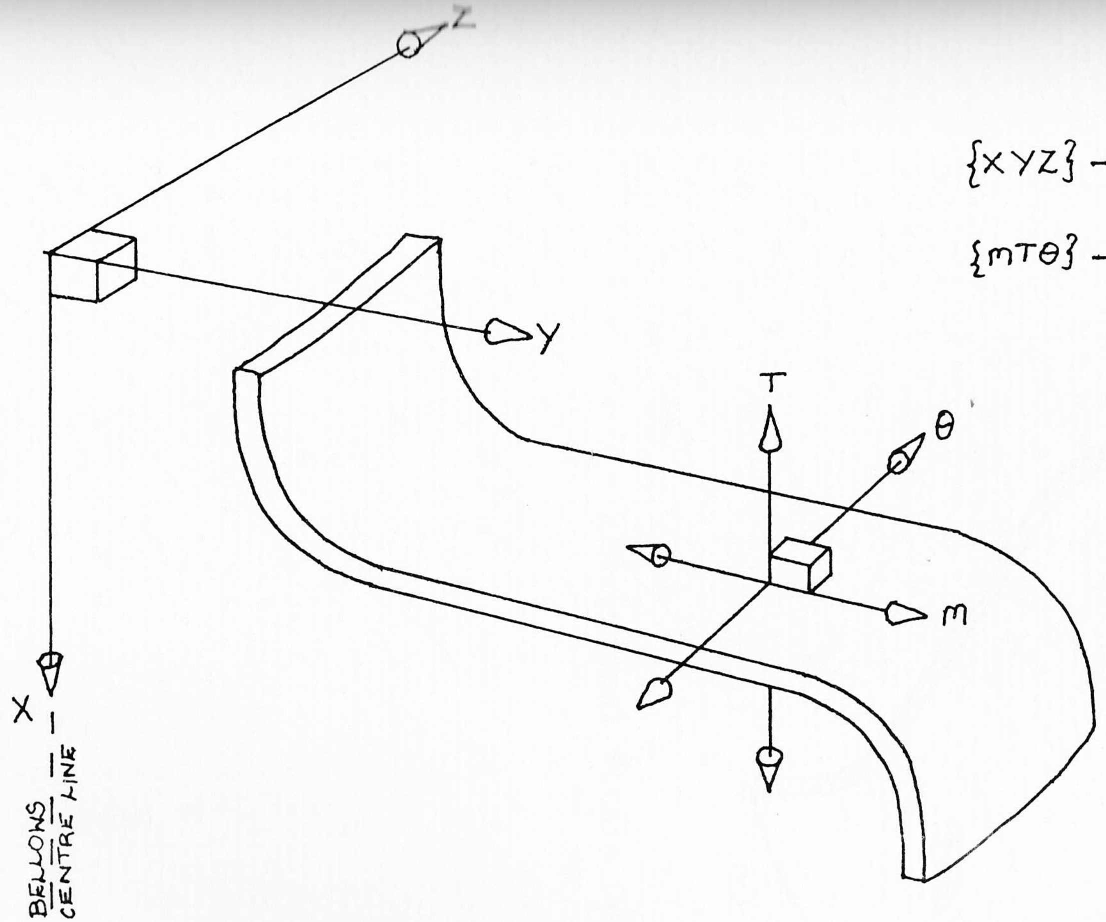
Figure 2.3



Thickness=22mm
P.C.D.=210mm
Nominal Size=125mm
B.S.4504 Table 16/8

Flange Dimensions.

Figure 2.31



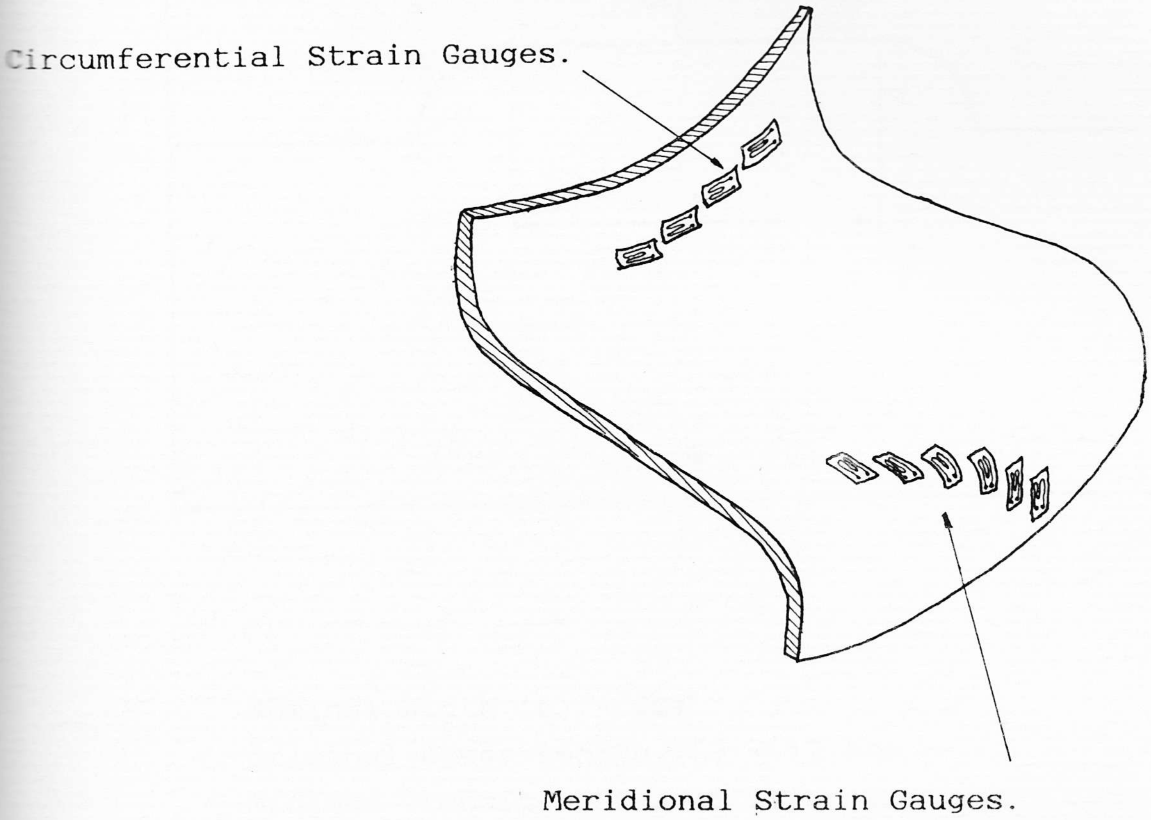
{XYZ} —— Global Cartesian Co-ordinate System.

{mTθ} —— Local Cartesian Co-ordinate System.

Note: The generator plane is meshed in the global XY plane. The orientation of the axis set (MTO) relative to the global XYZ axis set is a function of the position vector P such that the M-axis and the O-axis are tangential to the bellows surface and the T-axis is normal to the surface.

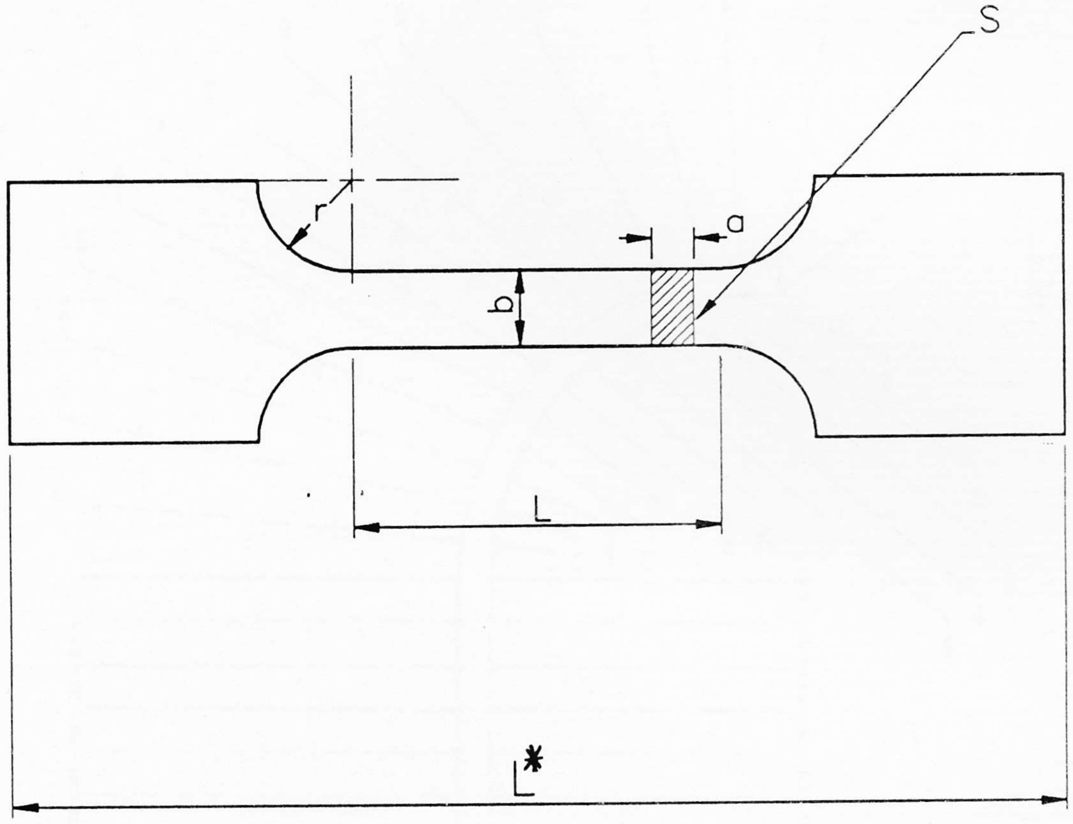
Axis Sets Used in this Project.

Figure 2.411



Strain Gauge Positioning.

