ANSYS/ASME: Potential Issue with Thermal Expansion Calculations

In many finite element (FE) systems, thermal expansion calculations are based on mean coefficients of linear thermal expansion (or for brevity coefficients of expansion), α, and these may be a function of temperature. The thermal strains used in the FE calculation are then determined from Eq (1), where $\Delta T = T_f - T_i$ is a uniform temperature change from the initial temperature, T_i to the final temperature T_f. Clearly, for accuracy, the mean coefficient of expansion should be appropriate for the temperature range being considered, i.e., $\alpha = \alpha|_{T_i}^{T_f}$.

Thermal Strain $$\varepsilon_T = \alpha \Delta T \quad (1)$$

where $\Delta T = T_f - T_i$

Since the data available to the engineer may not be in the form of mean coefficients of expansion, Some FE systems, e.g., ANSYS, allow the user to input instantaneous values of the coefficients of expansion, α', or thermal strains, ε_T, both as a function of temperature. The software then converts these into mean coefficients of expansion.

In a recent study, the author had reason to check the integrity of these conversions and found that for the case where thermal strains are transformed into mean coefficients of expansion, the resulting mean coefficients of expansion appeared, on first inspection, to be incorrect.

This technical note presents the results of this study which was based on a full set of thermal expansion data (instantaneous and mean coefficients of thermal expansion together with the thermal strain) for a range of temperatures from a reputable source.

Background Theory

It is common in finite element help manuals to indicate the coefficients of expansion on a plot of thermal strain versus temperature as shown in Figure 1.

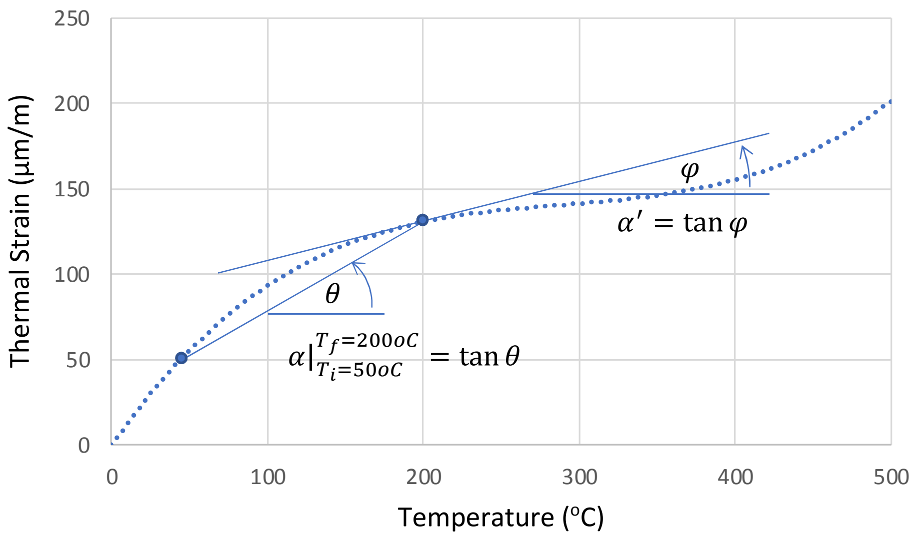


Figure 1: Coefficients of expansion from thermal strain

The instantaneous coefficient of expansion is simply the gradient of the thermal strain curve, as expressed in Eq (2).

Instantaneous Coefficient of Expansion

$$\alpha' = \frac{\partial \varepsilon_T}{\partial T} \tag{2}$$

The average coefficient of expansion is, from Figure 1, the gradient of the secant line cutting the thermal strain curve at the appropriate temperatures or the integral of the instantaneous value over the temperature range of interest divided by the temperature range as shown in Eq (3).

Average Coefficient of Expansion

$$\alpha = \frac{\int_{T_i}^{T_f} \alpha' dT}{T_f - T_i} \tag{3}$$

Given the thermal strain as a function of temperature the instantaneous coefficient of expansion can be determined from Eq (2) and then the mean coefficient of expansion from Eq (3).

