ECMM108 Advanced Structural Engineering

Appendix: Automated Yield Line Technique – Demonstration of EFE-YL

Introduction

A demonstration of an automated or computerised technique for the yield line analysis of ductile plates is to be given as implemented in the software EFE-YL. The example chosen for consideration is problem 4 from Assignment 4 for this module and is shown in Figure 1. The reinforcement is isotropic (equal strength in all directions both in sagging and hogging). Two edges are simply supported and a UDL is applied to a strip as shown in the figure.

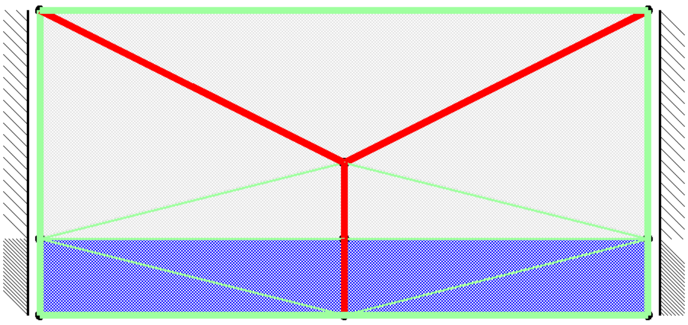


Figure 1: Geometry, boundary conditions and loading (Problem 4 from Assignment 4)

The yield line pattern assumed for the assignment, which may not be the correct pattern, is as shown in Figure 1 and comprises a symmetric 'Y' pattern of sagging (red) yield lines.

Model Construction

The automated technique used in EFE-YL requires that the model be meshed with triangular elements – see underlying mesh in Figure 1. The edges of the elements form potential yield lines and the particular configuration chosen by EFE-YL on solution is that which minimised the collapse load – recall that the method is an upper-bound technique.

Geometric Optimisation

The suggested yield line pattern of Figure 1 has a number of potential geometric degrees of freedom. The bifurcation point can move in the plane, i.e. it can have two degrees of freedom, and the bottom point can move along the edge, i.e. it can have a single degree of freedom. To maintain a 'Y' pattern with a vertical stem the horizontal degree of freedom of the bottom point should be coupled or made a slave of the horizontal position of the bifurcation point. Figure 2 shows the geometric variables used.

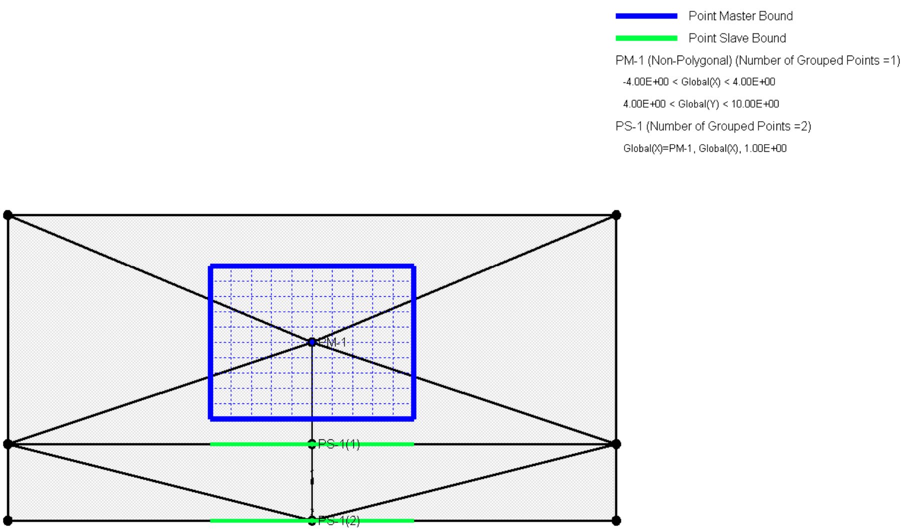


Figure 2: Geometric variation of 'Y' yield line pattern

A geometric optimisation problem has now been defined where the x and y position of the bifurcation point are variables (bounded by the blue box). For each x,y position there will be a different collapse load (load factor)but the optimum position for the bifurcation point is that which minimises the collapse load.

