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Structural Analysis of the Infinite Table

The so-called Infinite Table was posted in LinkedIn and whilst it was considered to be pleasing aesthetically, some concern was raised over its stiffness and strength. The structure is a single square hollow section (SHS) member formed into a closed-loop. In practice it has been made from 18 members welded together at 18 right-angled joints as shown in Figure 1.

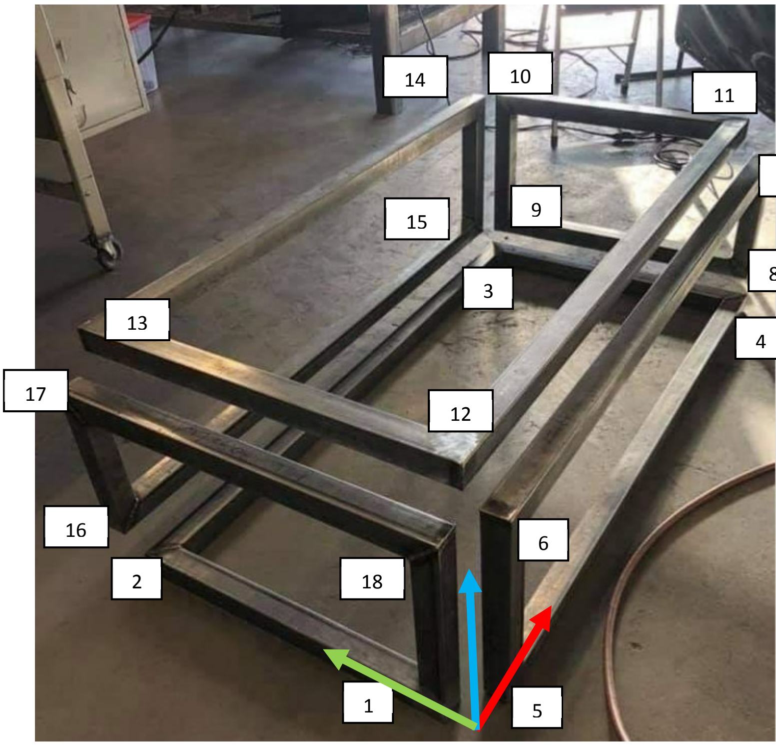


Figure 1: Infinite table with 18 joints

The table can be modelled with beam elements to assess its stiffness and strength. The model will be constructed to the dimensions given in Table 1.

Dimension	Symbol	Value, [m]
Length	l	1.00
Width	w	0.50
Height	h	0.40
Offset	Δ	0.08

Table 1: Model dimensions

The members are assumed to be made from a structural steel with properties given in Table 2.

Property	Value
Elastic modulus, [GPa]	210
Poisson's ratio	0.3
Density, [kg/m^3]	7800
Yield stress, [MPa]	275

Table 2: Material properties (structural steel)

The offset for the structural members, Δ = 80mm, has been chosen as twice the side dimension of the SHS. With the offset and table dimensions defined, the coordinates of the 18 points can be determined. Figure 2 illustrates the points as seen down the x axis.