In general, thermal expansion data is defined relative to some datum or reference temperature where the thermal strain is assumed to be zero. This temperature might be defined as T_0. The practising engineer is may be interested in establishing the thermal strain due to a temperature rise from T_i to T_f. If $T_i \neq T_o$ then the approach shown in Eq (4) is adopted.

$$\varepsilon_T\big|_{T_i}^{T_f} = \varepsilon_T\big|_{T_0}^{T_f} - \varepsilon_T\big|_{T_0}^{T_i} = \alpha\big|_{T_0}^{T_f}(T_f - T_0) - \alpha\big|_{T_0}^{T_i}(T_i - T_0) \tag{4}$$

A Practical Example

For this example, the temperature dependent thermal expansion properties for a range of aluminium alloys, as published in ASME Boiler & Pressure Vessel Code, II, Part D, 2010 (p714), will be used. These material properties are reproduced in Figure 2.

TABLE TE-2
THERMAL EXPANSION FOR ALUMINUM ALLOYS

Temperature, °C	Coefficients for Aluminum Alloys		
	A	B	C
20	21.7	21.7	0
50	23.3	22.6	0.7
75	23.9	23.1	1.3
100	24.3	23.4	1.9
125	24.7	23.7	2.5
150	25.2	23.9	3.1
175	25.7	24.2	3.7
200	26.4	24.4	4.4
225	27.0	24.7	5.1
250	27.5	25.0	5.7
275	27.7	25.2	6.4
300	27.6	25.5	7.1
325	27.1	25.6	7.8

GENERAL NOTES:

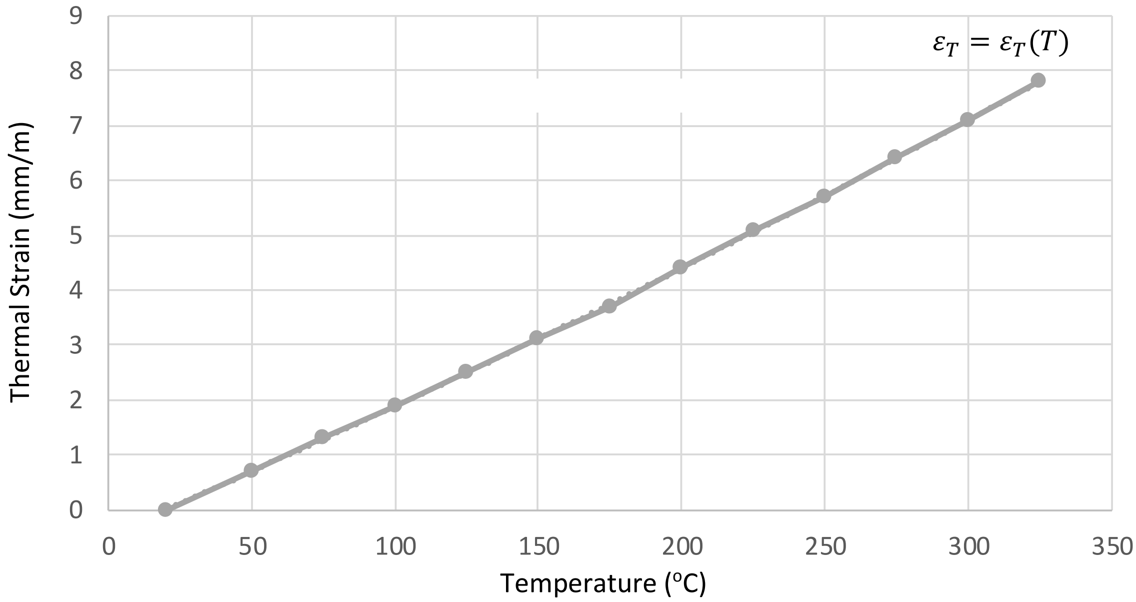
(a) Aluminum alloys represented by these thermal expansion coefficients include:

