# ANSYS: Potential Issue with Element Table Operation

Under certain conditions, the finite element (FE) method recovers the exact theoretical solution for the average stress over the domain of the mesh. Thus, if the theoretical stress field is $\sigma$ and the finite element stress field is $\tilde{\sigma}$ then Eq(1) holds, where $V$ is the volume of the FE model.

**Equality of Average Stress**

$$\frac{1}{V}\int \tilde{\sigma} dV = \frac{1}{V}\int \sigma dV \quad (1)$$

The conditions required for this equality to hold include:

- That boundary tractions and body loads are applied in a consistent manner.
- That the integration is taken over the complete model.
- That the integration scheme used by the FE system is an appropriate one. Numerical evidence shows that full or reduced Gauss quadrature, [1], is appropriate whereas some selective integration schemes are not.

The reader will realise that the FE solution, whilst generally approximate for a given mesh, is capable of converging to the theoretical solution with appropriate mesh refinement. Thus, even for a coarse FE mesh, the approximate FE solution must have some characteristics in common with the theoretical solution and it turns out that the way it does it is as shown in Eq(1).

From an engineers' viewpoint, it might seem that Eq(1) is of little practical use, however, there are a couple of cases where this equality can be of value to the practising engineer:

- For rotating discs the average hoop stress is often used, sometimes erroneously, to predict the plastic burst speed, [2].
- Under certain conditions, the average shear stress in a beam modelled as a two or three-dimensional continuum can be used to provide the exact shear stress at the centroid of the beam cross section, [3].

## Integrating FE Stresses

Let us consider a two-dimensional continuum which is modelled with standard, low-fidelity, conforming finite elements (CFE). Such CFE systems typically offer lower and higher degree, four and eight noded, elements for this purpose. The lower order element is usually provided with reduced (1x1) and full (2x2) Gauss quadrature (integration) schemes. For the higher order element usually only reduced (2x2) integration is offered. The Gauss quadrature scheme replaces the integral by a summation of FE stresses at $n \cdot n = n^2$ Gauss or integration points over element *e* as shown in Eq(2).

**FE Integration for Average Stress**

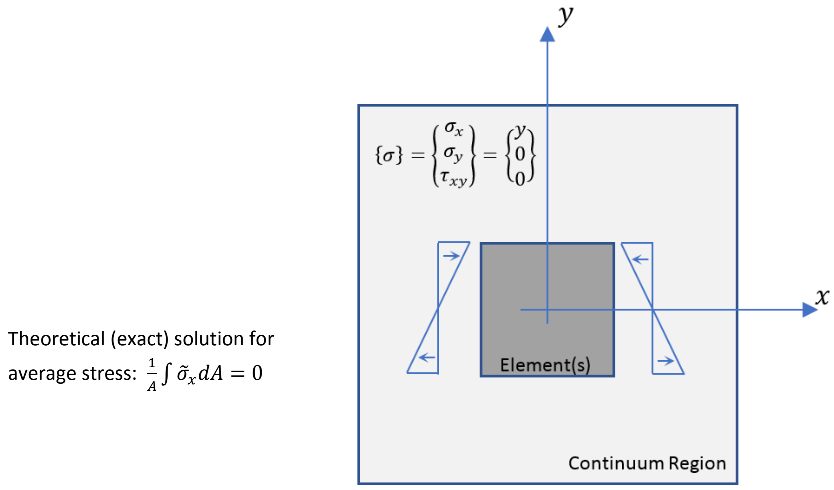
$$\frac{1}{A}\int \tilde{\sigma} dA = \frac{1}{A}\sum_{i=1}^{ne}\sum_{j=1}^{n^2}(\tilde{\sigma} \cdot J \cdot \omega)_j \quad (2)$$

*A* is the area, $\omega$ are the integration weightings, $J$ is the Jacobian and *ne* is the number of elements in the mesh.

The average stress over the mesh of elements is obtained by summing the elemental values for all the elements in the mesh – remember the equality of Eq(1) only holds for the complete mesh.

