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Limit Analysis Using Finite Element Techniques

Seminar for the Advanced Structural Engineering Module

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Aims

To demonstrate upper and lower bounds to collapse load factors of reinforced concrete slabs utilising finite element techniques with EFE through a number of numerical examples.

A Single Basic Fan Mechanism

In the finite element technique, yield lines can only be positioned along interfaces between elements or between elements and boundary lines. Each node that is free to deflect gives rise to a basic local fan mechanism, usually involving "circumferential" hogging yield lines around the boundary of a local patch of elements which all share the node, and "radial" sagging yield lines. The associated collapse mechanism has the form of an inverted pyramid, or pitched roof, and the mode of collapse can be easily appreciated by an animated view of the mechanism.

It should be emphasized that an animated view is based on scaling infinitesimal deflections so that they can easily be seen. If relative deflections were actually to develop as illustrated, then we would definitely be in the realm of "large displacements" and significant membrane stresses would be present due to the stretching of the slab. It is often claimed that although yield line solutions are theoretically "unsafe" due to their upper bound property, membrane stresses may in fact develop and provide an extra reserve of strength. At present this is difficult to quantify without performing a full incremental non-linear analysis.

The finite element technique as applied to limit analysis seeks to optimise/minimise the load factors corresponding to all possible combinations of basic local mechanisms.

(a) Model (b) Yield lines and displaced shape

Figure 1: Basic Fan Mechanism

A Two-Way Spanning Slab

This example illustrates a technique for adapting a mesh in an optimisation process aimed at minimising the lower bounds. The slab, which is homogeneous with isentropic reinforcement, is simply supported on three sides and loaded with a UDL, [1].

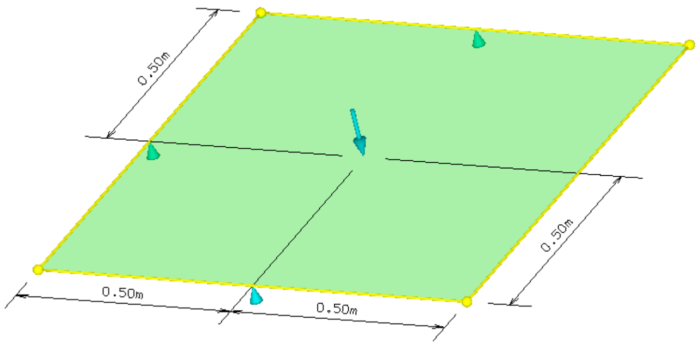
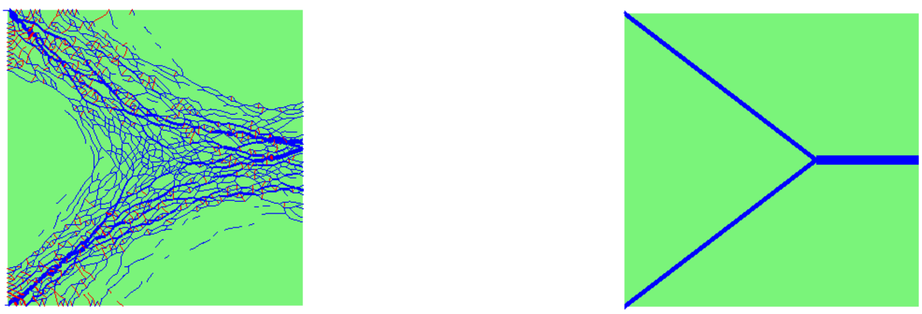


Figure 2: Model for two-way spanning slab

In this technique we start with a fine unstructured, or irregular, mesh of triangular elements. Typical element size should be smaller than the crack spacing anticipated in the neighbourhood of a yield line – say one order of magnitude less than the thickness of the slab, but this aspect is still under investigation. Since EFE presents the yield lines, which are multiple and locally in various directions, with thicknesses scaled according to the rotation (or relative work done), a realistic impression is given of the crack patterns associated with a mechanism.



(a) Refined Unstructured Mesh (λ=17.61) (b) Optimised Mesh (λ=14.14)

Figure 3: Results for two-way spanning slab

The collapse mechanism so obtained by EFE provides good qualitative information about the relative deflections, and can be used to adapt the finite element mesh so as to have far fewer elements, but with their edges approximately aligning with the main yield lines.

After the mesh is adapted (by the engineer interacting with the computer, or by an automated process), the simplified mesh is reanalysed, but now with the option of exploiting algorithms that optimise the positions of the nodes so as to minimise the load factor. These algorithms thus involve us with geometric optimisation.