Figure 2.412



Nominal Width (b) = 3mm

Original Gauge Length (L) = 12.5mm

Minimum Transition Radius (r) = 6mm

Approx. Total Length (L^*) = 50mm

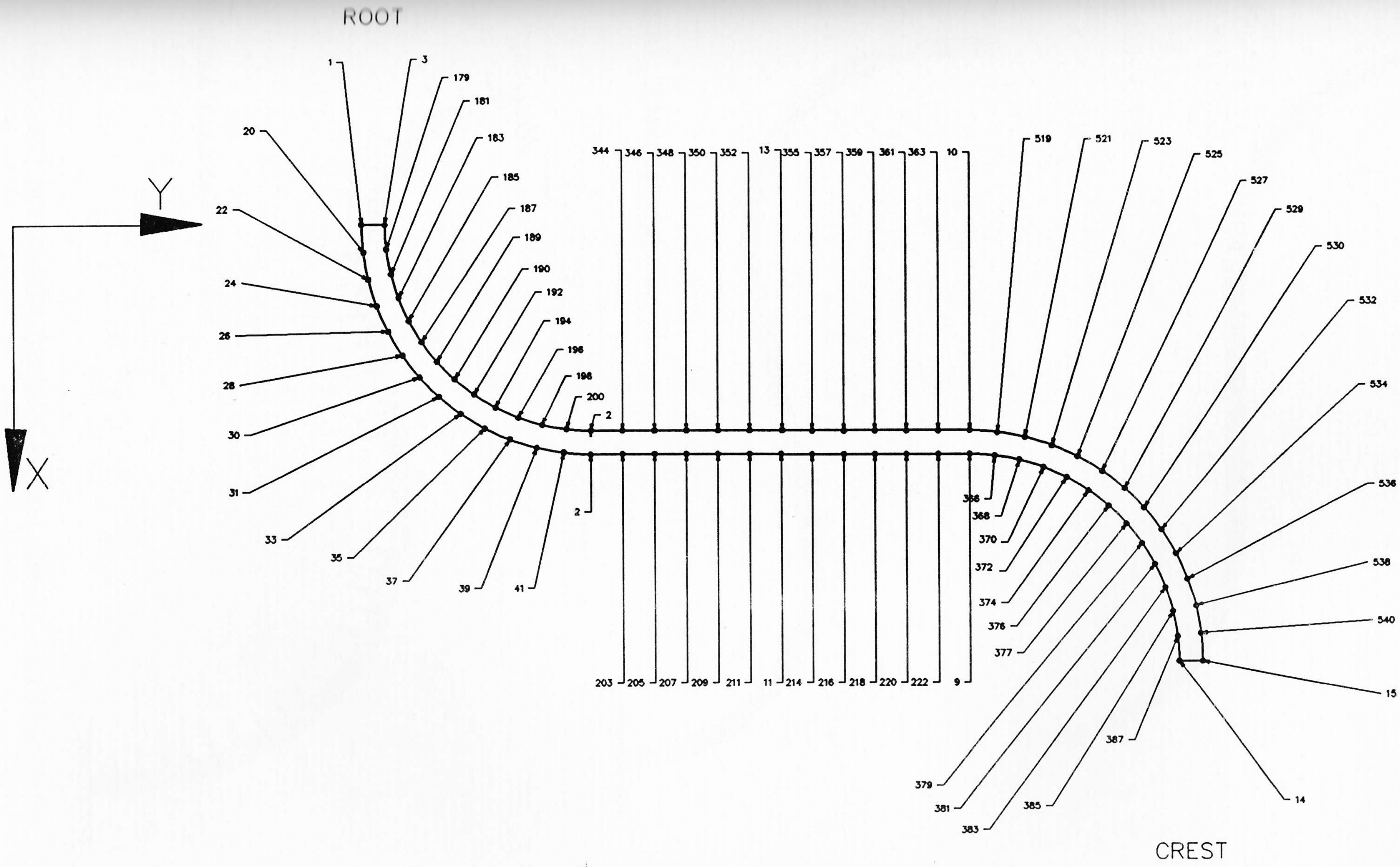
Specimen Thickness (a) = 0.9576mm

Original Cross Sectional Area (S) = 2.873×10^{-6} mm

The dimensions used for this test specimen are the preferred dimensions as stated by B.S.18 .

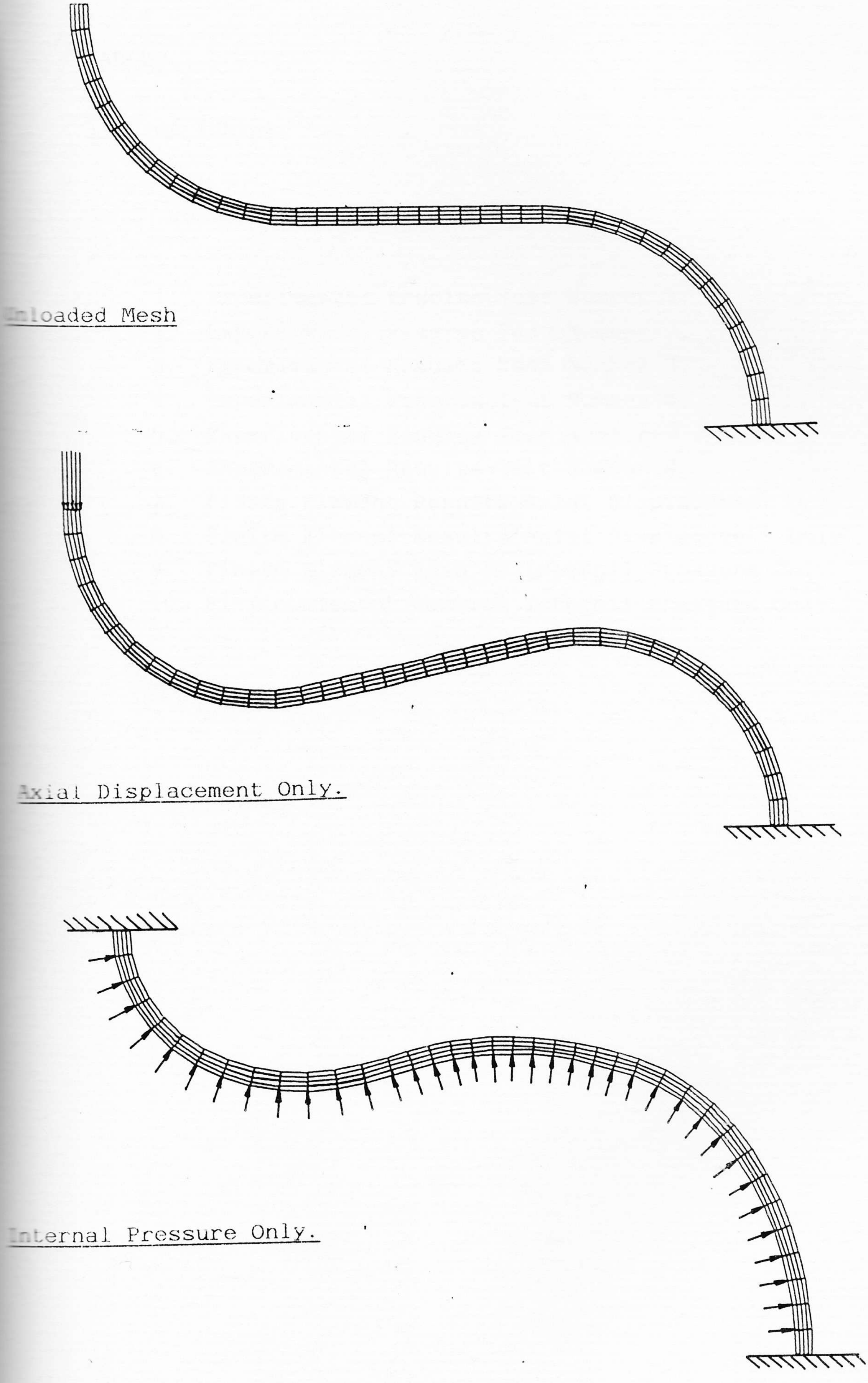
Tensile Test Specimen Data.

Figure 2.55



Finite Element Mesh Detailing Nodes of Interest.

Figure 3.8



Loads and Restraints Applied to the Finite Element Mesh.

Figure 3.81

TABLES.

List of Tables.

1. Experimental Results Test Number 1.
2. Experimental Results Test Number 2.
3. Experimental Results Test Number 3.
4. Experimental Results Test Number 4.
5. Experimental Results Test Number 5.
6. Experimental Results Test Number 6.
7. Finite Element Results Axial Displacement Only.
8. Finite Element Results Axial Displacement Only.
9. Finite Element Results Internal Pressure Only.
10. Finite Element Results Internal Pressure Only.

Table Nº1

Test Nº1	Nominal Pressure Value (KPa) 0				
Displacement (mm)	Load (N)	Strain Readings ($\mu\varepsilon$)			
		$\varepsilon_{\theta C}$	$\varepsilon_{\theta R}$	ε_{mR}	ε_{mC}
0.0	0.0	0	0	0	0
1.987	144	+118	-114	-79	+60
4.050	280	+242	-227	-154	+118
6.000	402	+353	-331	-224	+179
8.513	550	+497	-463	-314	+249
10.062	635	+580	-541	-369	+298

Table Nº2

Test Nº2	Nominal Pressure Value (KPa) 50				
Displacement (mm)	Load(N)	Strain Readings ($\mu\varepsilon$)			
		$\varepsilon_{\theta C}$	$\varepsilon_{\theta R}$	ε_{mR}	ε_{mC}
0.0	1006	-26	-8	-101	-71
2.008	1149	+91	-120	-173	-15
4.037	1285	+210	-229	-244	+43
6.058	1405	+326	-334	-313	+102
8.046	1521	+441	-436	-381	+158
10.051	1641	+548	-536	-449	+217

Table Nº3

Test Nº3	Nominal Pressure Value (KPa) 100				
Displacement (mm)	Load (N)	Strain Readings ($\mu\varepsilon$)			
		$\varepsilon_{\theta C}$	$\varepsilon_{\theta R}$	ε_{mR}	ε_{mC}
0.0	1948	-48	-15	-190	-140
2.008	2110	+69	-127	-264	-86
4.037	2235	+187	-235	-335	-28
6.058	2355	+303	-341	-404	+30
8.046	2484	+413	-443	-475	+87
10.051	2586	+524	-543	-541	+144

Table Nº4

Test Nº4	Nominal Pressure Value (KPa) 150				
Displacement (mm)	Load (N)	Strain Readings ($\mu\varepsilon$)			
		$\varepsilon_{\theta C}$	$\varepsilon_{\theta R}$	ε_{mR}	ε_{mC}
0.0	2978	-76	-22	-289	-213
2.008	3112	+41	-133	-360	-157
4.037	3251	+158	-242	-433	-100
6.058	3365	+274	-349	-503	-43
8.046	3484	+388	-450	-570	+12
10.051	3601	+494	-551	-642	+68

Table Nº5

Test Nº5	Nominal Pressure Value (KPa) 200				
Displacement (mm)	Load (N)	Strain Readings ($\mu\varepsilon$)			
		$\varepsilon_{\theta C}$	$\varepsilon_{\theta R}$	ε_{mR}	ε_{mC}
0.0	3948	-99	-29	-380	-284
2.008	4081	+18	-140	-452	-228
4.037	4215	+135	-249	-524	-171
6.058	/	/	/	/	/
8.046	/	/	/	/	/
10.051	/	/	/	/	/

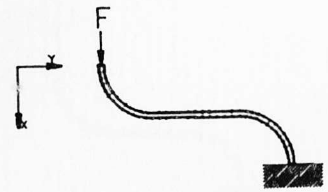
Table Nº6

Test Nº6	Zero Axial Displacement				
Nominal Pressure Value (KPa)	Load (N)	Strain Readings ($\mu\epsilon$)			
		$\epsilon_{\theta C}$	$\epsilon_{\theta R}$	ϵ_{mR}	ϵ_{mC}
0.0	5	0	0	0	0
25	502	-12	-5	-51	-38
50	1018	-27	-9	-100	-73
75	1502	-37	-12	-145	-109
100	1969	-50	-15	-189	-141
125	2487	-61	-19	-237	-180
150	2983	-76	-22	-284	-212
175	3456	-85	-25	-326	-248
200	3945	-100	-29	-372	-281

Node Nº	σ_θ (MPa/mm)	σ_m (MPa/mm)
1	-8.44	7.41
20	-8.25	7.69
22	-7.69	8.50
24	-6.75	9.72
26	-5.47	11.3
28	-3.94	13.0
30	-2.19	14.7
31	-0.34	16.1
33	1.48	17.1
35	3.16	17.6
37	4.50	17.3
39	5.41	16.4
41	5.78	14.7
2	5.34	12.1
203	4.72	9.69
205	4.19	7.56
207	3.63	5.47
209	3.07	3.44
211	2.50	1.41
11	1.92	-0.634
214	1.35	-2.60
216	0.76	-4.56
218	0.17	-6.50
220	-0.41	-8.41
222	-1.02	-10.3
9	-1.57	-12.0

Node Nº	σ_θ (MPa/mm)	σ_m (MPa/mm)
366	-1.95	-13.8
368	-1.88	-15.2
370	-1.38	-15.9
372	-0.53	-16.0
374	0.54	-15.7
376	1.73	-14.9
377	3.19	-13.8
379	4.13	-12.6
381	5.16	-11.4
383	6.00	-10.3
385	6.66	-9.44
387	7.03	-8.88
14	7.19	-8.69