EFE-YL can perform this optimisation automatically but it is useful to obtain a feel for the nature of the objective function, i.e. the objective function terrain. This has been done by setting the bifurcation point in the centre of the bounding box (as in Figure 2) and independently varying the two geometric variables between their respective bounds. The resulting plot of load factor against position is shown in Figure 3. The values in this figure are normalised so that all results fit onto a single plot. The load factors and collapse mechanisms at various key points are shown in the figure.

When the x position of the bifurcation point is varied the result is symmetric with a minimum at the central value of x. This is to be expected as the problem (reinforcement, loading and boundary conditions are symmetric. The variation of load factor with the y position of the bifurcation point appears to be a composite curve of two parts which join some three-quarters of the way between the minimum and maximum values of y. The two parts of the composite curve correspond to different collapse mechanisms with the mechanism for higher y values involving hogging yield lines (green) and no longer being a 'Y' pattern.

The results shown in Figure 3 were generated by moving the bifurcation point to the desired position and then performing an analysis. The results indicate a minimum value of the collapse load to occur for a central value of x and a minimum value of y. EFE-YL can conduct such optimisation automatically and if this is done the procedure converges to the optimum solution just identified.

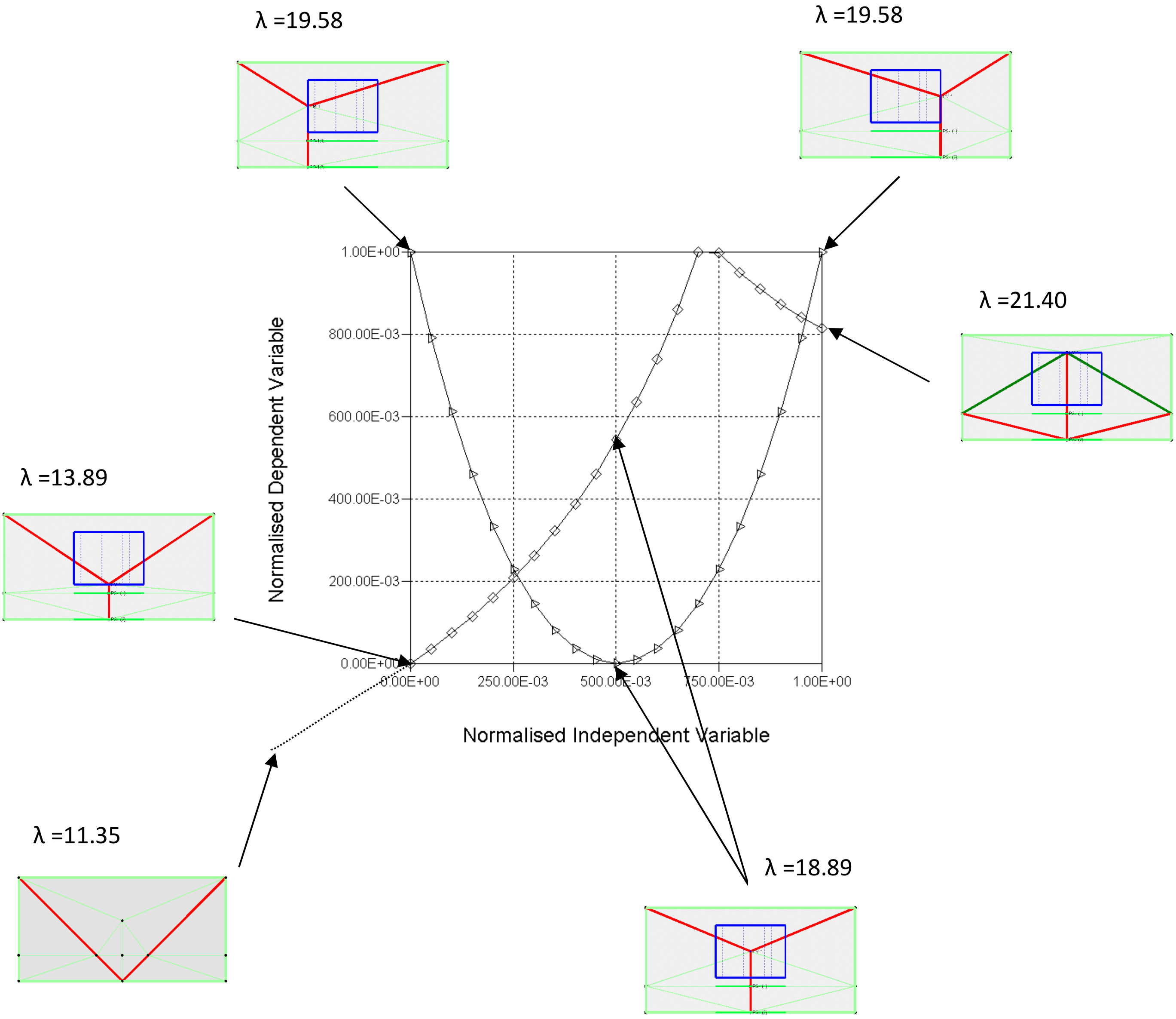


Figure 3: Plot of Load Factor λ against geometric variable

Thus far we have assumed a mode of collapse mechanism (the 'Y' shaped mode) and, recognising that this possesses some geometric variability, we have found the optimum position for the bifurcation point. A question remains as to whether or not the chosen mode of collapse was actually the correct one – in other words is there another collapse mechanism lurking in the wings that has a lower collapse load?

Mesh Refinement as a Method for Locating the Critical Collapse Mode