The Collins Anisotropic Bridge Deck

This example is based on a slab forming the deck of a road bridge that was the subject of limit analyses and experimental tests, as reported in a paper by Jackson and Middleton at Cambridge University, [2]. A 10th scale model tested in the laboratory simulated a 15m wide deck spanning 5.6m onto simple supports. The loading considered consisted of a pair of wheel loads, as UDL over square areas of side length 300mm. Since the main span is in the shorter direction, the slab is reinforced more lightly in the direction parallel to the supports, hence it is anisotropic.

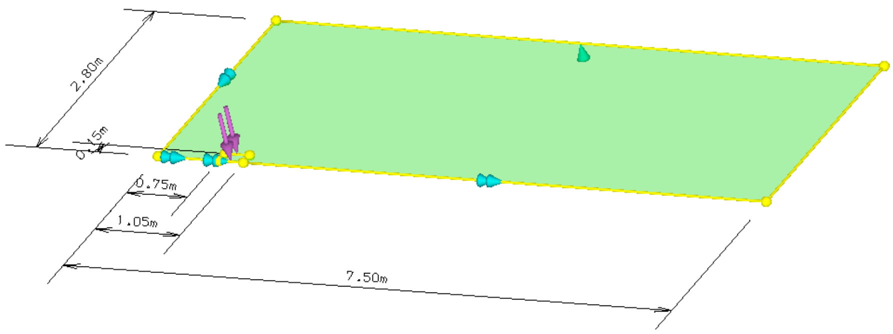


Figure 4: Model for Collins bridge slab (Quarter model)

The adaptive yield line technique is repeated for this problem, using symmetry to model one quadrant of the slab. A fine unstructured mesh provides an impression of the collapse mechanism, and this mesh is simplified for the geometric optimisation phase.

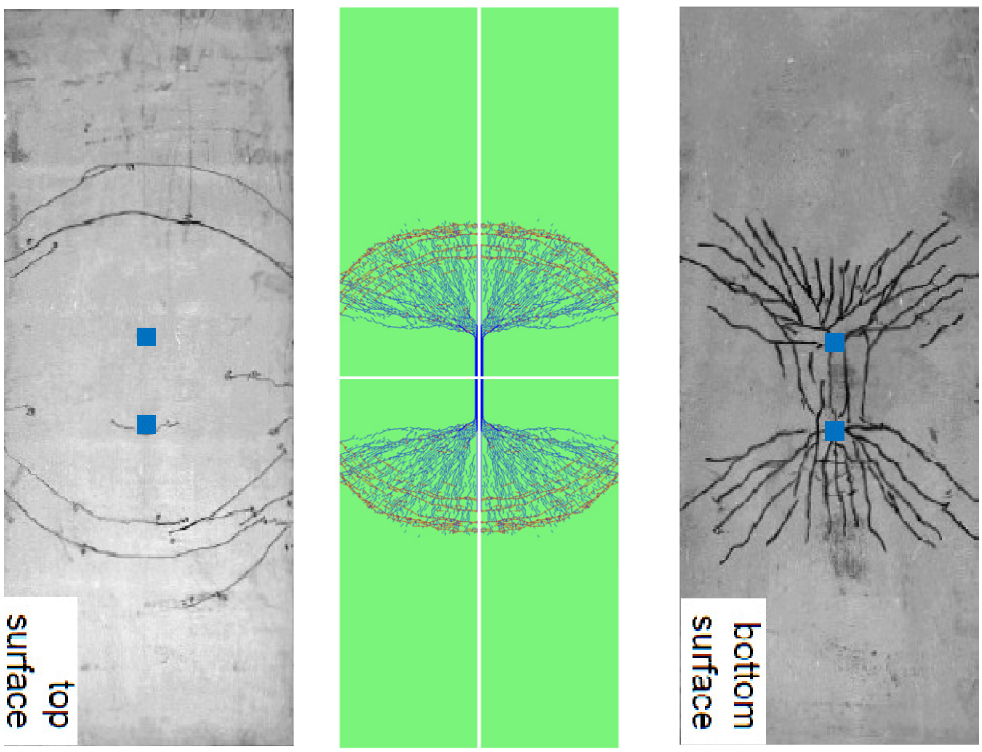


Figure 5: Results for the Collins bridge slab

The Verulam Floor Slab Problem

This example was developed in answer to an enquiry raised in the Verulam section of [3] concerning the effective width of slabs to be considered in designing for a concentrated load. As in the previous bridge deck problem, the slab was assumed to essentially span one way between parallel simple supports. In this case the slab is 12m wide and the span is 6m and 0.2m thick. We consider the application of a central 100kN load concentrated over a square of side length 200mm. The flexural strength was initially assumed to be isotropic, but this case was followed by others in which the reinforcement in the direction parallel to the supports is reduced, so making the slab anisotropic.