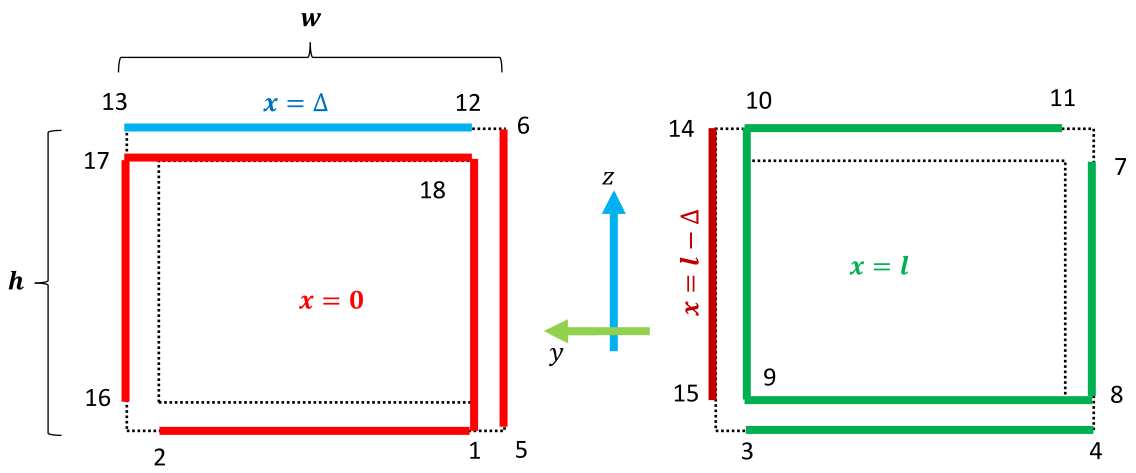


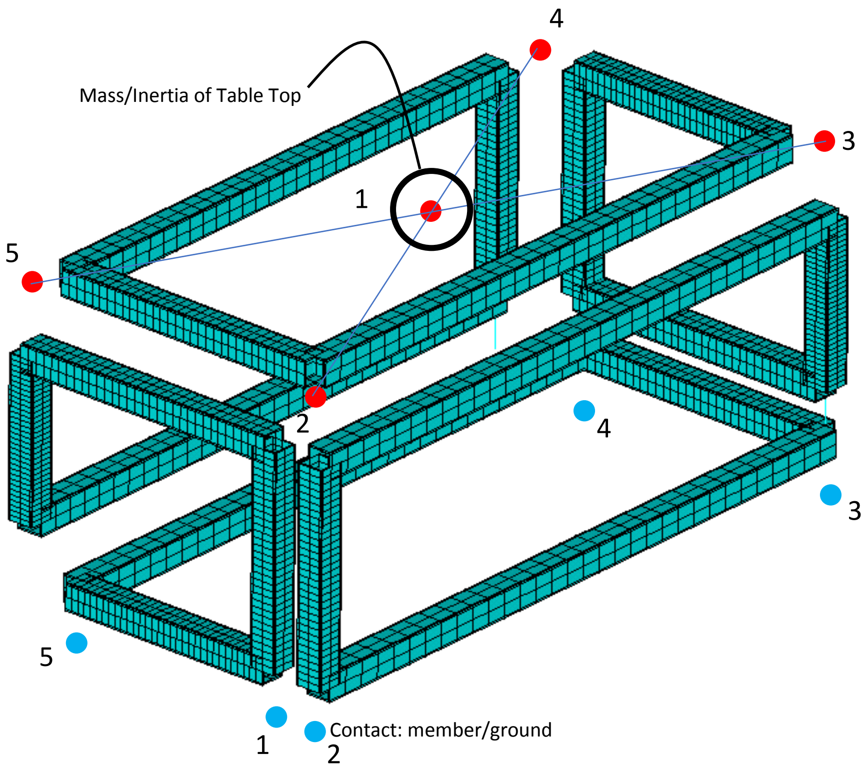
Figure 2: Images of infinite table along x axis for both ends

Using Figure 2 the coordinates of the 18 points are determined as shown in Table 3.

Point	x	y	z
1	0	Δ	0
2	0	$w-\Delta$	0
3	l	$w-\Delta$	0
4	l	0	0
5	0	0	0
6	0	0	$h-\Delta$
7	l	0	$h-\Delta$
8	l	0	Δ
9	l	$w-\Delta$	Δ
10	l	$w-\Delta$	h
11	l	Δ	h
12	Δ	Δ	h
13	Δ	w	h
14	$l-\Delta$	w	h
15	$l-\Delta$	w	Δ
16	0	w	Δ
17	0	w	$h-\Delta$
18	0	Δ	$h-\Delta$

Table 3: Coordinates of the 18 points

A finite element model was created using the coordinates given in Table 3 and the dimensions presented in Table 1. The model thus generated is shown in Figure 3 and uses 32 cubic shear deformable beam elements per member. This level of mesh refinement is an over-kill for this model but it avoids any question that the model is insufficiently refined.



The nodes identified as red circles are loading points. Node number 1 is coupled to all the nodes on the top surface of the table so that it moves as a plane – idealisation based on a stiff table top. Node numbers 2 to 5 are coupled to node number 1 to form loading points at the four corners. The five supports points, shown as blue circles, are connected to ground through contact elements so as to allow a vertex to lift should it so desire. The mass and inertia of the table top are applied to node number 1 which lies at the centre of the cuboid table top.