A03560	A93003	A95254
A24430	A93004	A95454
A91060	A95052	A95456
A91100	A95083	A95652
A92014	A95086	A96061
A92024	A95154	A96063

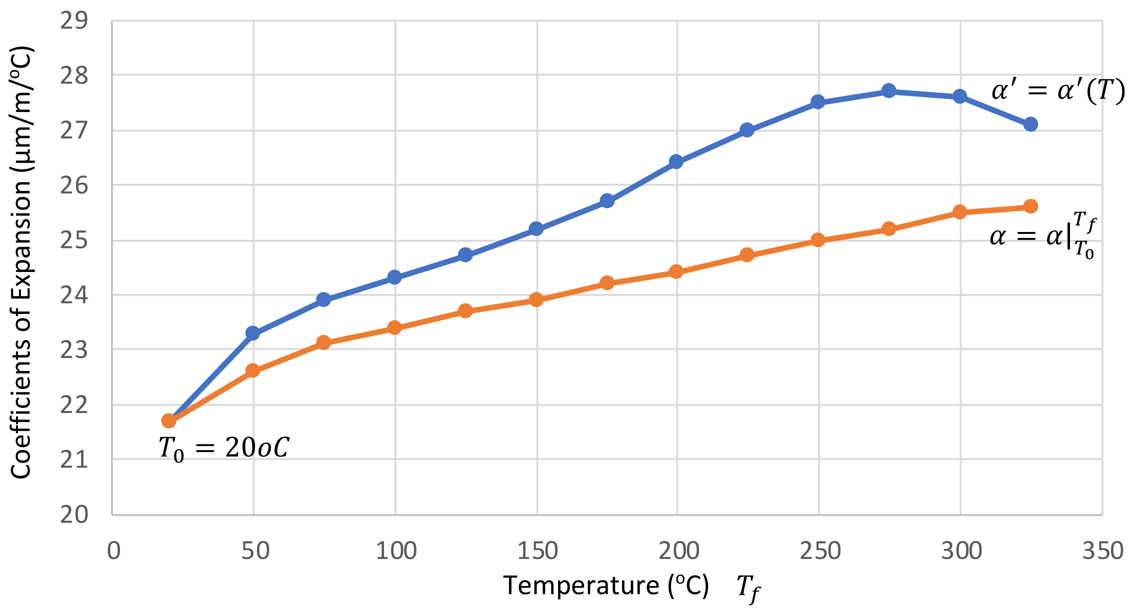
(b) Coefficient A is the instantaneous coefficient of thermal expansion × 10^{-6} (mm/mm/°C). Coefficient B is the mean coefficient of thermal expansion × 10^{-6} (mm/mm/°C) in going from 20°C to indicated temperature. Coefficient C is the linear thermal expansion (mm/m) in going from 20°C to indicated temperature.

Figure 2: Temperature dependent thermal expansions properties for aluminium alloys

As stated in note (b), the instantaneous and mean coefficients of expansion are presented in the columns denoted A and B, respectively, and the thermal strain in the column denoted C. It is unclear from the notes how these values were derived. However, it is assumed that the thermal strains are measured data and the coefficients of expansion have been derived from this data using Eq (2) and Eq (3). It is possible, likely, that the thermal strains have been processed, for example by curve fitting, to produce a (C^1) continuous representation of thermal strain as a function of temperature – this being useful in ensuring that the evaluation of Eq (2) leads to unique values at temperatures where the thermal strain is measured. The data from ASME is plotted in Figure 3.



(a) Thermal strain as a function of temperature



(b) Coefficients of expansion as functions of temperature

Figure 3: Temperature dependent thermal expansion data from ASME

Assuming that the ASME data is consistent, i.e., that the quantities are related by Eq (2) and Eq (3), then it is a simple matter to check the results from ANSYS. As noted previously, ANSYS maps instantaneous thermal coefficients into mean coefficients, i.e., column A into column B presumably using Eq (3). If the data is provided in the form of thermal strains then it will map this into mean coefficients, i.e., column C into column B presumably using Eq (2) followed by Eq (3).