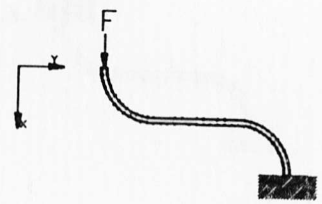
TABLE 7: Finite Element Results
Axial Displacement Only



Node Nº	σ_θ (MPa/mm)	σ_m (MPa/mm)
3	-12.50	-7.47
179	-12.47	-7.75
181	-12.41	-8.56
183	-12.25	-9.81
185	-12.00	-11.4
187	-11.63	-13.1
189	-11.06	-14.8
190	-10.31	-16.3
192	-9.41	-17.3
194	-8.34	-17.8
196	-7.22	-17.7
198	-6.13	-16.7
200	-5.13	-15.1
4	-4.41	-12.9
344	-3.94	-10.9
346	-3.41	-8.84
348	-2.86	-6.72
350	-2.32	-4.69
352	-1.77	-2.63
13	-1.20	-0.625
355	-0.64	1.40
357	-0.06	3.38
359	0.51	5.31
361	1.10	7.25
363	1.68	9.16
10	2.36	11.3

Node Nº	σ_θ (MPa/mm)	σ_m (MPa/mm)
519	3.34	13.6
521	4.47	15.1
523	5.72	15.8
525	6.94	16.0
527	8.13	15.7
529	9.16	14.9
530	10.00	13.9
532	10.69	12.7
534	11.22	11.5
536	11.56	10.4
538	11.81	9.50
540	11.94	8.97
15	11.97	8.78

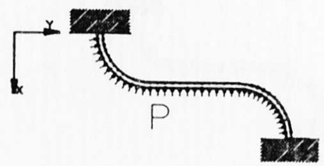
TABLE 8: Finite Element Results
Axial Displacement Only



Node Nº	σ_θ (MPa/MPa)	σ_m (MPa/MPa)
1	13.7	275
20	17.1	273
22	27.7	268
24	44.1	258
26	64.7	241
28	87.3	217
30	110	184
31	129	141
33	144	87.7
35	151	26.7
37	151	-42.4
39	144	-113
41	129	-183
2	111	-243
203	96.5	-285
205	83.3	-319
207	72.6	-345
209	64.1	-362
211	57.9	-371
11	54.2	-372
214	52.6	-364
216	53.6	-349
218	57.0	-325
220	62.8	-293
222	70.9	-253
9	82.9	-202

Node Nº	σ_θ (MPa/MPa)	σ_m (MPa/MPa)
366	97.6	-133
368	107	-65.5
370	112	3.45
372	112	60.5
374	107	114
376	97.7	159
377	85.8	197
379	72.3	226
381	58.9	247
383	46.7	262
385	37.1	272
387	31.0	278
14	28.9	279

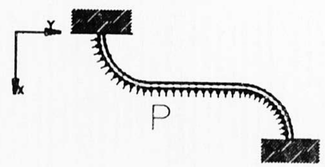
TABLE 9: Finite Element Results
Internal Pressure Only



Node Nº	σ_θ (MPa/MPa)	σ_m (MPa/MPa)
3	-145	-296
179	-141	-295
181	-129	-290
183	-110	-280
185	-84.6	-265
187	-53.3	-242
189	-17.5	-210
190	21.2	-168
192	61.0	-116
194	100	-55.2
196	136	13.9
198	168	83.8
200	195	154
4	217	225
344	232	275
346	242	312
348	251	340
350	257	360
352	261	371
13	263	374
355	262	368
357	259	354
359	254	332
361	246	303
363	236	265
10	224	221

Node Nº	σ_θ (MPa/MPa)	σ_m (MPa/MPa)
519	204	159
521	177	91.7
523	145	26.2
525	109	-36.1
527	71.1	-90.3
529	33.0	-137
530	-3.53	-175
532	-36.9	-205
534	-65.9	-227
536	-89.5	-243
538	-107	-254
540	-117	-259
15	-121	-261

TABLE 10: Finite Element Results
Internal Pressure Only



GRAPHS.

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2. Circumferential Strain at Convolution Crest V Axial Displacement.
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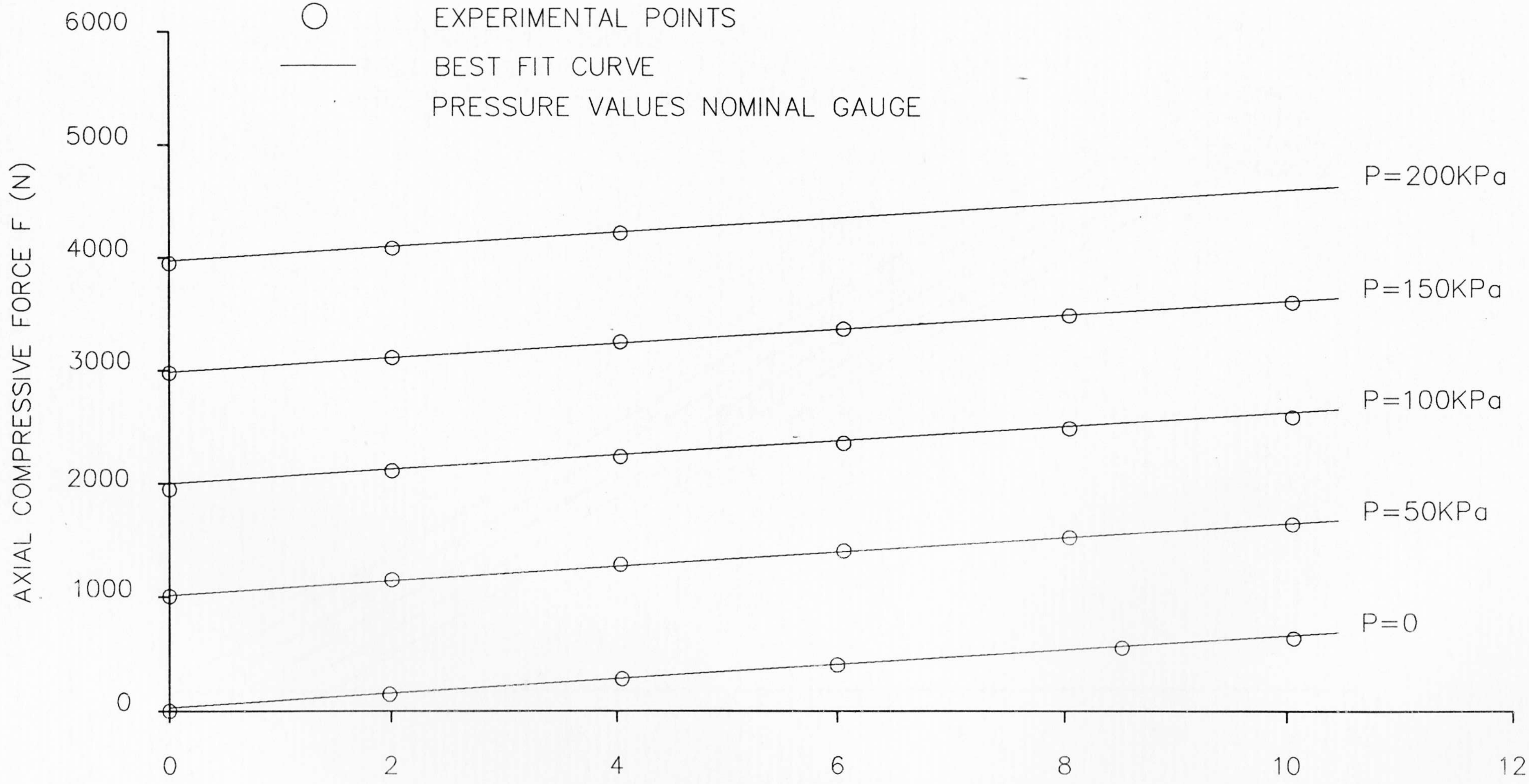
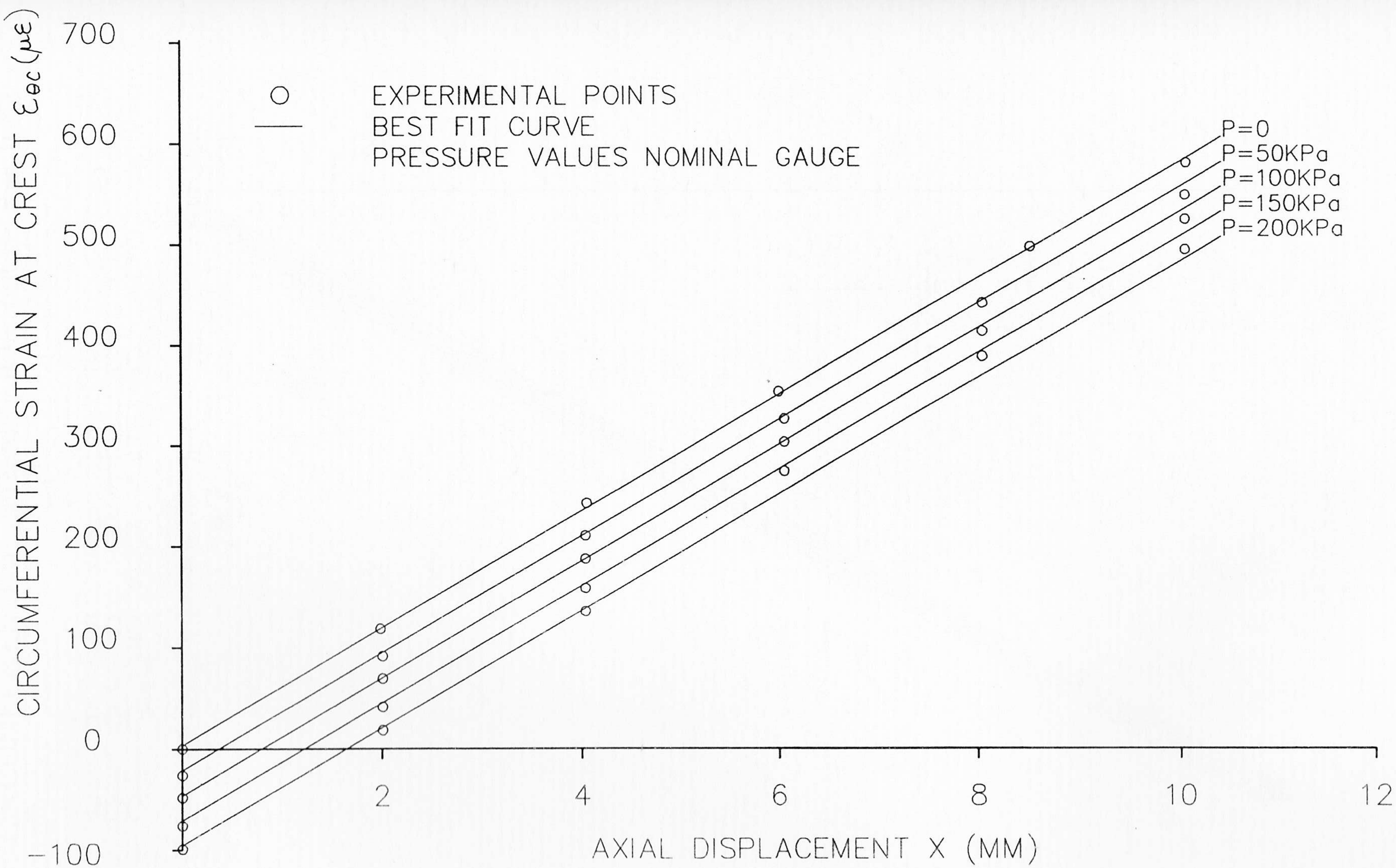
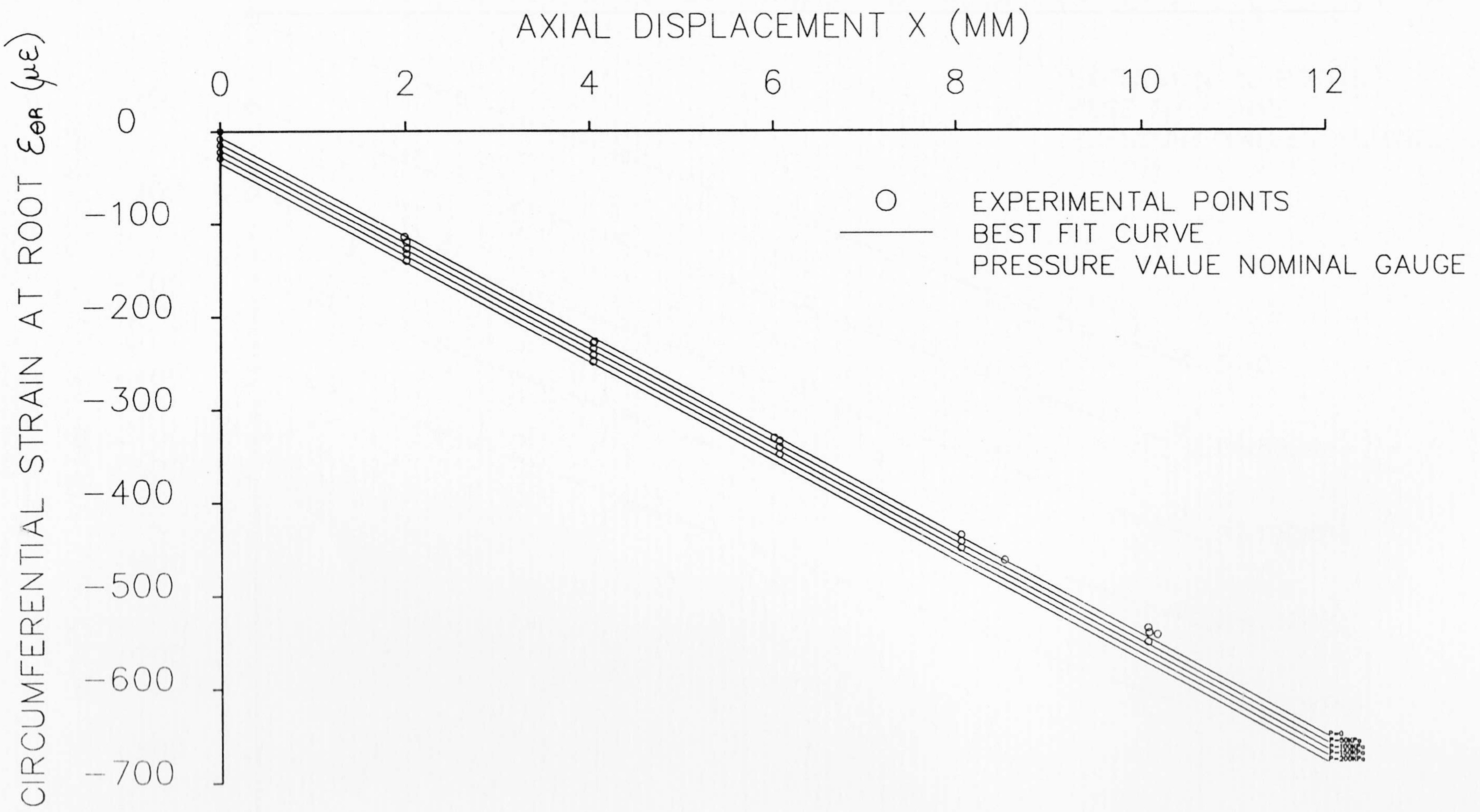


