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STRONG EQUILIBRIUM IN FEA – AN ALTERNATIVE PARADIGM?

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ABSTRACT

This paper presents a brief review of some recent and current research in the formulation and application of hybrid equilibrium elements, with emphasis on the design and assessment of structures.

Equilibrium is a key requirement in design or assessment although the precise meaning of the term is somewhat ambiguous in practice. This ambiguity played its part in the collapse of the Sleipner offshore oil platform in 1991 [1]. So the question arises: why don't we take more advantage of "strong" equilibrium via assumed stress fields in equilibrium finite elements (EFE), as opposed to "weak" equilibrium as derived from assumed displacement fields in compatible finite elements (CFE)?

The seeds of strong equilibrium in EFE were actually sown some 50 years ago [2], but software development for commercial use took the easier path towards CFE. So much investment has been put into CFE that there is now little knowledge of alternatives, and there is resistance to changing the status quo unless industry demands it. However, in general industry is unaware of strong equilibrium as a possible alternative! It should be noted that equilibrium models also have an important role to play in dual analyses for the purposes of verification and error estimation.

The usual form of the hybrid equations is given in Eq. (1) [3], as well as the derivation of the element stiffness ***K*** matrix which would allow incorporation into commercial types of FE software.

$$\begin{bmatrix} -\boldsymbol{F} & \boldsymbol{D}^T \\ \boldsymbol{D} & 0 \end{bmatrix} \begin{Bmatrix} \hat{\boldsymbol{s}} \\ \hat{\boldsymbol{v}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \hat{\boldsymbol{g}} \end{Bmatrix} \Rightarrow \boldsymbol{K}\hat{\boldsymbol{v}} = \{\hat{\boldsymbol{g}}\} \text{ where } \boldsymbol{K} = \boldsymbol{D}\boldsymbol{F}^{-1}\boldsymbol{D}^T \tag{1}$$

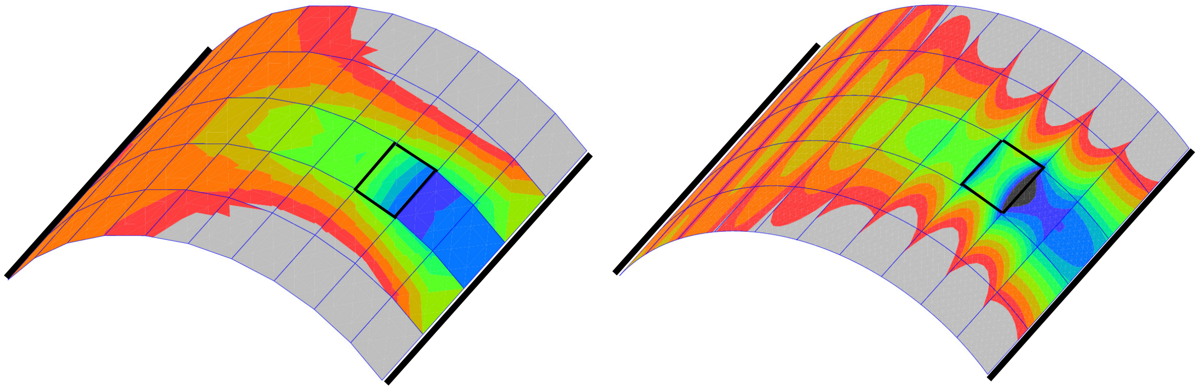
In Eq. (1) $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{v}}$ vectors collect the internal stress field and side displacement field parameters respectively; the $\hat{\boldsymbol{g}}$ vector contains the load parameters which are dual to the displacement ones. Recent developments and applications are discussed in [3,4,5]. This paper will focus on two areas which should be of interest to practicing engineers:

Limit analyses of plates –lower bound (safe) solutions based on different yield criteria: Nielsen for reinforced concrete slabs [4], and current research on von Mises for steel plates. This area is of particular relevance to future emphasis on efficient use of resources with the need to reduce energy use and consequent benefits to climate change [6].

Load paths in the assessment of masonry arches – the arch ring has been modelled using meshes of flat shell elements [5] (i.e. with sufficient elements to capture the main geometric features). Current equilibrium models are restricted to elements with linear geometry which implies that curved surfaces are approximated by piecewise linear ones. The isoparametric or isogeometric concepts used with compatible models are more problematic when imposing strong equilibrium.

Figure 1 compares typical results for circumferential compressive membrane forces in an arch of constant radius, thickness and width, obtained from equilibrium and compatible models of degree 2 for moment and displacement fields respectively. The models have simple supports at the abutments, and support a vertical load uniformly distributed over the single element indicated by the solid outline.

Although deflected shapes of the sides of the elements are very similar, as are the total strain energies of the models, the distributions of stress-resultants show significant differences. Reasons for these differences will be discussed in the presentation. However the practicing engineer may be more interested in load paths and understanding the pattern of associated stress-resultants within a structure. Equilibrium models provide this information in a way which is completely consistent with a strong form of equilibrium. This is in contrast to output from compatible models where equilibrium is generally compromised by inappropriate displacement assumptions.



equilibrium forces $\leq$ 36.2kN/m compatible forces $\leq$ 51.8kN/m

Figure 1: *Contours of circumferential membrane forces. The same colour coding is used for each set of contours: tension: grey, compression > 0: red (zero) to blue (maximum).*

When we consider the dimensions of masonry arches it may be more useful to model with 3D elements. We will present some recent research ideas concerning the formulation of a hexahedral solid equilibrium element. This will be regarded as a super-element composed of 12 tetrahedral sub-elements whose external tractions are restricted to polynomial distributions on the faces of a hexahedron that ensure codiffusive tractions and strong equilibrium with internal stress fields as required by an equilibrium element. For addressing the question of load paths, the assumed traction distributions can be simplified to just linear ones representing stress-resultants.

It is concluded that EFE should be an essential tool for the structural engineer, either in design/assessment, or as a complement to conventional CFE for the purposes of error estimation.

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