It is a simple matter to force ANSYS to perform these conversions or transformations; data input as instantaneous coefficients of expansion or thermal strains are mapped into mean coefficients of expansion which can be extracted from the program and plotted as a function of temperature. The results from this exercise are shown in Figure 4.

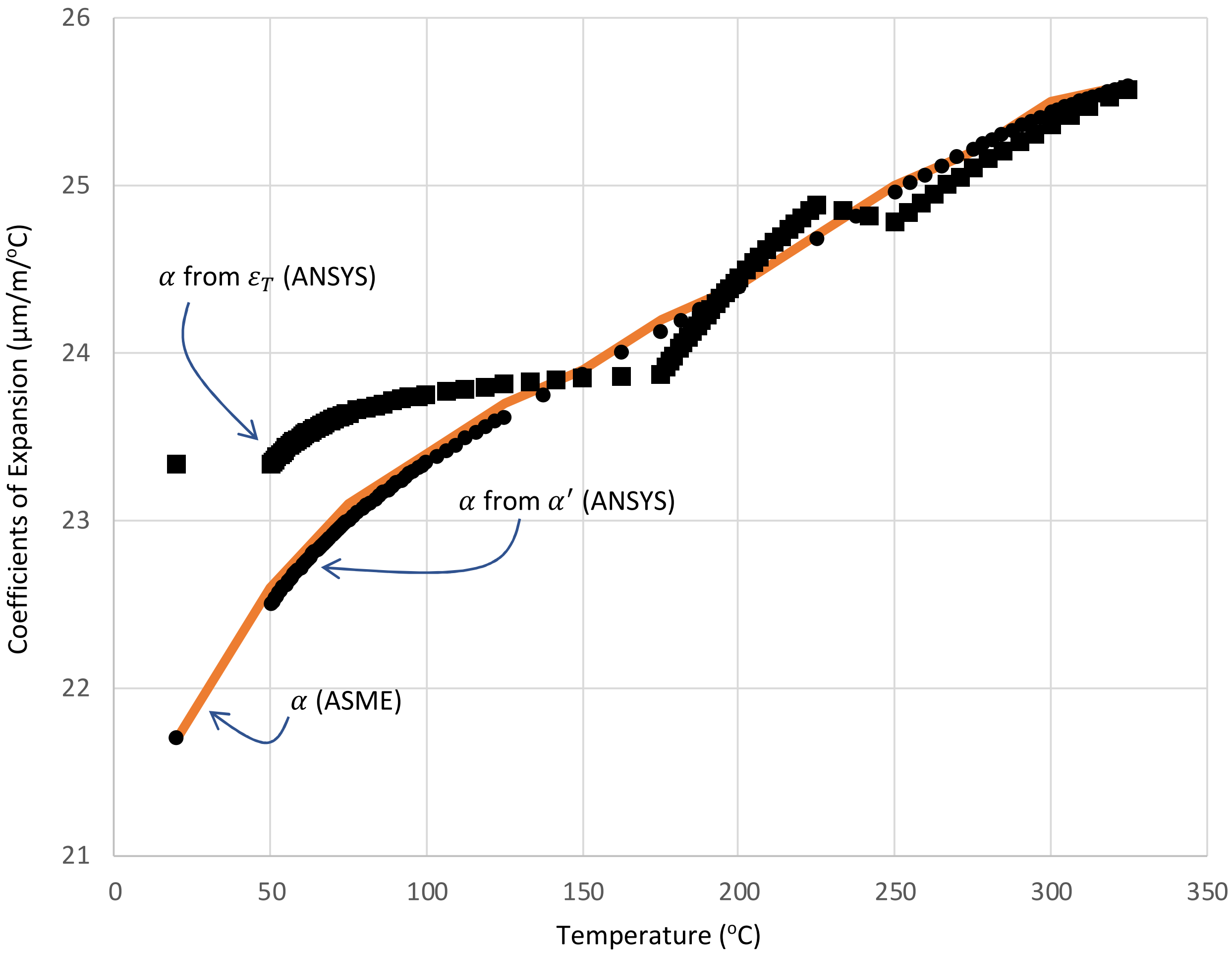


Figure 4: Mean coefficient of expansion from ANSYS

In Figure 4, the mean coefficient of expansion from ASME is plotted as a solid orange line. The solid black circles are results from ANSYS produced by inputting the instantaneous coefficients of expansion from ASME. The results appear reasonable although the distribution of data points is rather uneven.

The solid black squares in Figure 4 represent the mean coefficients of expansion calculated by ANSYS from the ASME thermal strains. The same uneven distribution of data points as noted above is observed for these results. However, of more concern is the lack of agreement between the ASME data and that calculated by ANSYS.

One might be forgiven, when seeing the results of Figure 4 to question the veracity of the mean coefficients of expansion from ANSYS. It turns out, however, that the mean coefficients of expansion