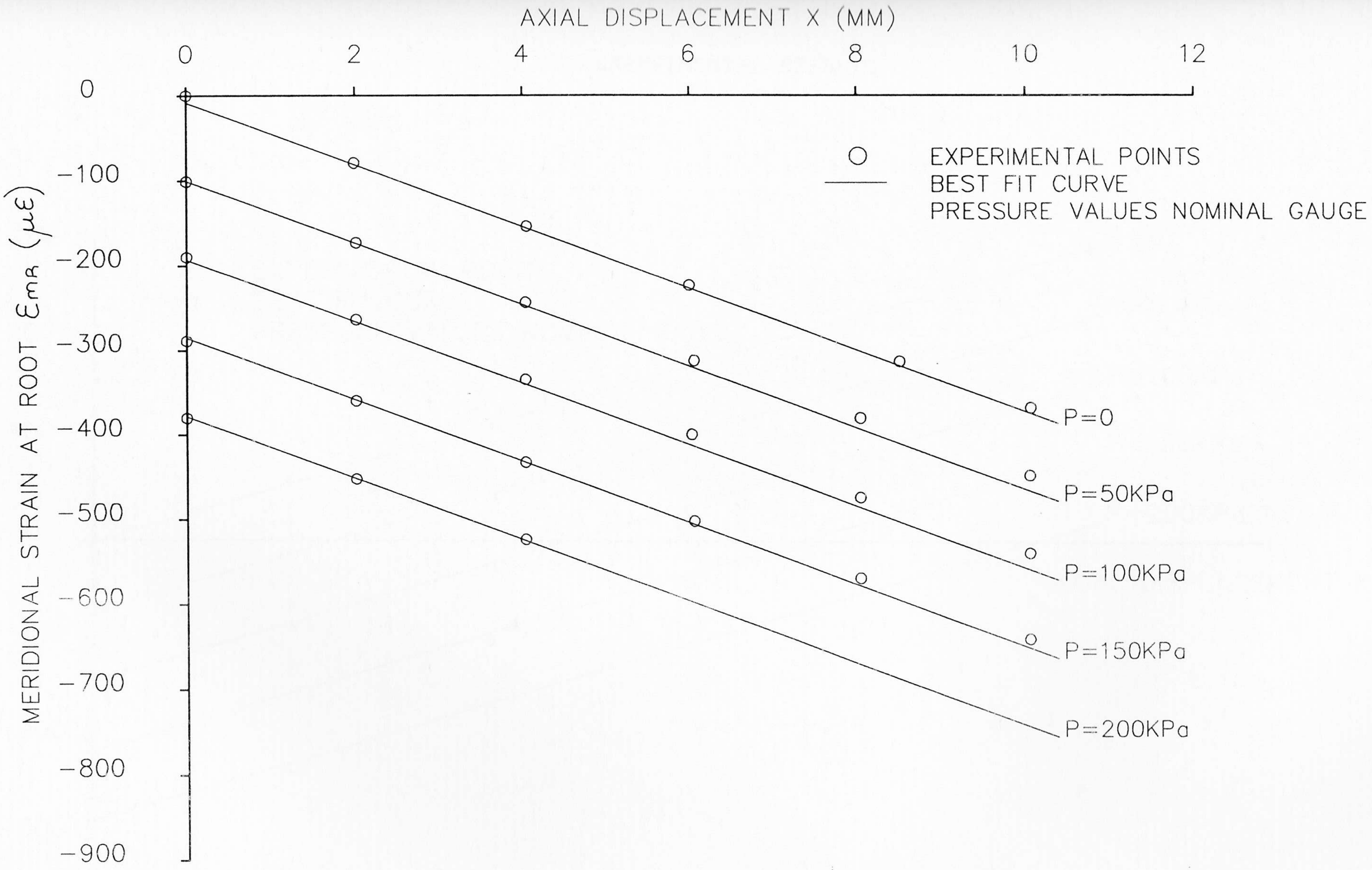
GRAPH 1: AXIAL FORCE V AXIAL DISPLACEMENT