In the absence of any knowledge of the critical collapse mechanism, mesh refinement can be a useful although, as will be demonstrated, not foolproof technique. The idea here is that we mesh the model with some initial mesh then observe the way in which the collapse mechanism changes as the mesh is progressively refined. Experience with other numerical techniques, e.g. elastic finite element analysis, may condition us to expect a monotonic convergence of load factor with mesh refinement. However, such monotonic convergence is not generally obtained with the yield line technique since the element themselves are rigid and it is the position and orientation of the yielding edges that describes the collapse mechanism. Different meshes will have different patterns of element edges which may or may not allow the series of meshes to converge.

A convergence run using mesh refinement is shown in Figure 4. The first column of the figure shows the mesh and resulting yield line pattern. The second column lists the number of elements specified to the mesh generator and the actual number of elements produced whilst the final column lists the load factor. The initial mesh uses the geometry of the previous model for the 'Y' shaped collapse mode with the bifurcation point located at the centre of the bounds.

The' convergence' observed in Figure 4 is not monotonic with the load factor oscillating with increasing numbers of elements. A number of meshes (20 and 40 specified elements) give the same results which has a load factor below that achieved with the other meshes. The collapse mechanism for these is a very simple one comprising a sagging yield line across the centre of the slab. Two highly refined meshes of 300 and 1000 specified elements were considered but as neither admits the simple central yield line in the mesh neither produces is able to better the lowest load factor. A 'minimal' mesh using the minimum number of elements possible to discretise the geometry whilst admitting the simple central yield line is shown in the last row of Figure 4 and is able to produce the lowest load factor.

The results of the mesh refinement study indicate that the critical collapse mechanism is that of the simple single sagging yield line running across the centre of the slab. This mechanism produces a significantly lower (approximately 50%) load factor than that of the assumed 'Y' pattern (5.56 as opposed to 11.35) and highlights the necessity for considering a range of possible collapse mechanisms when using the yield line technique.