produced by ANSYS from the original ASME thermal strain data is correct, at least at the original temperatures specified in the ASME data, in that these mean coefficients of expansion lead back to the original ASME thermal strains. At 50°C the thermal strain from ASME is 0.7mm/m. ANSYS converts this into a mean coefficient of expansion as per Eq (5).

$$\alpha|_{20oC}^{50oC} = \frac{0.7 - 0}{50 - 20} = 23.333\mu m/m/oC \tag{5}$$

This is different from the ASME value which is 22.6 μm/m/°C. However, if the ANSYS coefficient is used to calculate the thermal strain then, clearly, the correct value is recovered.

If, on the other hand, the ASME coefficient is used to calculate the thermal strain the result shown in Eq (6) is obtained.

$$\varepsilon_T|_{20oC}^{50oC} = 22.6 \cdot 30 = 0.678 \tag{6}$$

The percentage error in thermal strain calculated from the ASME mean coefficient of expansion is given in Eq (7).

$$e|_{20oC}^{50oC} = \frac{0.7 - 0.678}{0.7} = 3.14\% \tag{7}$$

The error in thermal strain as calculated using the ASME mean coefficient of expansion is shown for the entire temperature range in Figure 5.

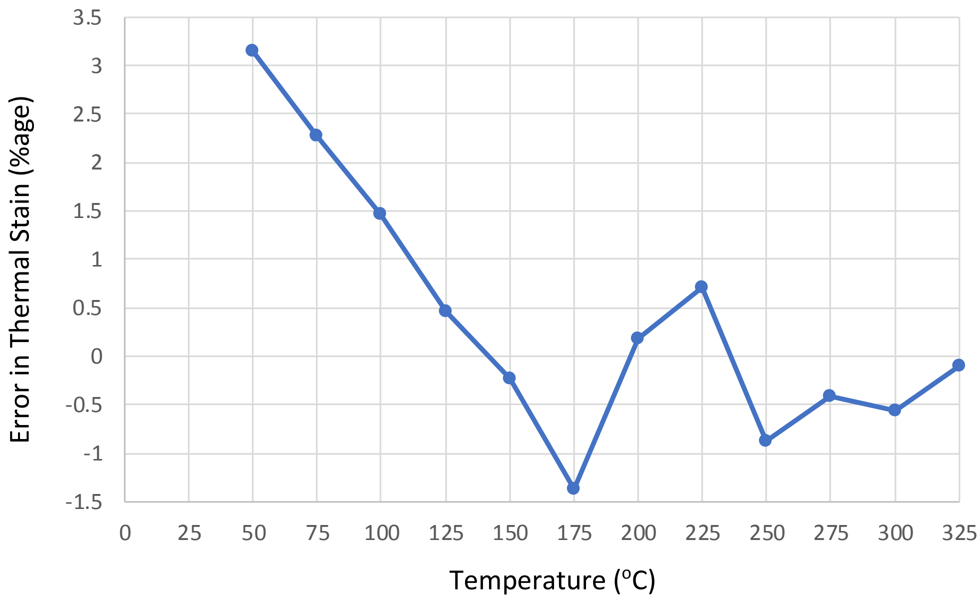


Figure 5: Error in ASME thermal strain as calculated from ASME mean coefficient of expansion

