Automated Yield-Line Analysis for the Limit Analysis of Plates

by
Dr A.C.A. Ramsay (Visiting Research Fellow)
Department of Civil & Structural Engineering, Nottingham Trent University, Nottingham, NG1 4BU, UK.

Introduction

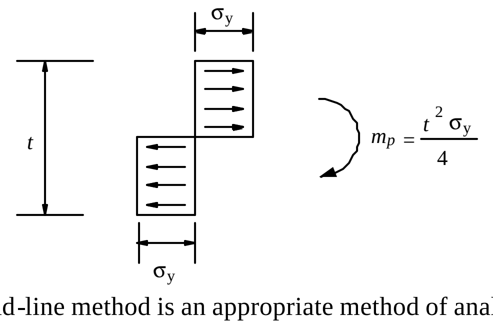
The motivation for writing this article came from the NAFEMS publication *Selected Benchmarks for Material Non-Linearity* [1] in which the elasto-plastic collapse of plate-type structures is considered using conventional finite elements. The results for two configurations of square plate under uniformly distributed load, one with simply supported edges (case NL7A) and the other with clamped edges (case NL7B) are presented in this publication.

Recent involvement in the development of automated methods for the limit analysis of plates encouraged the author to compare these solutions with those given by the yield-line method. These comparisons are presented together with a brief description of some recent developments in automated yield-line analysis.

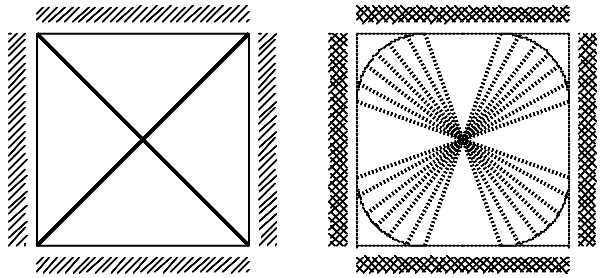
Yield-Line Analysis

The yield-line method, as originally proposed by Johansen [2[1]], involves the postulation of a yield-line pattern. The segments of the plate, which are delineated by the boundaries of the plate and the yield-lines, remain plane and are assumed rigid. The segments of the plate can rotate relative to each other about the yield-lines and the collapse load may be determined by equating internal virtual work done along the yield-lines by the relative rotation of the segments of the plate against the plastic moments of resistance along the length of the yield-lines to the external virtual work which is performed by the applied loads displacing through a displacement field that is compatible with the yield-line pattern.

For a plate of thickness t and with equal yield stress σ_y in tension and compression, the plastic moment of resistance per unit length m_p is given as:



The yield-line method is an appropriate method of analysis for standard configurations of plate in which the critical or exact fracture pattern is known *a priori*. This is the case for the two configurations of plate under discussion and the critical fracture patterns, which are taken from reference [3], are given below. In these figures single hatching represents simply supported edges whilst double hatching represents clamped edges. The bold lines represent positive (hogging) yield-lines and the dotted lines negative (sagging) yield-lines. With the plate thickness given as t = 0.4 and the yield stress as σ_y = 30, the plastic moment capacity per unit length is calculated as m_p = 1.2.



Case NL7A Case NL7B

The collapse loads corresponding to these yield-line patterns are then 0.0180 and 0.0322 respectively for the two configurations of plate. These values, which are the critical or exact values [3], are lower than those reported in reference [1] which are 0.0188 and 0.0385 respectively. There are, perhaps, two principal reasons for these differences. Firstly, the yield criteria are different. Whereas the finite element analysis of reference [1] chose Von Mises yield criterion, the yield-line analysis uses a simple 'square' yield criterion [3][*] . Secondly, the finite element model uses a relatively course mesh and since the method provides an upper-bound to the critical value of the collapse load, mesh refinement should lead to a reduction in the prediction of the collapse load.

The rigid-plastic assumption upon which the yield-line method is based provides significant simplification to the problem of determining the collapse load for plates. The method does, however, rely on *a priori* knowledge of the critical fracture pattern; knowledge which the finite element method does not require and which for more complicated configurations of plate is generally not available. In such cases a different approach is adopted.

Automated Yield-Line Analysis

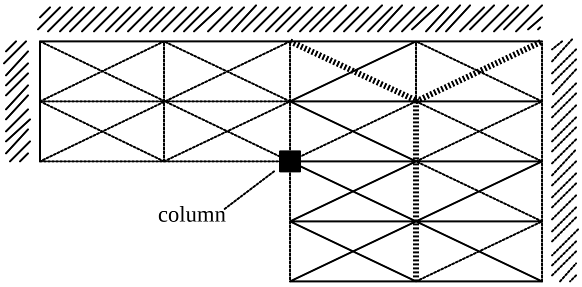
By virtue of the upper-bound theorem of plasticity, the yield-line method can be shown to provide solutions which, when not exact, give a collapse load that is greater then the critical value. As such, in the absence of the critical fracture pattern, a number of likely candidates can be tried and the one with the lowest collapse load being deemed nearest to the exact solution. Performing such searches by hand is tedious and, unless carried out exhaustively, is liable to lead to erroneous and unsafe solutions.