Is the single sagging yield line running across the centre of the slab the correct (true or exact) mechanism? It seems likely but this question may only be answered by somehow confirming that the moment distribution within the rigid regions nowhere violate the yield criterion. Since the yield line technique provides no information regarding the moment distribution within the elements this can be neither confirmed nor denied with this technique. In order to obtain the moment distribution within elements resort needs to be made to a lower-bound technique in which the moment field for the entire slab is defined. Such lower bound elements exist and are currently being incorporated into EFE with a view to providing bounded solutions to the limit analysis of ductile plate structures.

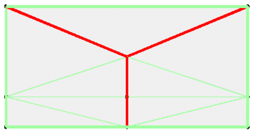
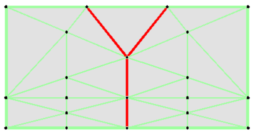
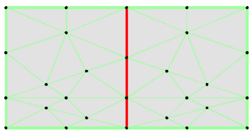
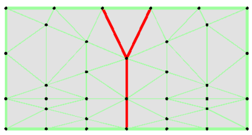
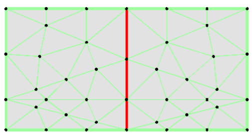
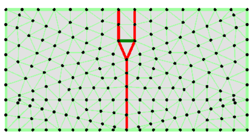
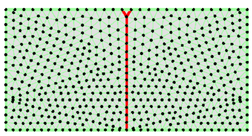
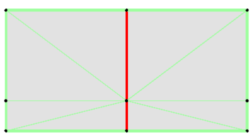
Mesh	Number of Elements specified (actual)	Load Factor
	1 (9)	18.89
	10 (29)	7.04
	20 (40)	5.56
	30 (49)	6.09
	40 (64)	5.56
	300	5.88
	1000	5.69
	1 (8)	5.56

Figure 4: Convergence of load factor with mesh refinement

A Case Study – St John's Wood

The yield line technique is recommended by the Concrete Centre [10] as being admissible for the design of reinforced concrete slabs and being acceptable through the appropriate Euro Codes.

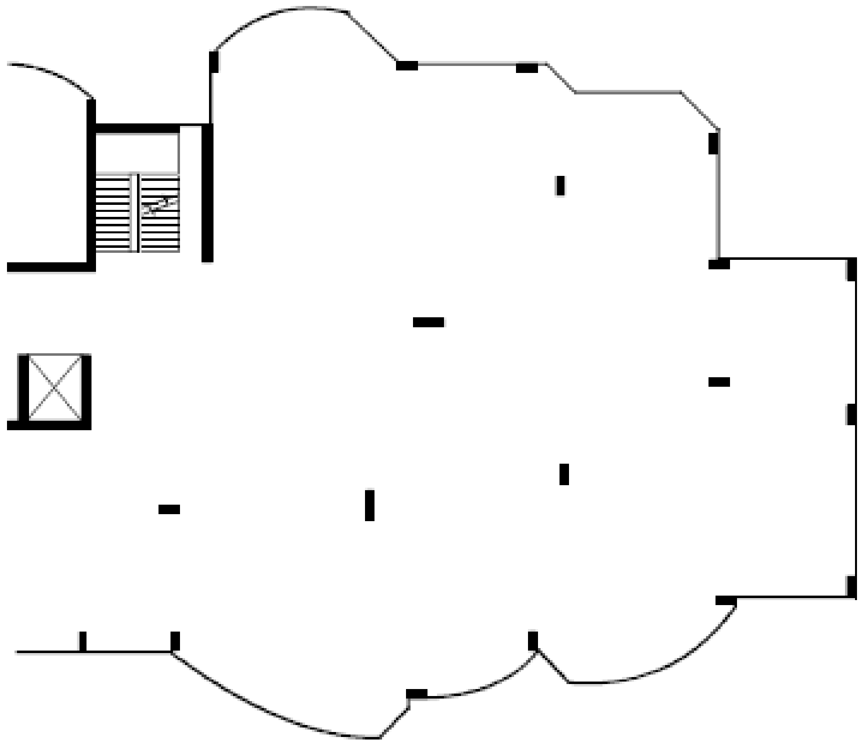


Figure 5: part floor plan of a typical floor at St John's Wood.

