

*An equilibrium shell element for folded plate structures*

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Abstract

In this paper we investigate the use of a quadrilateral flat shell hybrid equilibrium element to model plate and folded plate structures. This element is assembled from four triangular elements in order to avoid spurious kinematic modes [1]. Reissner-Mindlin theory is generally assumed for plate bending behaviour where the distinction between soft and hard kinematic boundary conditions must be recognised, and the presence of soft conditions usually results in narrow boundary layers containing large gradients of torsional moments and shear forces. Benchmark tests are reported in the context of linear elastic theory for separate membrane and plate bending problems, in order to demonstrate the performance of elements with varying degrees of stress or moment fields.

Flat shell elements with linear fields of membrane stress and transverse shear forces combined with quadratic fields of moment are then used to model a folded plate problem. This type of problem has extra complexity as regards the modelling of the interaction between plates along fold connections. When displacement elements are used with nodal degrees of freedom, drilling freedoms at nodes along a fold connection may be included so as to lead to a fully conforming solution [2,3]. The hybrid equilibrium element with Reissner- Mindlin theory has six rigid body kinematic freedoms for each side, but side tractions cannot generally be completely co-diffusive along a fold without invoking the concept of couple stresses similar to those in microcontinuum theories [4].

A simpler way to model such connections so as to maintain co-diffusive tractions is to release the torsional moments along a fold in a similar way as for soft supports. The implications for the kinematic aspects of the connections are explained in terms of an analogy with the vertebra of the spine thus demonstrating that such connections are physically possible, although unlikely in reality. The advantage of the simpler connection is that complete equilibrium of plate stress-resultants can be satisfied and an upper bound derived for the strain energy of a force driven problem. These connections are implemented for elements of degree 2 and results are compared with those from a model composed of conforming elements that exclude drilling freedoms. Although the equilibrium model is free from shear locking, a penalty is shown to be that for very thin plates boundary layers exist along the folds that can lead to severe discontinuities in moments and shear forces, although they remain statically admissible.

The paper concludes with proposals for further research into alternative ways of modelling the folds, and considering large displacements in a corotational formulation. The concept of a 1-D hybrid pseudo-equilibrium spine element is proposed for the purposes of transmitting non-codiffusive torsional and transverse shear stress resultants along a fold. The spine

element concept could also be used to connect flat shell hybrid equilibrium elements when bending is governed by Kirchhoff theory, and could be extended to represent a plate stiffener.

#### References

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This contribution should be related to theme: **A**- analysis of spatial structures