GRAPH 2: CIRCUMFERENTIAL STRAIN AT CONVOLUTION CREST V AXIAL DISPLACEMENT

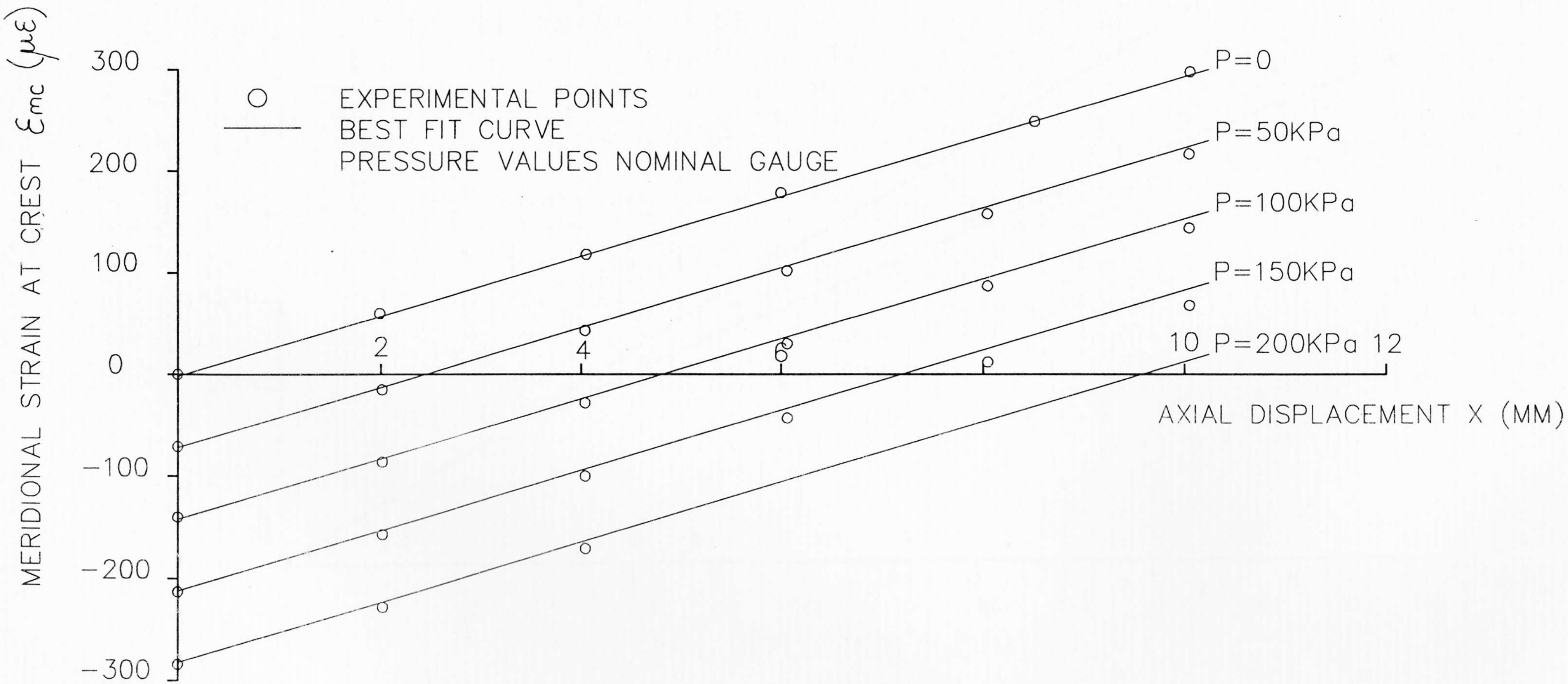


GRAPH 3: CIRCUMFERENTIAL STRAIN AT CONVOLUTION ROOT V AXIAL DISPLACEMENT

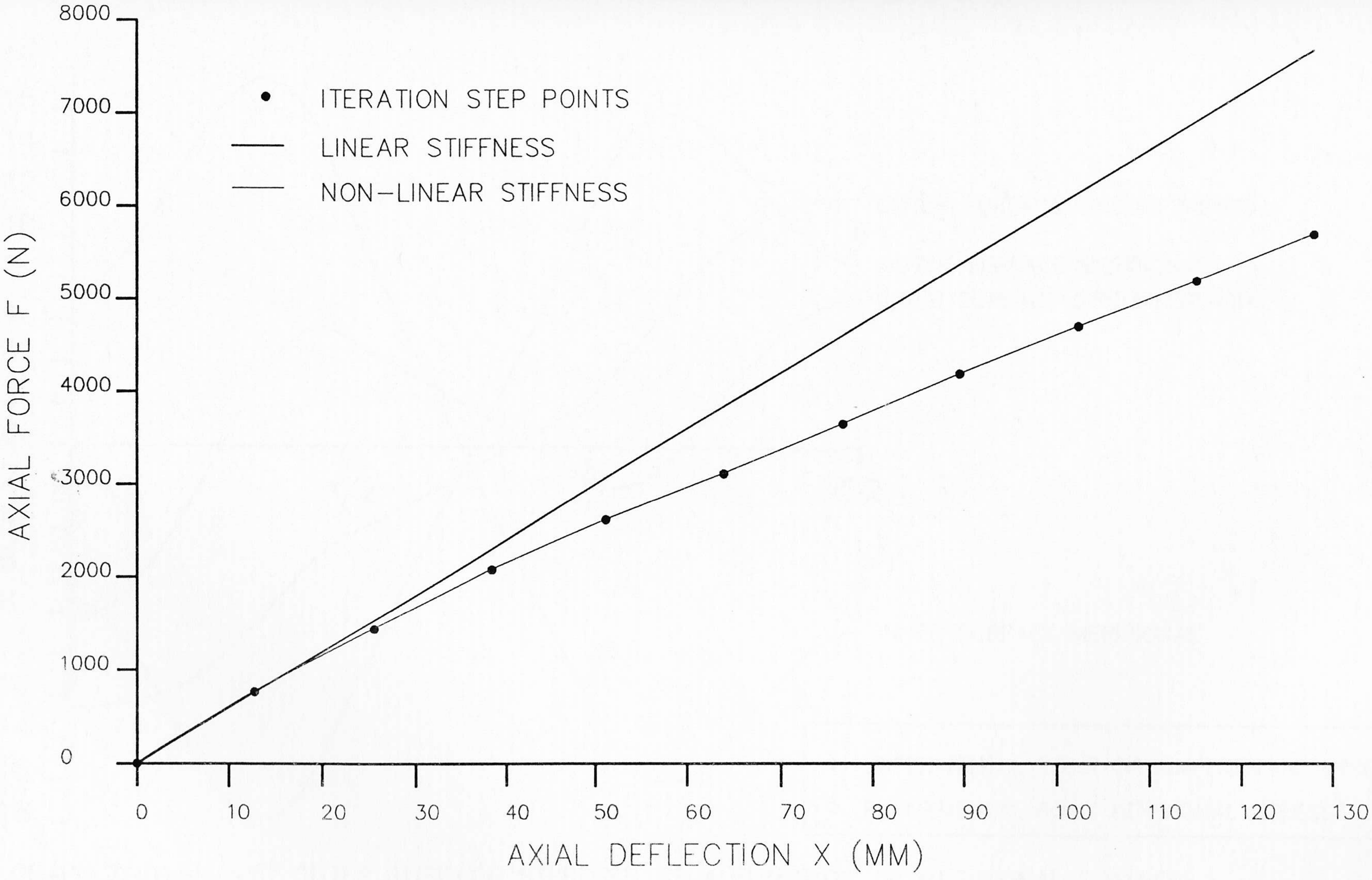


GRAPH 4: MERIDIONAL STRAIN AT CONVOLUTION ROOT V AXIAL DISPLACEMENT

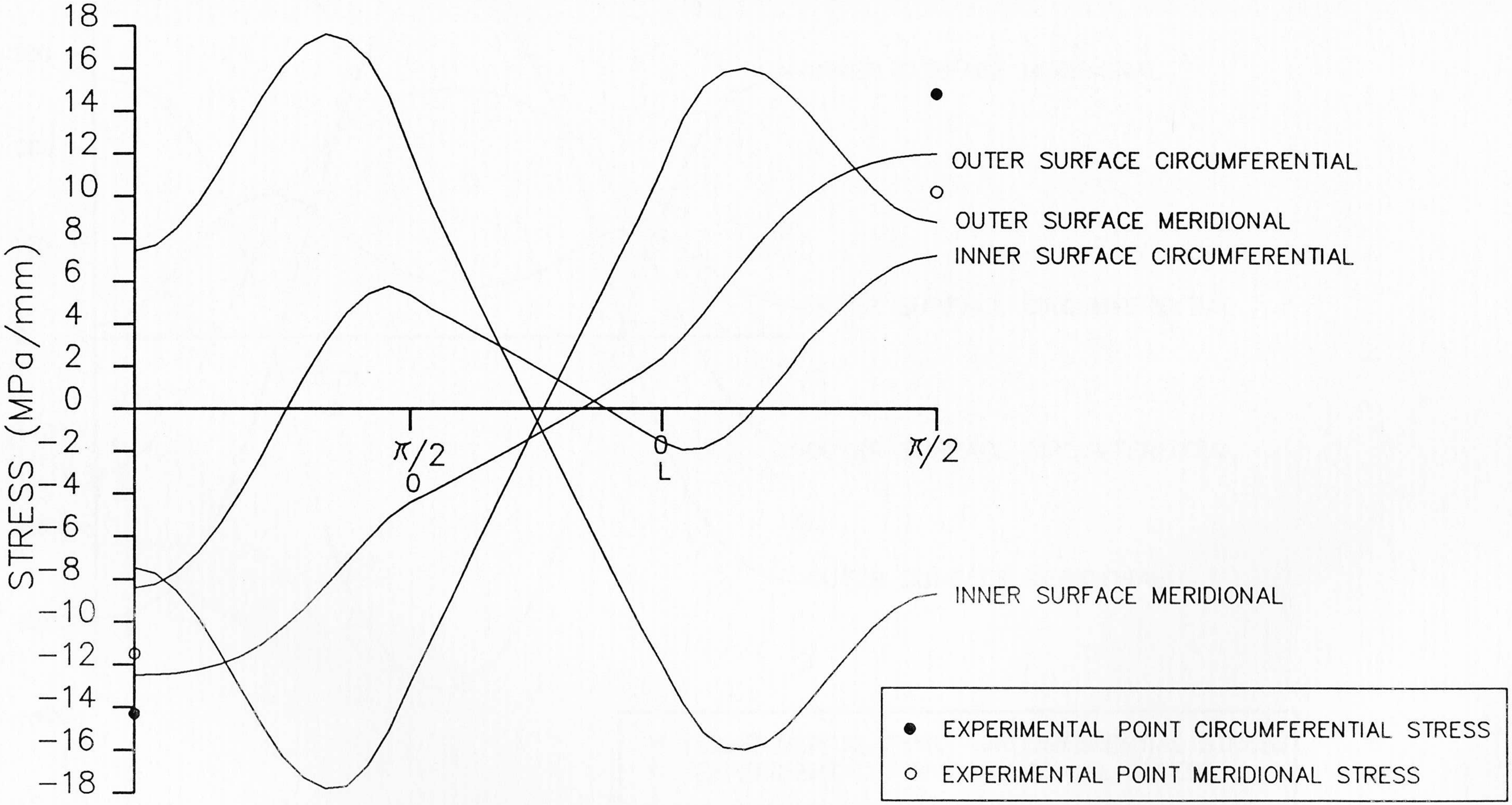
EXPERIMENTAL RESULTS



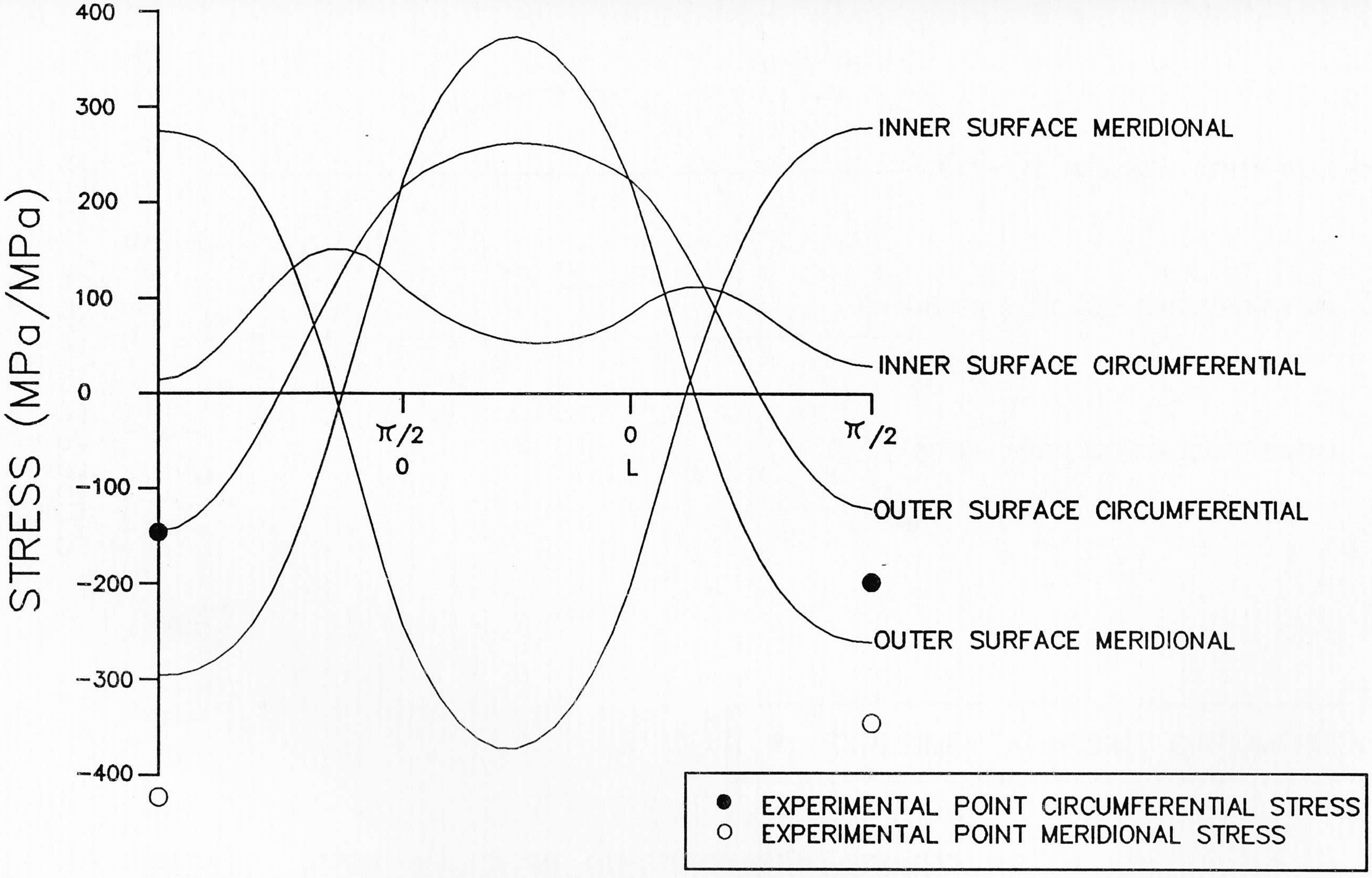
GRAPH 5: MERIDIONAL STRAIN AT CONVOLUTION CREST V AXIAL DISPLACEMENT



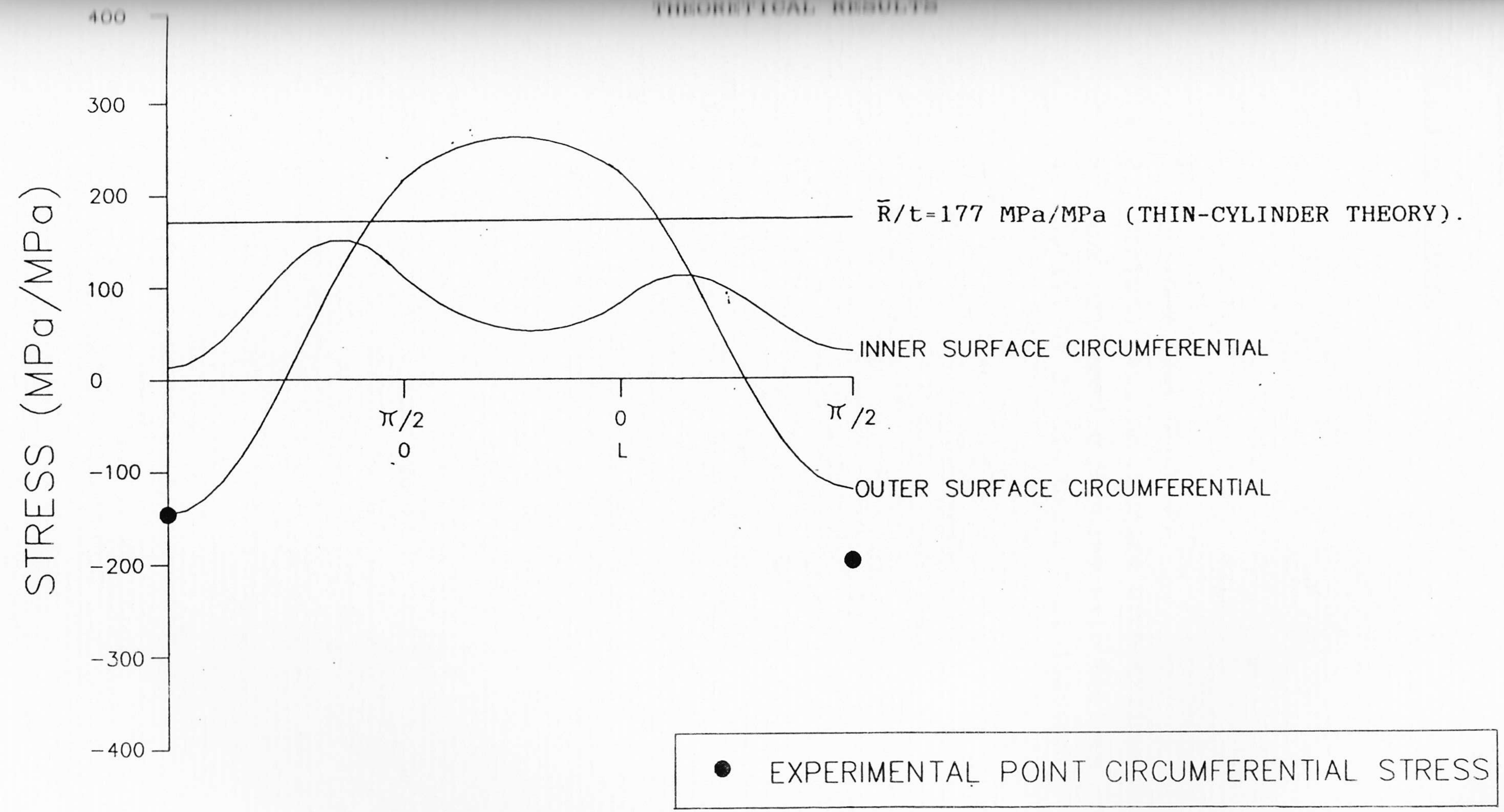
GRAPH 6: NON-LINEAR STIFFNESS BEHAVIOUR WITH LARGE DISPLACEMENTS



GRAPH 7: INNER AND OUTER SURFACE STRESSES AS A FUNCTION OF MERIDIONAL POSITION
(FINITE ELEMENT RESULTS FOR THE CASE OF AXIAL DISPLACEMENT ONLY.)



GRAPH 8: INNER AND OUTER SURFACE STRESSES AS A FUNCTION OF MERIDIONAL POSITION
(FINITE ELEMENT RESULTS FOR THE CASE OF INTERNAL PRESSURE ONLY)