Figure 5 shows a part floor plan of a 7-storey block of flats in London. The flat slab has a constant thickness of 250mm. The floor has an irregular geometry and an irregular array of support columns and column dimensions. The maximum span is about 7.5m.

Yield analyses were carried out using EFE-YL assuming a uniform slab with isotropic yield moments and a uniformly distributed load. Figure 6 summarises results with a unit applied load intensity and a moment capacity of 50kN/m obtained for a range of meshes, the specified numbers of elements ranged from 100 to 2000. The upper bound nature of the load factors is apparent, and we look for the least upper bound.

The results are then used to derive lower bounds for the moment capacity to support a specified load at ULS, i.e. 21.7kN/m^2. Now we look for the greatest lower bound.

Number of elements specified	Load factor for m = 50kNm/m	Required moment capacity kNm/m for a total design load of 21.7kN/m^2
100	27.392	
200	25.437	
300	24.092	
400	23.688	45.80
500	24.033	
600	23.175	46.82
700	23.884	
800	23.732	
900	23.052	47.07
1000	22.651	47.90, which compares to 47.2 from David Johnson [16]
1200	23.444	
1400	22.868	
1600	23.433	
1800	23.005	

Figure 6: results for a range of meshes with a UDL = 1kN/m^2 and a notional isotropic yield moment of 50kNm/m for both sagging and hogging moments.

Closure

The yield line technique offers a useful method for estimating the limit load of ductile plates such as reinforced concrete slabs. The method relies heavily on the engineer being able to correctly predict the critical collapse mechanism. For simple cases critical collapse mechanisms are well known and documented. For non-standard cases the engineer needs to be prepared to study a range of potential mechanisms in order to home-in on the critical one. Automated techniques such as EFE-YL make this exploration simpler and less prone to error but neither the hand technique nor the automated technique protect the engineer from assuming a non-critical collapse mechanism. As seen in the simple example shown in this document the difference between the collapse load for critical and non-critical mechanisms can be very significant and with the yield line technique a non-critical mechanism always gives an unsafe prediction of the collapse load.

Further research and development work is being conducted by RMA into providing complimentary lower bound solutions so that even where the true collapse mechanism is unknown the true collapse load may be bounded – hopefully with sufficient tightness to be of practical value.