## A Simple Example

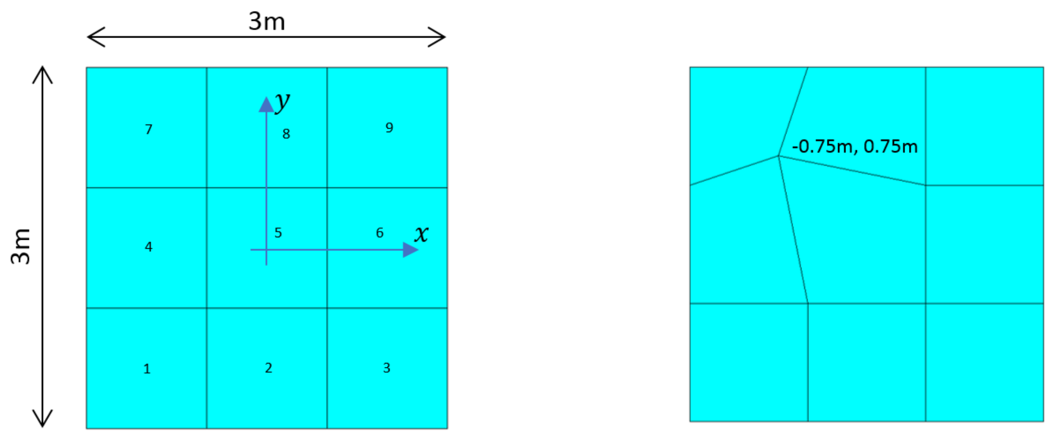
In Figure 1, the Continuum Region Element (CRE) Method of John Robinson is applied to a mesh of elements. The mesh is centred at the origin of the continuum region over which a stress field that satisfies the equilibrium conditions with zero body forces, and which has corresponding strains that are compatible, i.e., a statically and kinematically admissible stress field, is defined. The boundary tractions from the stress field are shown as they act on the mesh of elements – the mesh sees a moment being applied to left and right hand edges and is, thus, sometimes called the constant moment problem.




A square continuum region is shown in the $x, y$ plane. The stress field shown applies in this region and represents that corresponding to a constant moment in a beam under a point load. The stress field is both statically and kinematically admissible and so represents a known theoretical solution. The boundary tractions from this stress field acting on the finite elements are shown and there are no body forces to consider. These boundary tractions need to be applied in a consistent manner if the finite element solution is to converge to the theoretical solution.

**Figure 1**: Constant moment problem using CRE-Method

Consider a 3m by 3m square model meshed with 3x3=9 quadrilateral elements as shown in Figure 2. Two meshes will be considered, the first being a regular undistorted mesh and the second being distorted by moving one of the nodes as shown.




(a) Undistorted Mesh (b) Distorted Mesh

**Figure 2**: Meshes for the constant moment problem

In ANSYS the lower order element is called Plane182 and the higher order element Plane183. The model of Figure 1 was analysed in ANSYS using the meshes of Figure 2 and the FE stresses ($\tilde{\sigma}_x$) are shown in Figure 3 where it is seen that the only model capable of capturing the exact solution is that with the undistorted Plane183(Reduced) elements. Whilst the solution is independent of the elastic properties, the values used for this study were 210GPa for Young's modulus, and 0.3 for Poisson's ratio.

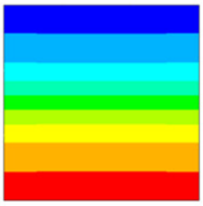
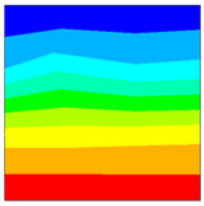
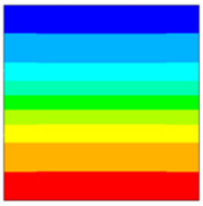
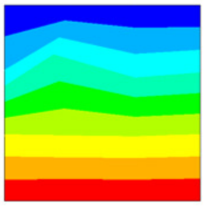
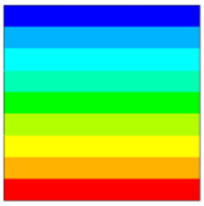
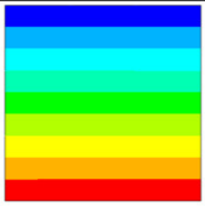
| | **Undistorted** | **Distorted** |
|---|---|---|
| **Plane182 (Reduced)** |  $-1.125 \le \sigma_x \le +1.125$ |  $-1.166 \le \sigma_x \le +1.151$ |
| **Plane182 (Full)** |  $-1.454 \le \sigma_x \le +1.454$ |  $-1.450 \le \sigma_x \le +1.453$ |
| **Plane183 (Reduced)** |  $-1.500 \le \sigma_x \le +1.500$ |  $-1.501 \le \sigma_x \le +1.501$ |