GRAPH 9: INNER AND OUTER CIRCUMFERENTIAL STRESSES AS A FUNCTION OF MERIDIONAL POSITION FOR THE CASE OF INTERNAL PRESSURE PLUS THE THIN-CYLINDER RESULT.

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1. Bellows Test Specimen.
2. Denison Testing Machine with Bellows in Position
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1. Error in Stiffness Due to Error in Material Thickness.
2. Finite Element (PAFEC) Data Files.
3. Manufacturing Drawings For Bellows Test Rig.
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5. Abbreviations Used in References.
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8. Axial Stiffness Finite Element Model of the Experimental Bellows Used by Turner and Ford.
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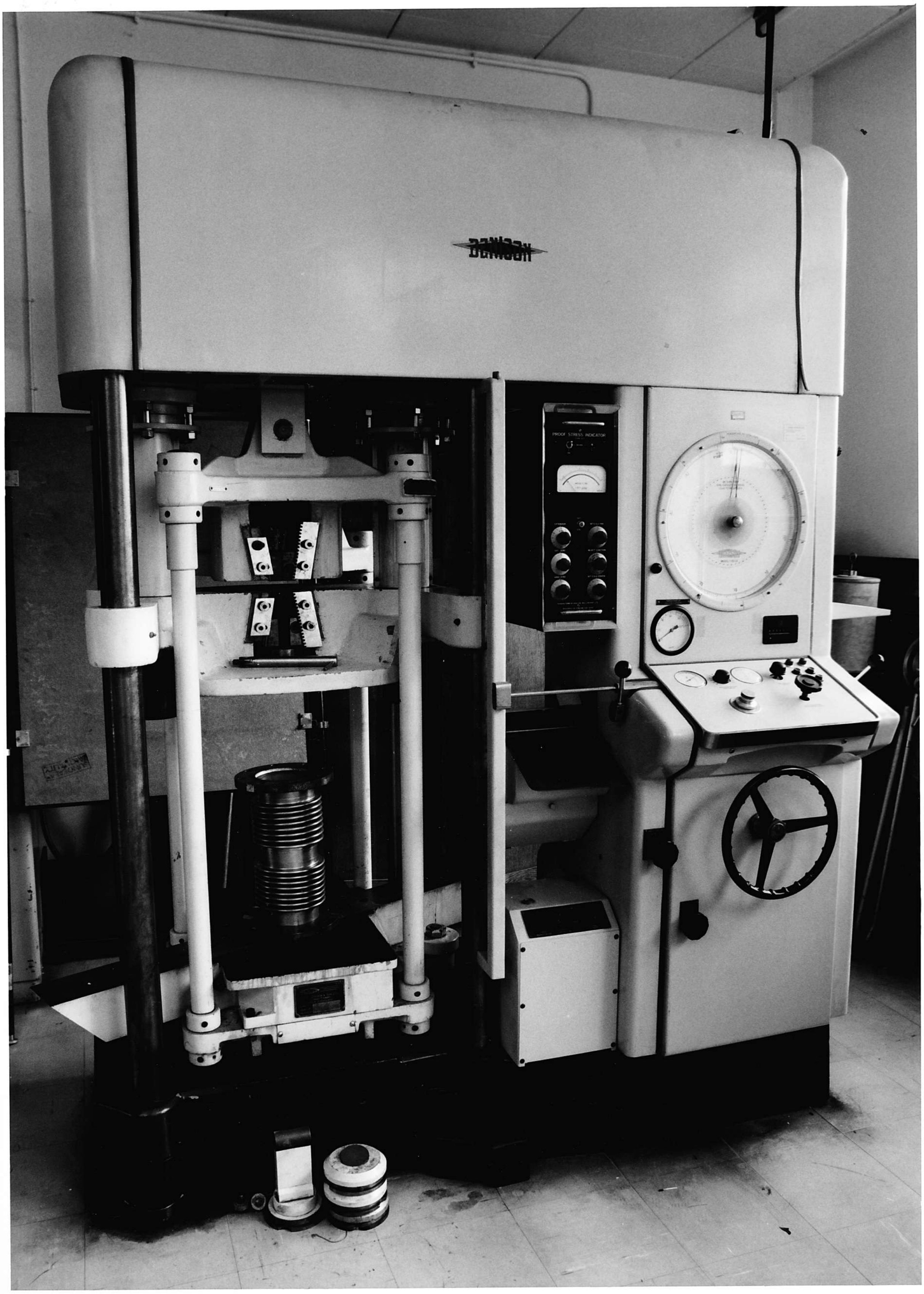
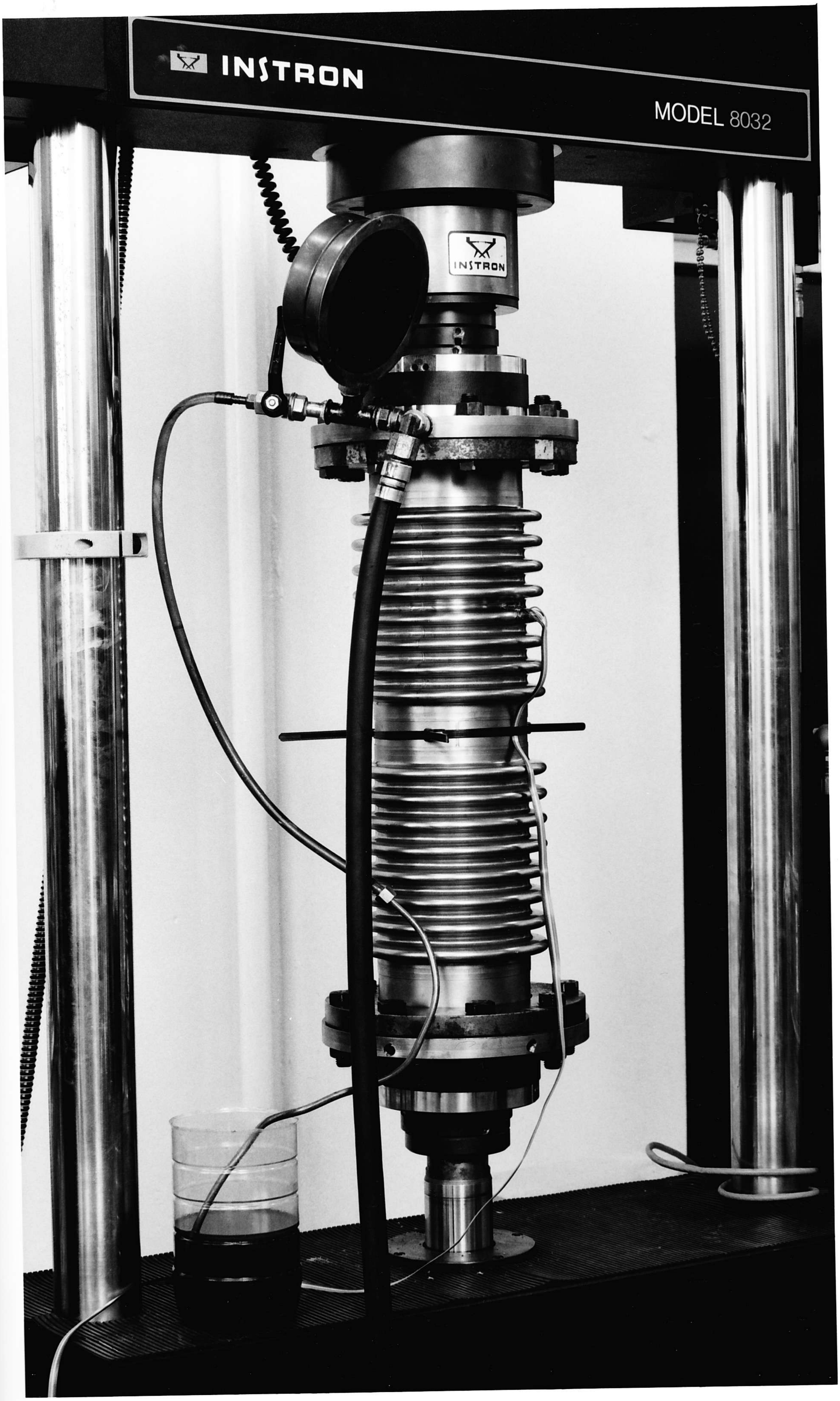

PROOF STRESS INDICATOR


INSTRON
MODEL 8032
INSTRON

APPENDIX 1.