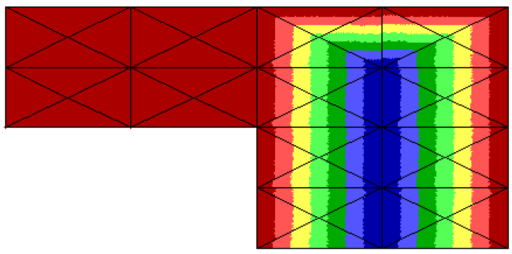
The concept of comparing the collapse loads for a number of different yield-line patterns was effectively automated by Munro & Fonseca [2[6]]. In their method, the plate is discretised as a mesh of rigid triangular elements for which the interfaces between elements and any moment-resisting

[*] a comparison between these two yield criteria for circular plates can be found in reference [4].

boundaries are considered as potential yield-lines. Linear programming is then used to determine the particular pattern with the lowest collapse load. This method is demonstrated below for a reinforced concrete landing.

The landing is simply supported on three edges with additional support being given by a corner column. The plastic moment of resistance is determined from the distribution of reinforcement. Top and bottom reinforcement are equal and assumed to be isotropic and uniform over the area of the landing so that the plastic moment of resistance at any point, and in any direction is characterised by the single parameter m_p which is the same in both sagging and hogging. The mesh used is as shown and a uniformly distributed load is applied.





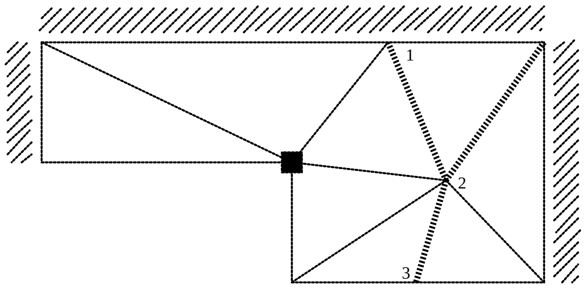
Of the possible fracture patterns defined by this mesh, the one with the lowest corresponding collapse load is shown above together with a contour plot of the normal displacement. The collapse load is $13.04m_p$.

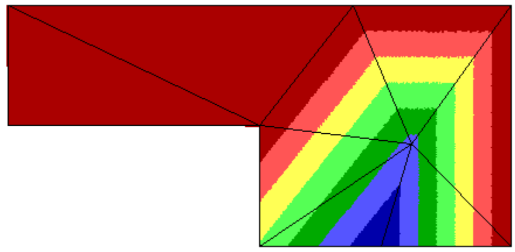
Because the selection of critical fracture pattern is based on a finite set of possible patterns as defined by the chosen mesh, it is possible for the critical fracture pattern not to be detected. The critical pattern will only be determined when the chosen mesh has element edges which coincide with the yield-lines of the critical fracture pattern and this situation cannot be guaranteed even with highly refined meshes[2^8].

Geometric Optimisation of Fracture Patterns

To help overcome this potentially unsafe situation, geometric optimisation of the fracture pattern can be carried out. An automated yield-line analysis, as discussed above, is performed in order to detect the correct mode shape of the critical fracture pattern. A further analysis is then carried out on a coarse mesh which is constructed so as to model accurately this mode of fracture pattern. The positions of the nodes defining this fracture pattern are treated as variables whose position is determined so as to minimise the collapse load [2^9,10].

Performing geometric optimisation of the fracture pattern already determined for the reinforced concrete landing results in the following solution.





The four geometric variables considered were the x-coordinates of nodes 1 & 3 and the x- and y-coordinates of node 2. The collapse load corresponding to the optimised fracture pattern is $9.12m_p$ which represents a significant 30% reduction on that previously predicted.

Closure

The methods discussed in this article provide an alternative approach to the limit analysis of plate-type structures. Whilst the results presented show the utility of these approaches, they are still the subject of research and although recent developments have lead to significant improvements [2], their effective use does still require a certain degree of engineering judgement which may not be required with conventional finite element analysis.

References

[1] D. Linkens, 'Selected Benchmarks for Material Non-Linearity', NAFEMS (R0026), 1993.
[2] A.C.A. Ramsay & D. Johnson, 'Geometric Optimisation of Yield-Line Patterns using a Direct Search Method', to be published later this year in Structural Optimization.
[3] E.H. Mansfield, 'Collapse Analysis of Rigid-Plastic Plates with a Square Yield Diagram', Proc. Roy. Soc. Series A. Vol 241, 311-337, 1957.
[4] R.H. Wood, 'Plastic & Elastic Design of Slabs & Plates', Thames & Hudson, London, 1961.

Note: [2^n] means reference number n in the list of references given in reference [2].

Acknowledgement

The author is grateful to Dr David Johnson of The Nottingham Trent University for reviewing this article.