Figure 3: FE model, with load, support and contact conditions

The table is supported on its base. In the FE model the five vertices on the base are attached to ground through contact elements which allow the vertex to lift should it need so to do. It will be assumed that the top is made from 8mm toughened glass so that for a density of 2500kg/m^3 the mass of the top would be 10kg. The three principal moments of inertia for the top are then calculated as shown in Table 4. The mass of table frame and top is then a massive 38.7kg!

Property	Value
m, [kg]	10
I_{xx}, [kgm^2]	0.34
I_{yy}, [kgm^2]	0.97
I_{zz}, [kgm^2]	1.04

Table 4: Mass and inertia for glass table top

The loading for the table will be considered as a 100kg person sitting on the table under gravitational acceleration, i.e., a 1kN load. This load will be applied independently to the five loading points identified in the figure.

The structural response of interest is considered to be the maximum vertical deflection and the maximum elastic stress. The analysis is a non-linear one including the contact elements and large deformations. The first analysis undertaken is a modal analysis conduced without pre-stress. The shapes and frequencies for the first two modes are shown in Figure 4.

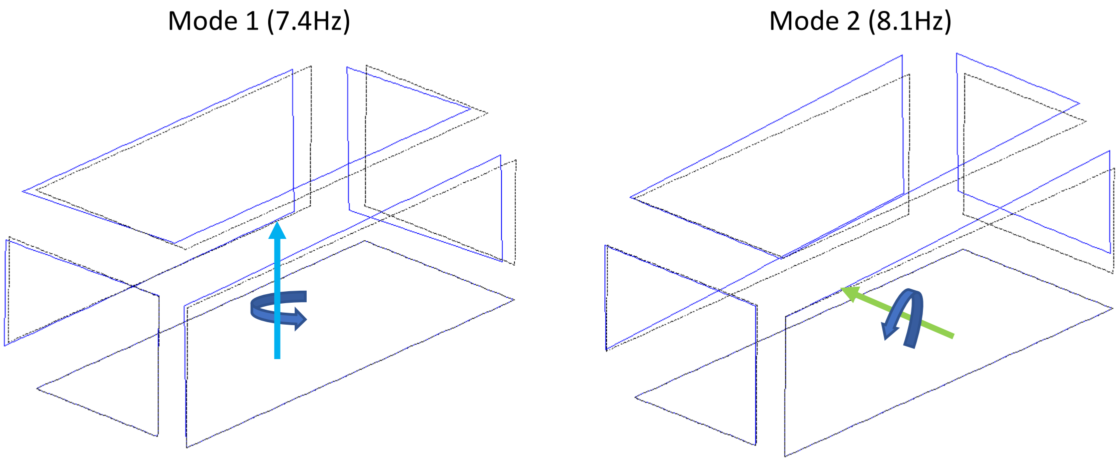


Figure 4: Natural frequencies and mode shapes

The displaced shapes and contour plots of von Mises (equivalent) stress are shown in Figure 5. The displacements of the table top and the maximum von Mises stress are shown for the 6 load cases in Table 5. The displacement of the table top is characterised by the vertical displacement at the centre of the top, Uz, and the two in-plane rotations, Rx and Ry of the same point. The maximum vertical displacement for the entire table, Uz(max) is also reported in the table.

		Live Loads on Points, [kN]					Kinematic Response				Static Response
Load Case	Self-Weight	1	2	3	4	5	Uz, [mm]	Rx, [degrees]	Ry, [degrees]	Uz (max), [mm]	Svm (max), [MPa]
0	Yes	0	0	0	0	0	3.8	-0.1	0.2	6.2	26
1	Yes	1	0	0	0		18.4	-0.1	0.5	23.7	106
2	Yes	0	1	0	0	0	15.8	0.8	-0.8	24.5	**168**
3	Yes	0	0	1	0	0	19.5	0.6	1.0	30.8	132
4	Yes	0	0	0	1	0	19.6	-0.8	1.1	**32.3**	129
5	Yes	0	0	0	0	1	15.5	-0.8	-0.7	25.0	164

Table 5: Displacements and stresses

The reaction forces for the six load cases are shown in Table 6. They have been rounded up to the nearest Newton. The sum is given in the last column.