Error in Stiffness Due to Error in Material Thickness.

The analysis outlined in this appendix results in an equation that may be used to evaluate the relative error in the calculated stiffness of a bellows expansion joint when there is an error in the assumed thickness used in the calculation of δt existent in m out of the n convolutions.

The stiffness of a bellows expansion joint may be written :-

$$K_{EB} = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots \frac{1}{K_i}} \qquad -(1)$$

Where K_i is the stiffness of the i^{th} convolution

For a set of bellows with such an error in the assumed thickness the actual stiffness will be :-

$$K_{EB\delta t} = \frac{K_t K_{\delta t}}{(n-m)K_{\delta t} + mK_t} \qquad -(2)$$

Where K_t = Stiffness of corrugation of correct thickness.

$K_{\delta t}$ = Stiffness of corrugation of incorrect thickness.

Now, let

$$\delta K_{EB} = K_{EB\delta t} - K_{EB}$$

Then,

$$\frac{\delta K_{EB}}{K_{EB}} = \frac{K_{EB\delta t}}{K_{EB}} - 1 \qquad -(3)$$

Where $K_{EB\delta t}$ = Stiffness of Bellows with Incorrect Thickness

Substituting (2) into (3) results in an equation which expresses the relative error in stiffness as a function of the number of convolutions which do not have the assumed thickness.

$$\frac{\delta K_{EB}}{K_{EB}} = \frac{nK_{\delta t}}{(n-m)K_{\delta t} + mK_t} - 1 \qquad -(4)$$

It has already been noted that the stiffness of a convolution is proportional to the cube of the convolution thickness. Clearly then :-

$$\left.\begin{aligned} K_t &\ \alpha\ t^3 = At^3 \\ K_{\delta t} &\ \alpha (t\pm\delta t)^3 = A(t\pm\delta t)^3 \end{aligned}\right\} \qquad -(5)$$

Where A = Constant of Proportionality.

Substituting (5) into (4) yields :-

$$\frac{\delta K_{EB}}{K_{EB}} = \frac{m\delta t^3+3mt^2\delta t-3mt\delta t^2}{nt^3+n\delta t^3+3nt^2\delta t+3nt\delta t^2-m\delta t^3-3mt^2\delta t-3mt\delta t^2}$$

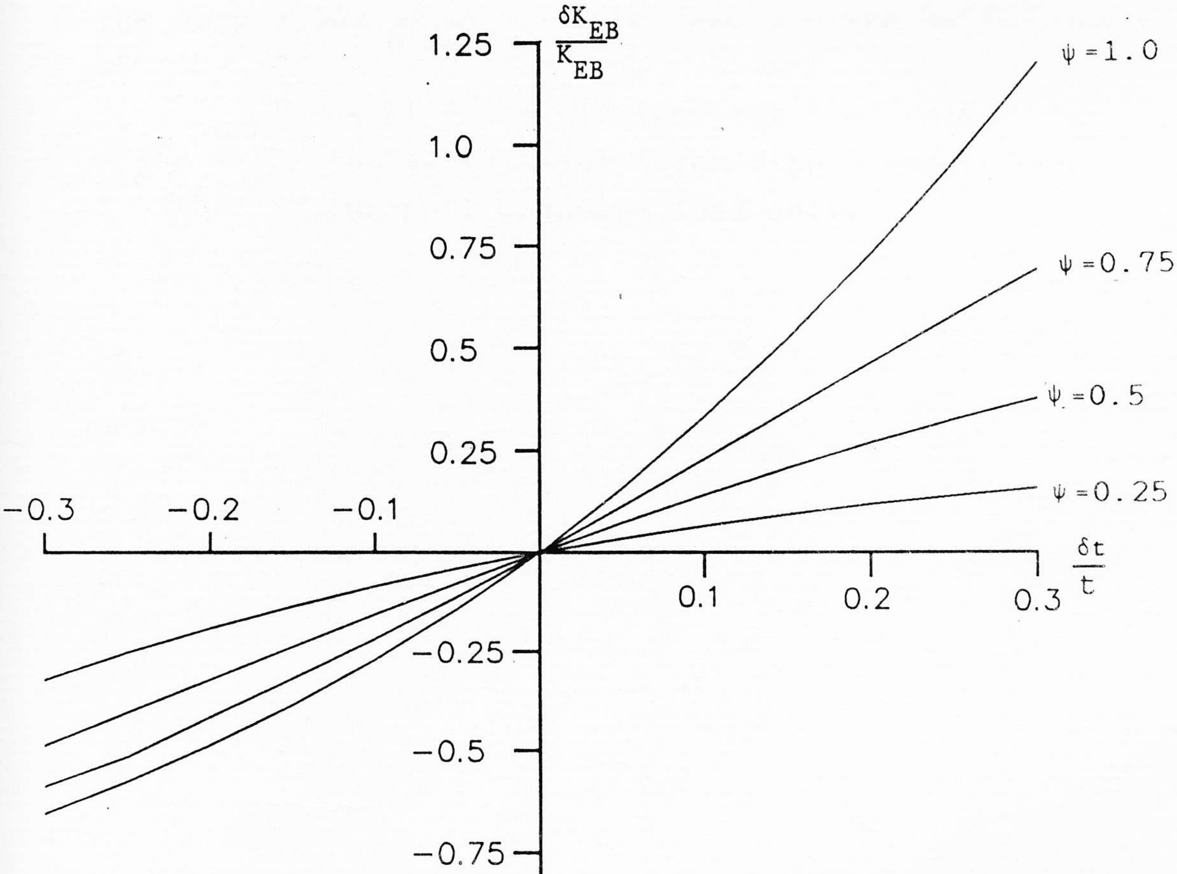
By letting, $\frac{\delta t}{t} = \phi$

and

$$\frac{m}{n} = \psi$$

$$\frac{\delta K_{EB}}{K_{EB}} = \frac{\psi(\phi^3+3\phi^2+3\phi)}{(1-\psi)(\phi^3+3\phi^2+3\phi)+1}$$

This final eqution represents the relative error in the stiffness of the bellows unit as a function of two non-dimensional ratios and as such a plot of the characteristic curves will be equally applicable to any set of bellows. Such a plot is shown on the following page. It must be realised, however, that this relationship was evaluated on the assumption that the constant of proportionality in equations (5) was constant. This assumption is clearly only valid for small values of δt.



δK_EB / K_EB
1.25
1.0
0.75
0.5
0.25
−0.25
−0.5
−0.75
−1.0
−0.3
−0.2
−0.1
0.1
0.2
0.3
δt / t
ψ = 1.0
ψ = 0.75
ψ = 0.5
ψ = 0.25

APPENDIX 2.

Finite Element (PAFEC) Data Files.

This appendix contains the finite element (PAFEC) data files that were developed in this project. No explanation of the format of these files is given however the reader who is interested in this aspect of the work is directed to the references that the author used in the formulation of the data files (references 4 & 5).

The data files shown in this appendix are as follows:-

1. Axial force/displacement load only.
2. Iterative axial force/displacement load.
3. Internal pressure load only.

APPENDIX 3.

Manufacturing Drawings for Bellows Test Rig.

This appendix contains the following:-

1. Parts List.
2. Materials List.
3. General Assembly Drawing.
4. Sub-assembly Drawings.
5. Parts Drawings.

These drawings are the preliminary design drawings for the test rig that was not actually manufactured.

APPENDIX 4.

Correspondence With Bellows Manufacturers.

This appendix contains the following:-

1. The addresses of all bellows manufacturers contacted during this project.
2. Copies of all correspondences between the author and the above mentioned bellows manufacturers.

APPENDIX 5.

ABBREVIATIONS USED IN REFERENCES.

Int.J.Mech.Sci- International Journal of Mechanical Sciences.

Proc.Inst.Mech.Eng- Proceedings of the Institution of Mechanical Engineers.

Phillips Res.Rep.- Phillips Research Reports

A.S.M.E.- American Society of Mechanical Engineers.

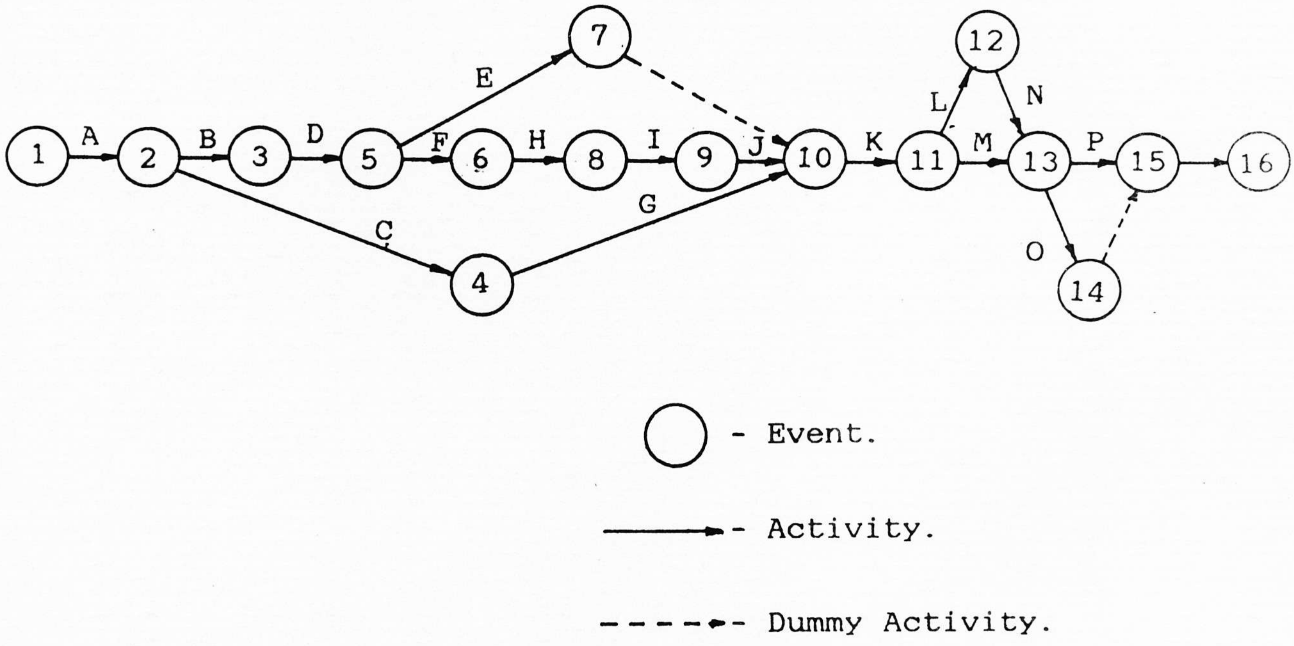
J.Appl.Mech.- Journal of Applied Mechanics

J.Math.Phys.- Journal of Mathematics and Physics.

Quart.Appl.Math.- Quarterly of Applied Mathematics

APPENDIX 6.

NETWORK ANALYSIS OF PROJECT.



ACTIVITIES.