**Figure 3**: Finite element stress ($\tilde{\sigma}_x$) for the constant moment problem

To get ANSYS to calculate the average stress the Element Table features are used. Firstly, SX and VOLU need to be added to the table and then a new term SX multiplied by VOLU – this quantity is called I in Figure 4. This quantity is then summed over the elements in the mesh and divided by VOLU to provide the average stress.

```
SUM ALL THE ACTIVE ENTRIES IN THE ELEMENT TABLE

TABLE LABEL     TOTAL
SX       -0.240116
VOLU       9.00000
I       -0.334070E-02
SXAVE    -0.371189E-03
```

**Figure 4**: ANSYS Element Table result for average stress for distorted mesh of Plane182(Full) elements

The average stress (SXAVE) reported by ANSYS and shown in Figure 4 is different from the theoretical value which is zero. Table 1 presents the results for the six meshes considered and indicates whether the calculation of the average stress through the ANSYS Element Table features are exact or approximate. The weights for the Gauss quadrature schemes are also shown in the table.

| | $n$ | ω | **Undistorted** | **Distorted** |
|---|---|---|---|---|
| **Plane182 (Reduced)** | 1 | 2 | Exact | Exact |
| **Plane182 (Full)** | 2 | 1 | Exact | Approximate |
| **Plane183 (Reduced)** | 2 | 1 | Exact | Approximate |

**Table 1**: Quality of ANSYS calculation of the average stress & weights for integration schemes

The reason why ANSYS only provides an approximate answer to the average stress when the elements are distorted and when a 2x2 Gauss quadrature scheme is adopted is that the software appears to adopt an approximate method rather than exact method. The calculation for the distorted mesh of Plane182(Full) elements is shown in Table 2. The approximate expression used by ANSYS to calculate the area weighted stress for an element is compared with the exact method in Eq(3). The approach used by ANSYS is equivalent to calculating the average stress from the four Gauss points and then multiply this by the area of the element. When a single Gauss point is used, or when the element is undistorted, the approximate approach used by ANSYS leads to the correct solution but this is not the case, as shown, when a 2x2 integration scheme is used and the element is distorted.

**Integration used in ANSYS**

$$\sum_{j=1}^{n^2} (\tilde{\sigma}_x \cdot J \cdot \omega)_j \neq \frac{1}{n^2} \sum_{j=1}^{n^2} (\tilde{\sigma}_x)_j \cdot \sum_{j=1}^{n^2} (J \cdot \omega)_j \qquad (3)$$