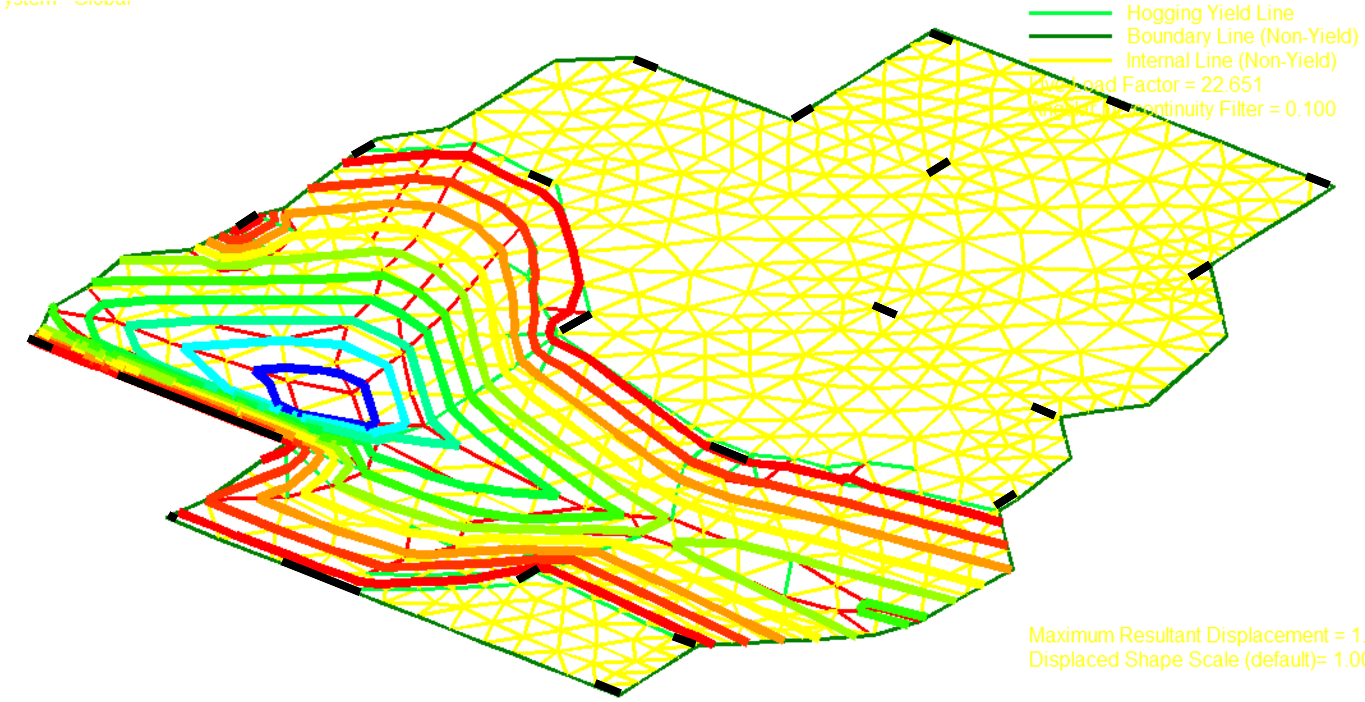


Figure 7: an isometric view of a collapse mechanism of the slab with column positions included

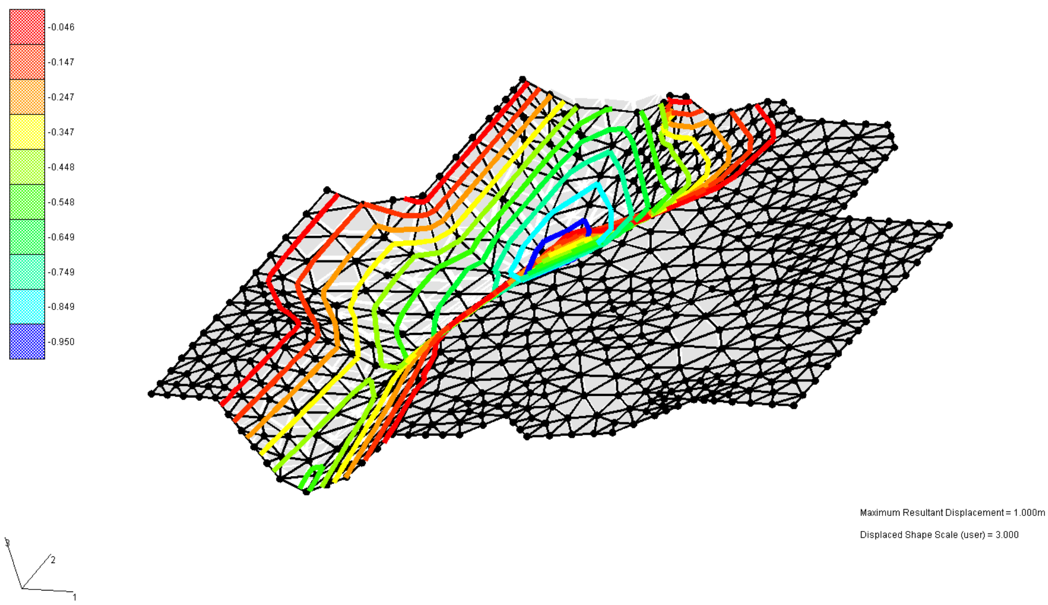


Figure 8: another isometric view of a collapse mechanism

Figures 7 and 8 show different views of the deflected form of a collapse mechanism together with contour lines of vertical deflection.