A. Receive Objectives.
B. Delivery of Bellows.
C. Literature Survey.
D. Measurement of Bellows.
E. Develop Finite Element Models.
F. Design Test Rig.
G. Evaluate Analytical Models.
H. Delivery of Test Rig.
I. Experimental Stiffness.
J. Experimental Stress/Strain.
K. Correlate Results to Date.
L. Obtain Sample of Bellows Material.
M. Obtain More Accurate Measurements.
N. Test Material Sample.
O. Update Finite Element Models.
P. Update Anlytical Models.

APPENDIX 7.

Extracts From the Standards of the E.J.M.A.

This appendix contains a number of extracts from the standards of the Expansion Joint Manufacturers Association. In addition a typical manufacturers bellows data sheet is shown.

APPENDIX 8.

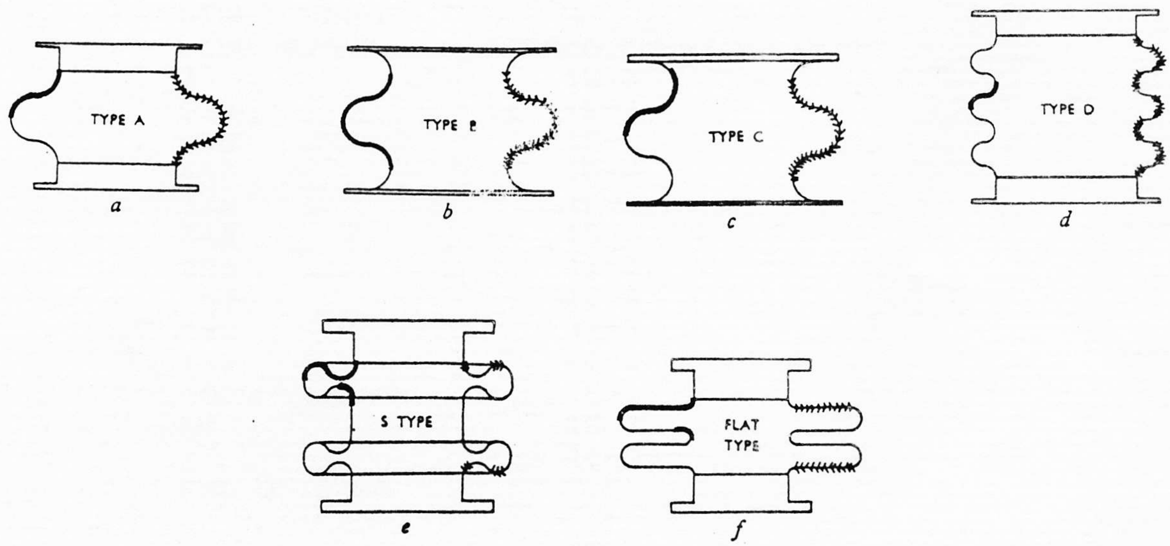
Axial Stiffness Finite Element Model of the Experimental Bellows Used by Turner and Ford (22)

In this appendix the axi-symmetric finite element model developed in this project is run with geometric and material data from Turner and Fords experimental work. In addition the N.A.A empirical formulation for axial stiffness is evaluated for these bellows. The results from these analyses are shown together with Turner and Ford's experimentally observed stress distributions.

A8.1 Geometric and Material Properties of Experimental Bellows.

R=178.1	
t=1.524	
r=129.586	all dimensions in mm
r=226.614	
L=0	
E=207 GPa	
ν=0.3	

The above properties are for the type A bellows shown in the following diagram.



Gauge Locations and Thickness Surveys

Gauges.
Thickness survey in three planes.

APPENDIX 9.

Numerical Harmonic Analysis and Post-Processing of Result

Whenever a periodic function is described by a set of paired numerical values or by a graphical plot it is usually possible to numerically integrate the curve and solve for the coefficients of a Fourier Series. This process is known as numerical harmonic analysis. In this way it is possible to represent the component stress distributions either from experimental observation or from a finite element analysis as a Fourier Series. The technique of numerical harmonic analysis can be found in many engineering mathematics texts* and is therefore not shown here. For the purpose of this report the author intends to show the advantages afforded by performing such an analysis on a set of results.

If numerical harmonic analysis is performed on the finite element results one would end up with eight Fourier Series' representing the inner and outer surface meridional and circumferential stresses as functions of pressure and functions of axial displacement. For example the outer surface circumferential stresses could be represented as:-

$$\sigma(x) = \sum_{i=0}^{n} (\alpha_i x \operatorname{Cos} i m) \qquad \sigma(p) = \sum_{i=0}^{n} (\beta_i p \operatorname{Cos} i m)$$

By the principle of superposition one could add these two expressions resulting in an expression for the outer surface circumferential stress as a function of both axial displacement and internal pressure:-

$$\sigma(x,p) = \sum_{i=0}^{n} (\alpha_i x + \beta_i p) \operatorname{Cos} i m$$

* Stroud, K.A (1986) 'Further Engineering Mathematics' Macmillan.

Performing this analysis on the outer surface circumferential stress distribution given by the finite element analysis, the following coefficients are found up to and including the third harmonic:-

$$\sigma(x,p)=93.5p-0.72x-(9.1p+11.2x)\text{Cos}m-(203p-0.5x)\text{Cos}2m-(1.5p+1.6x)\text{Cos}3m-(28.5p+0.01x)\text{Cos}4m$$

The reader who is interested may plot this function out and he will find, as indeed the author found, that this function which is a infinite series truncated at the third harmonic represents with reasonable accuracy the stress distributions given by the finite element analysis.

The main advantage that this method of analysis affords is that with such a mathematical description of stress distribution one can use mathematical procedures to maximise, or minimise, the function and this may be of importance when tackling the problem of elastic failure in bellows.

Looking at the process of finding maxima and minima we first note that the general equation for the stress at a point is a function of three independent variables, ie $\sigma=\sigma(x,p,m)$. In reality the bellows will be required to work within certain limits of axial displacement and internal pressure and if we include the implied range for m we can see that:-

$$\left.\begin{array}{l} a\leq x\leq b \\ c\leq p\leq d \\ 0\leq m\leq 2\pi \end{array}\right\}$$

The process for finding turning points is now simply to equate all the first partial derivatives of the equation to zero and solve the resulting equations within the given ranges. This process although conceptually simple may be quite complicated in terms of mathematics. However, numerical computer packages such as N.A.G. routines are available for solving problems of this type.

APPENDIX 10.

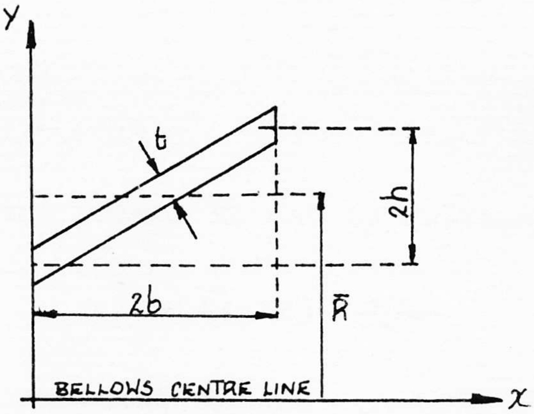
Mathematical Description of Boundary Curves.

The work shown in this appendix was undertaken by the author at a time during the project when there was a possibility of studying the optimisation of bellows design in terms of convolution geometry. In the event, due to shortage of time, this work was not completed. However, the author did make a start on this work and this appendix shows a method by which convolution geometries may be accurately described. The format chosen for this appendix is to illustrate the need for such a method and then to propose a solution to this problem.

The Problem.

When defining nodal coordinates for a convolution geometry the process is greatly simplified if one can find an accurate mathematical description for the curves of the convolution. If the convolution is considered to be of constant thickness then the two boundary curves must, by definition, be a constant normal distance apart. With the simpler convolution geometries, those that are descibed by linear equations, this posses no problem in terms of mathematics. When the boundary curves are of a higher degree, for example parabolic or elliptic curves, then the mathematics becomes slightly more involved.

As an example of the simple solution the triangular convolution geometry is studied. By application of simple rules of geometry an equation descibing the two boundary curves can be written:-

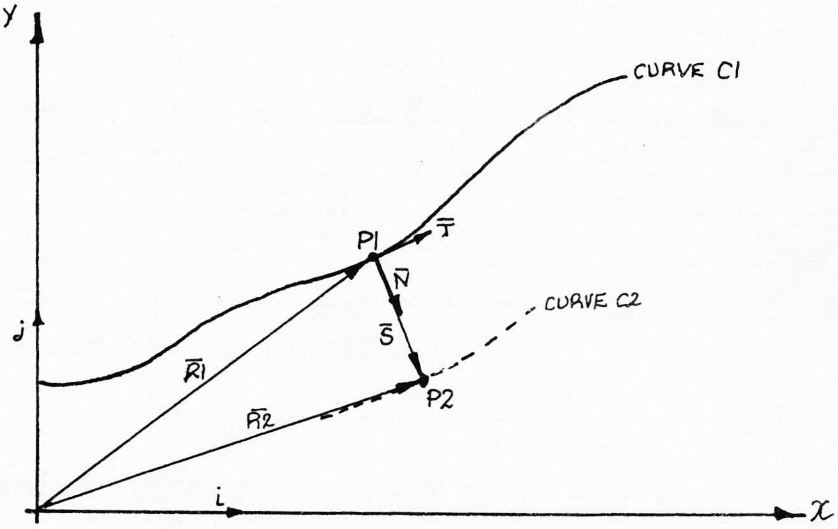


$$y=hx/b+\bar{R}-h\pm t/(2\cos(\tan^{-1}(h/b)))$$

This solution is achieved by translating the equation for one of the boundary curves a constant distance down the y-axis. It should be apparent to the reader that this method will not work with curves that are not linear The following section outlines a method that may be used for non-linear convolution geometries.

The Solution.

Given the problem as outlined above one can write the problem mathematically as:-



For any point P1 on curve C1 there must exist a point P2 on curve C2 that is a constant normal distance (t) from P1.

Representing the curve C1 in a parametric form with u as the parameter one can write:-

$$y=f(x) \text{ thus } x=g(u) \text{ and } y=h(u)$$

The position vector $\overline{R1}$ can be written:-

$$\overline{R1}=g(u)i+h(u)j$$

A unit tangential vector to the curve C1 at point P1 can be written as:-

$$\overline{T}=\overline{R1}'/|\overline{R1}'| \quad =(A(u)^2+B(u)^2)^{\frac{1}{2}}(i\text{Cos}\theta+j\text{Sin}\theta)$$

$$\text{where } \theta=\text{Tan}^{-1}\left.\frac{dy}{dx}\right|_{P1}$$

The unit tangential vector can now be rotated such that it lies in the direction of $\overline{S}$ by a rotation of $\pi/2$ in the clockwise direction. Hence calling this new vector $\overline{N}$ (note that it is not a true unit normal vector since it does not always point towards the centre of curvature of the curve) one can write:-

$$\overline{N}=(A(u)^2+B(u)^2)^{\frac{1}{2}}(i\text{Cos}(\theta-\pi/2)+j\text{Sin}(\theta-\pi/2))$$

The magnitude of the vector $\overline{S}$ is known ie $|\overline{S}|=t$ and one can therefore write:-

$$\overline{S}=t\overline{N}$$

Utilising the loop equation for the vectors $\overline{R1}$, $\overline{R2}$ and $\overline{S}$ we can write the vector $\overline{R2}$ as:-

$$\overline{R2}=\overline{R1}+t(A(u)^2+B(u)^2)^{\frac{1}{2}}(i\text{Cos}(\theta-\pi/2)+j\text{Sin}(\theta-\pi/2))$$

This equation represents the result we were looking for since it relates points on curve C1 to points on curve C2 in the defined manner.