Closure

This technical note illustrates the sort of care needed by the practising engineer when using published thermal expansion data in a finite element program. Published data may have been processed in such a manner that it can lead to errors in the thermal strain calculated by a finite element system. In this case, data published by ASME was used. The thermal strains are the basic data in the sense that these were originally measured during experiment. In producing instantaneous and mean coefficients of expansion, ASME has processed the original thermal strain data to produce a continuous curve of thermal strain versus temperature. This curve is then used to produce the coefficients of expansion reported. The coefficients of expansion based on the processed thermal strain data are such that they do not recover exactly the original thermal strain data. In the case considered (aluminium alloys) the maximum error was just over 3%. The implications of this error would, clearly, need to be considered on a case by case basis given the particular engineering application being considered.

Given that the processing undertaken by ASME introduces an error into the thermal strain calculation the best advice to the practising engineer would be to use only the basic thermal strain data. However, not all finite element systems allow this with at least some systems only offering the facility for dealing with mean coefficients of expansion. In contrast, ANSYS does allow thermal strain to be used in defining thermal expansion behaviour. Whilst it processes it into mean coefficients of expansion, it does so in a manner that is consistent, at least at the data points provided, with the original thermal strain data.

If engineers are forced to use mean coefficients of expansion in their finite element system then they might consider, where possible, calculating these directly from basic thermal strain data rather than relying on processed data of uncertain provenance. ASME might assist in this process by listing, in their table of data, the maximum error in thermal strain associated with using the coefficients of expansion listed in their data tables.

Acknowledgement

The author had some useful correspondence with Jeremy Theler of Seamplex. Jeremy has prepared a technical note which will be available on his company website:

On the Evaluation of Thermal Expansion Coefficients, SP-WA-17-TN-F38B

Appendix

The graphical output from ANSYS for the thermal strain (THSX) input to the software and the mean coefficient of expansion (ALPX) calculated by the software are shown in Figure 6.

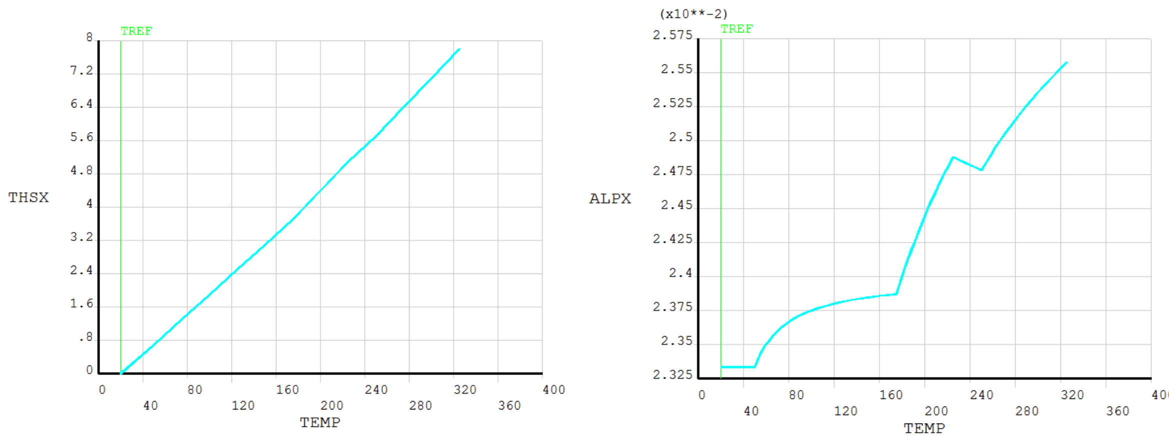


Figure 6: Graphical output from ANSYS