

Efficient Design through Lower-Bound Limit Design

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There is an urgent need for engineers to become more efficient in their use of materials if the world's targets for reducing emissions are to be met. The built environment alone is estimated to account for around 50% of all carbon emissions, [1]. Partial factors of safety adopted in the Eurocodes are such that a unit utilisation is deemed acceptable to account for uncertainties in material strength and applied loads, yet buildings are regularly designed inefficiently with utilisations of 0.5 or less.

Many of the materials used in engineering are ductile, e.g., steel and reinforced concrete and to ignore the ability of such materials to redistribute load is to use material inefficiently. If formulated through the lower-bound theorem of plasticity, with strong equilibrium, then through limit analysis a design can be guaranteed *a priori* to be safe and through limit design to be both safe and efficient.

RMA recently undertook a limit analysis of a heavily loaded reinforced concrete slab. There was a concern that the strength of the slab had been compromised by the misplacement of some of the reinforcement. However, limit analysis showed that even without the misplaced steel, the slab had 27% more capacity than was actually required. Limit design of the slab might have brought the utilisation down to unity thereby saving considerable amounts of concrete and steel having to be manufactured and transported to site. The balcony slab shown in Figure 1 demonstrates how by simply rotating the reinforcement through 45 degrees, 50% of the steel may be discarded without having any significant influence on the strength of the structure, [2].

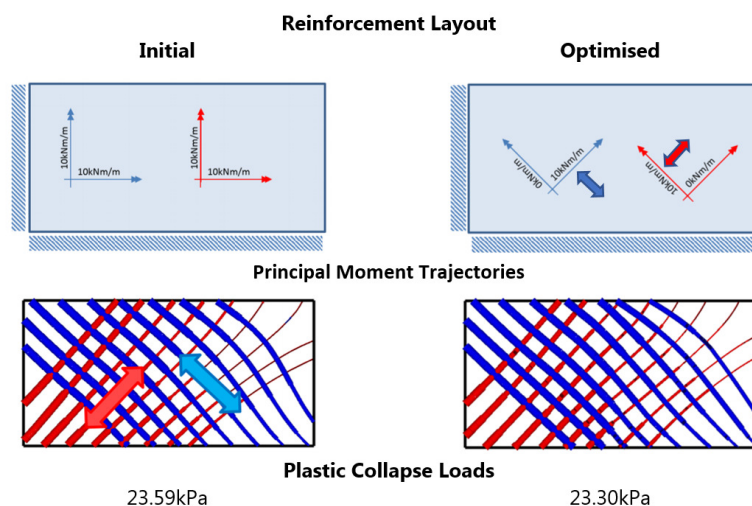


Figure 1: Optimised reinforcement for a rectangular balcony slab

In mechanical engineering safety crucial artefacts such as pressure vessels and high-speed rotating discs, both of which are governed by the Lamé equations, seem highly amenable to limit analysis and design. In this context, RMA have recently developed the Lamé Finite Element (LFE) for the linear-elastic analysis of such components. The element can also be used to predict the plastic limit load and, as the LFE is a strict equilibrium element, it does so in a safe manner. Figure 2 illustrates how the LFE may be used to produce design charts for a rotating disc.