| | $\tilde{\sigma}_x$ | | | | | Exact | ANSYS | Distorted |
|---|---|---|---|---|---|---|---|---|
| Element | GP1 | GP2 | GP3 | GP4 | Sum | $\sum_{i=1}^{n^2} (\tilde{\sigma}_x \cdot J \cdot \omega)_i$ | $\frac{1}{n^2}\sum_{i=1}^{n^2} (\tilde{\sigma}_x)_i \cdot \sum_{i=1}^{n^2} (J \cdot \omega)_i$ | |
| 1 | 1.22410 | 1.22880 | 0.65667 | 0.65203 | 3.76160 | 0.94040 | 0.94040 | No |
| 2 | 1.24020 | 1.23350 | 0.64150 | 0.64822 | 3.76342 | 0.94086 | 0.94086 | No |
| 3 | 1.23470 | 1.24110 | 0.66984 | 0.66346 | 3.80910 | 0.95228 | 0.95228 | No |
| 4 | 0.30514 | 0.32127 | -0.32798 | -0.45635 | -0.15792 | -0.01142 | -0.03948 | Yes |
| 5 | 0.28128 | 0.22947 | -0.47232 | -0.35323 | -0.31480 | -0.11940 | -0.09838 | Yes |
| 6 | 0.32092 | 0.31810 | -0.32281 | -0.31998 | -0.00377 | -0.00094 | -0.00094 | No |
| 7 | -0.87632 | -0.71206 | -1.23040 | -1.28450 | -4.10328 | -0.79002 | -0.76937 | Yes |
| 8 | -0.69395 | -0.75930 | -1.22690 | -1.22940 | -3.90955 | -0.96042 | -0.97739 | Yes |
| 9 | -0.66314 | -0.67374 | -1.23950 | -1.22890 | -3.80528 | -0.95132 | -0.95132 | No |
| Sum | | | | | **-0.96048** | **0.00000** | **-0.00334** | |
| Sum/Area | | | | | | **0.00000** | **-0.00037** | |
| | $J \cdot \omega$ | | | | | | | |
| Element | GP1 | GP2 | GP3 | GP4 | Sum | | | |
| 1 | 0.25000 | 0.25000 | 0.25000 | 0.25000 | 1.00000 | | | |
| 2 | 0.25000 | 0.25000 | 0.25000 | 0.25000 | 1.00000 | | | |
| 3 | 0.25000 | 0.25000 | 0.25000 | 0.25000 | 1.00000 | | | |
| 4 | 0.25000 | 0.28608 | 0.25000 | 0.21392 | 1.00000 | | | |
| 5 | 0.31250 | 0.27642 | 0.31250 | 0.34858 | 1.25000 | | | |
| 6 | 0.25000 | 0.25000 | 0.25000 | 0.25000 | 1.00000 | | | |
| 7 | 0.18750 | 0.15142 | 0.18750 | 0.22358 | 0.75000 | | | |
| 8 | 0.25000 | 0.28608 | 0.25000 | 0.21392 | 1.00000 | | | |
| 9 | 0.25000 | 0.25000 | 0.25000 | 0.25000 | 1.00000 | | | |
| Sum | | | | | **9.00000** | | | |

The average stress shown in the ANSYS column agrees with that calculated through the ANSYS Element Table feature and is seen to be incorrect – it should be zero.

**Table 2**: Calculation of average stress for distorted mesh of Plane182(Full) elements

## Closure

This technical note has demonstrated how ANSYS appears to adopt an approximate approach for the calculation of such quantities as the average stress when using the Element Table features of the software. The reason for this is surmised to be that, as a legacy code, ANSYS was written at a time when any reduction in floating point operations (FLOPS) was worth having even if, as in this case, the accuracy might be compromised. Clearly with modern computer power this should not be an issue and the engineer would certainly prefer to have accurate values for such quantities. There is, it is believed, with certain software vendors, a reluctance to make any change to the software so as to ensure that when a customer runs a legacy database, the same result as was achieved by earlier versions of the software is obtained.

As the error obtained using the approximate approach in this example is rather small, it is not considered to be normally of practical engineering interest. However, the fact that this error exists at all demonstrates leads the engineer to question what other calculations have been fudged within the software he/she is using!

## References

[1] http://math2.uncc.edu/~shaodeng/TEACHING/math5172/Lectures/Lect_15.PDF

[2] Angus Ramsay, NAFEMS Benchmark Challenge Number 8, NAFEMS Benchmark Magazine, January 2017.
http://www.ramsay-maunder.co.uk/knowledge-base/projects/nafems-benchmark-challenge/nbc-number-8/

[3] Angus Ramsay, NAFEMS Benchmark Challenge Number 6, NAFEMS Benchmark Magazine,
http://www.ramsay-maunder.co.uk/knowledge-base/projects/nafems-benchmark-challenge/nbc-number-6/