		Live Loads on Points, [kN]						Reaction Forces, [N]						
Load Case	Self-Weight	1	2	3	4	5		1	2	3	4	5		Total, [N]
0	Yes	0	0	0	0	0		0	53	105	81	141		380
1	Yes	1	0	0	0	0		0	560	253	389	178		1380
2	Yes	0	1	0	0	0		283	323	356	0	418		1380
3	Yes	0	0	1	0	0		211	156	651	151	211		1380
4	Yes	0	0	0	1	0		0	106	225	524	524		1379
5	Yes	0	0	0	0	1		0	235	0	318	830		1383

Table 6: Reaction forces

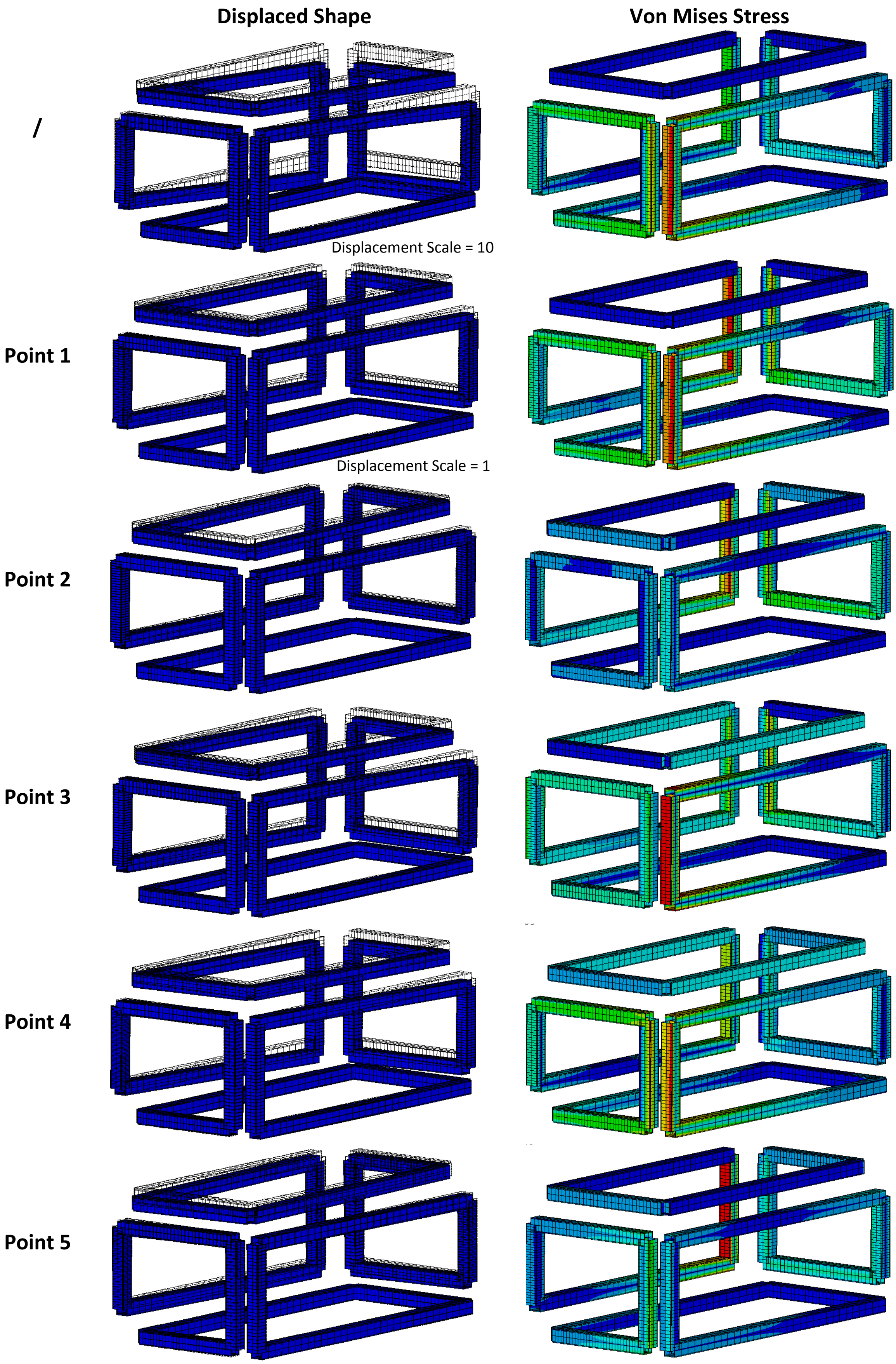


Figure 5: Displaced shape and equivalent stress contours

Closure

The stability of a table requires it to have at least three legs distributed sensibly around the periphery of the table top. If the table top is rectangular then four legs are required if the table is going to be stable for an arbitrarily positioned vertical load. If a leg is defined as a vertical support taking load to ground then the infinite table has two legs and rather than just transmitting an axial load to ground it also transits bending moments which are equilibrated by reactions through up to five of the vertices on which the base is supported. The structure of the infinite table is, therefore, unlike a standard configuration where the legs act as columns with minimal bending. This makes the infinite table rather flexible with a deflection under self-weight of over 6mm and, depending on which corner of the table you sit on, up to 32mm vertical movement. Whilst this is a large deflection, the stresses remain within the yield stress for the steel frame so that the deflections are elastic and the table will return to its original shape when the load is removed.

The infinite table attracted a lot of interest when it was posted on LinkedIn. Many appreciated its interesting, almost, Escheresque form whilst some of a structural engineering bent commented on its structural inefficiency. There is, of course, a link between beauty and structural efficiency which we see and appreciate whenever we look to what has evolved in nature. Nothing in nature is wasted. However, for the infinite table, which is practically speaking excessively flexible and massive at over 38kg we can conclude that it makes an inefficient use of materials. The infinite table may be compared to a conventional table as shown in Figure 6. This table, which was taken from the John Lewis website, has similar dimensions to the infinite table, is made of the same materials and weights about half that of the infinite table. It is likely also that the maximum deflection for this table will be less than 1mm.