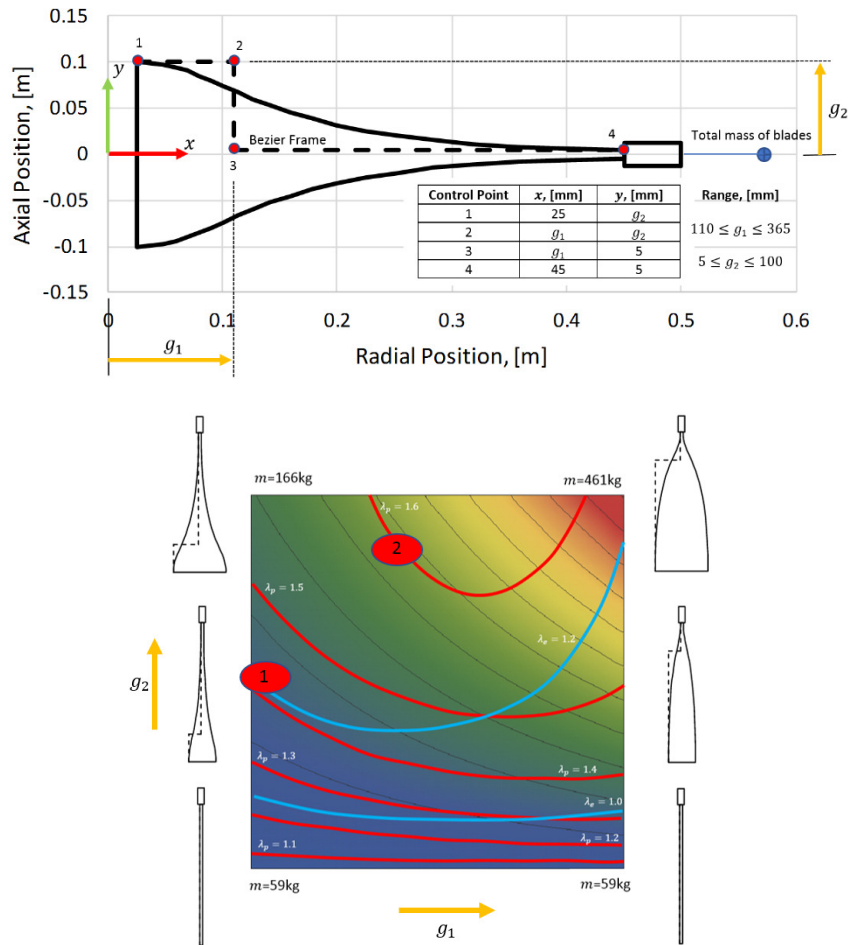


Figure 2: Design charts for a rotating bladed disc

Further R&D is currently being undertaken to develop the mathematical programming techniques to enable limit analysis and design of such components. Whereas with iterative or incremental plastic analysis, fairly fine meshes are required to accurately predict the plastic limit load, with limit analysis highly accurate solutions are obtained using a single element. The design chart for thick-walled pipes shown below was generated using the LFE and before limit analysis had been fully explored, [3].

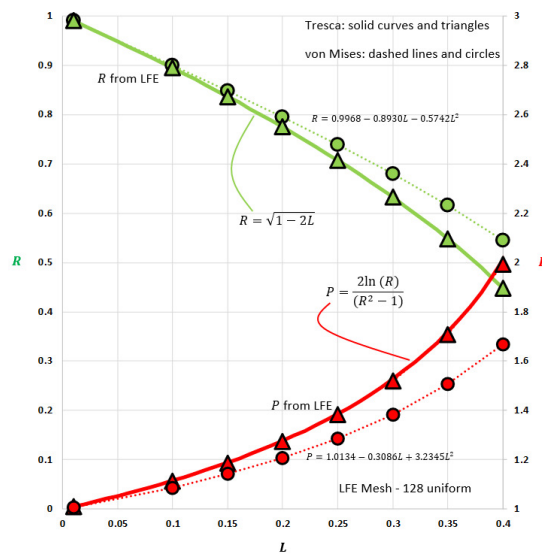


Figure 3: Design chart for internally pressurised cylinders

The presentation for NWC-2021 will be based around limit analysis and design and it is anticipated that results for the geometric optimisation of rotating discs through limit design will be available for discussion.

References

[1] Minimising Energy in Construction (MEICON) is an EPSRC funded project involving researchers at the universities of Cambridge and Bath, (<https://www.meicon.net/>)

[2] Ramsay, A.C.A. & Maunder, E.A.W., *Equilibrium Finite Elements for RC Slab Design*, Structure Magazine (US), July 2017.

<https://www.structuremag.org/?p=11774>

<https://www.ramsay-maunders.co.uk/knowledge-base/publications/equilibrium-finite-elements-for-rc-design/>

[3] Ramsay, A.C.A., *Safe and Economical Simulation for the Built Environment*, NAFEMS Benchmark Magazine, April 2021.

<https://www.ramsay-maunders.co.uk/downloads/Simulating-Tomorrow.pdf>