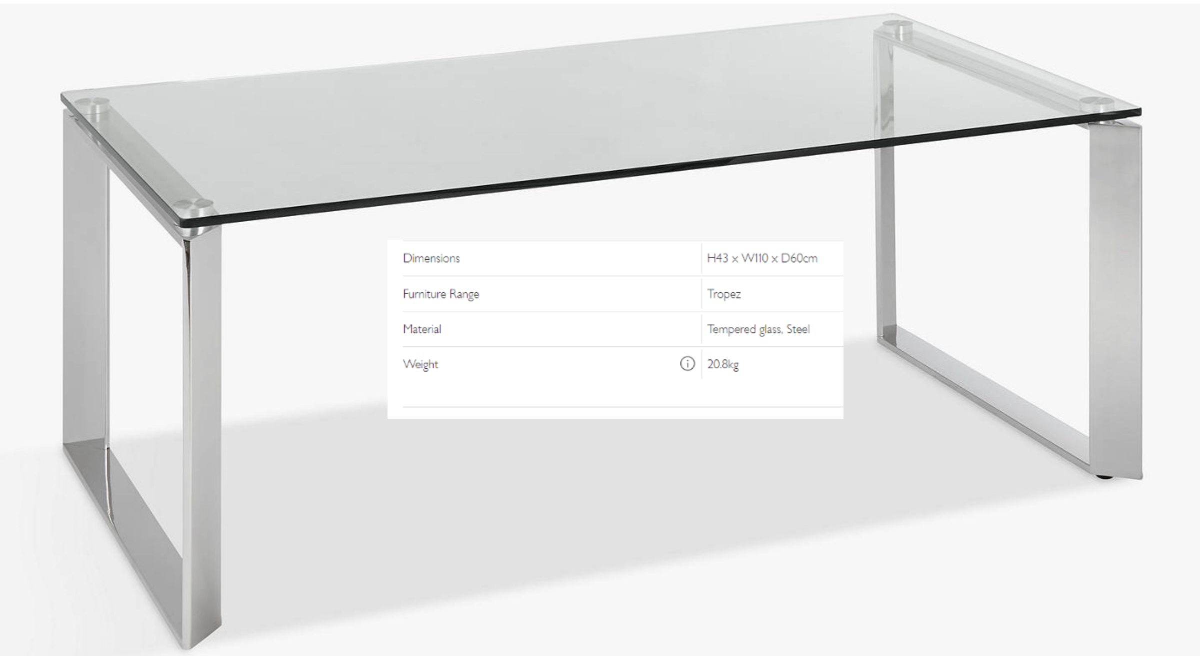
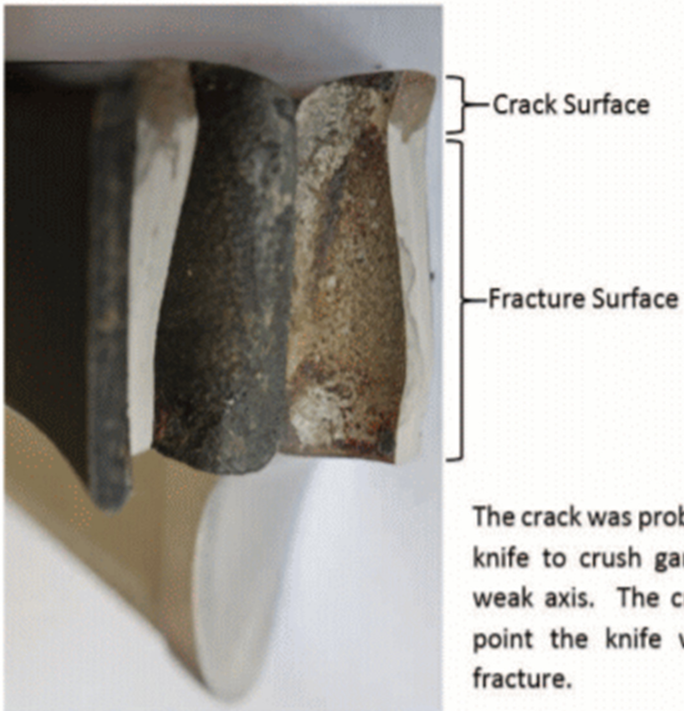
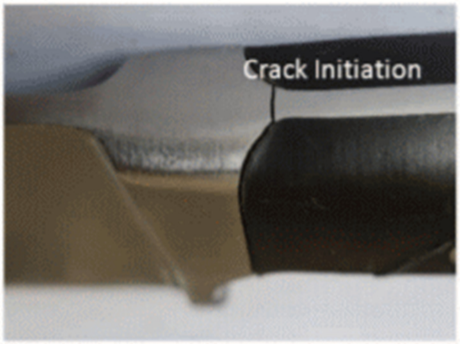


Figure 6: Conventional table of similar proportions to the infinite table

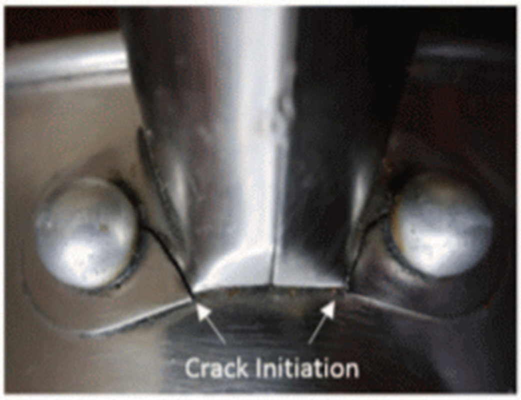
It is probably, therefore, reasonable to conclude that the infinite table, whilst curious, is not beautiful and, further, that it will not anytime soon be heading for mass production.

As a coda to this technical note, and because some in the LinkedIn post appeared to ignore the requirement for fatigue assessment of domestic artefacts, I have included Figure 7 which shows two examples from my own kitchen demonstrating fatigue failure.





The crack was probably initiated due to regular use of the knife to crush garlic which caused bending about the weak axis. The crack grew to the level seen at which point the knife was dropped and suffered a brittle fracture.



The cracks initiated at the sharp corners where the tabs for the two rivets used to join the handle to the pan were bent outwards from the pre-formed handle. The crack grew to the extend shown before the reduction in stiffness became noticeable. Had the pan continued to be used then it is likely that a ductile fracture would have been observed with the handle tearing from the pan.

Figure 7: Fatigue failure in kitchen utensils