The adaptive yield line technique leads to the collapse mechanism as shown, and the simplified mesh leads to a reduction in load factor. This is also confirmed, or compared to, a hand solution as published in the Verulam section of [4].

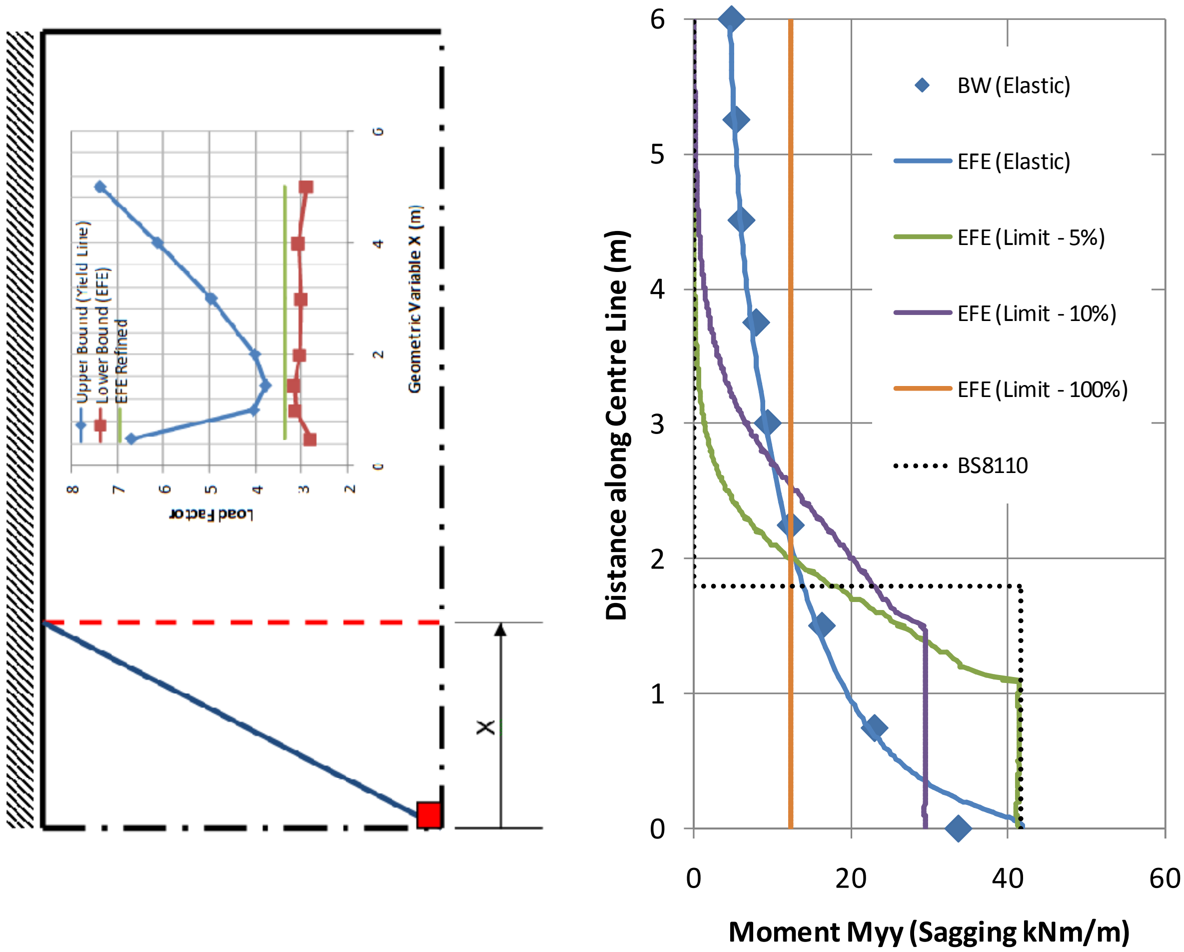


Figure 6: Results for the Verulam slab [A]

Lower bound equilibrium solutions were also obtained, and enabled the closeness of upper and lower bounds to be compared. This example clearly shows the effects of allowing moment redistribution from distributions based on elastic behaviour assuming isotropic elastic properties. The distributions of bending moment in the span direction are shown across a midspan section. The yield utilisation ratio plots indicate where the full yield strength of the slab has been reached after moment redistribution.

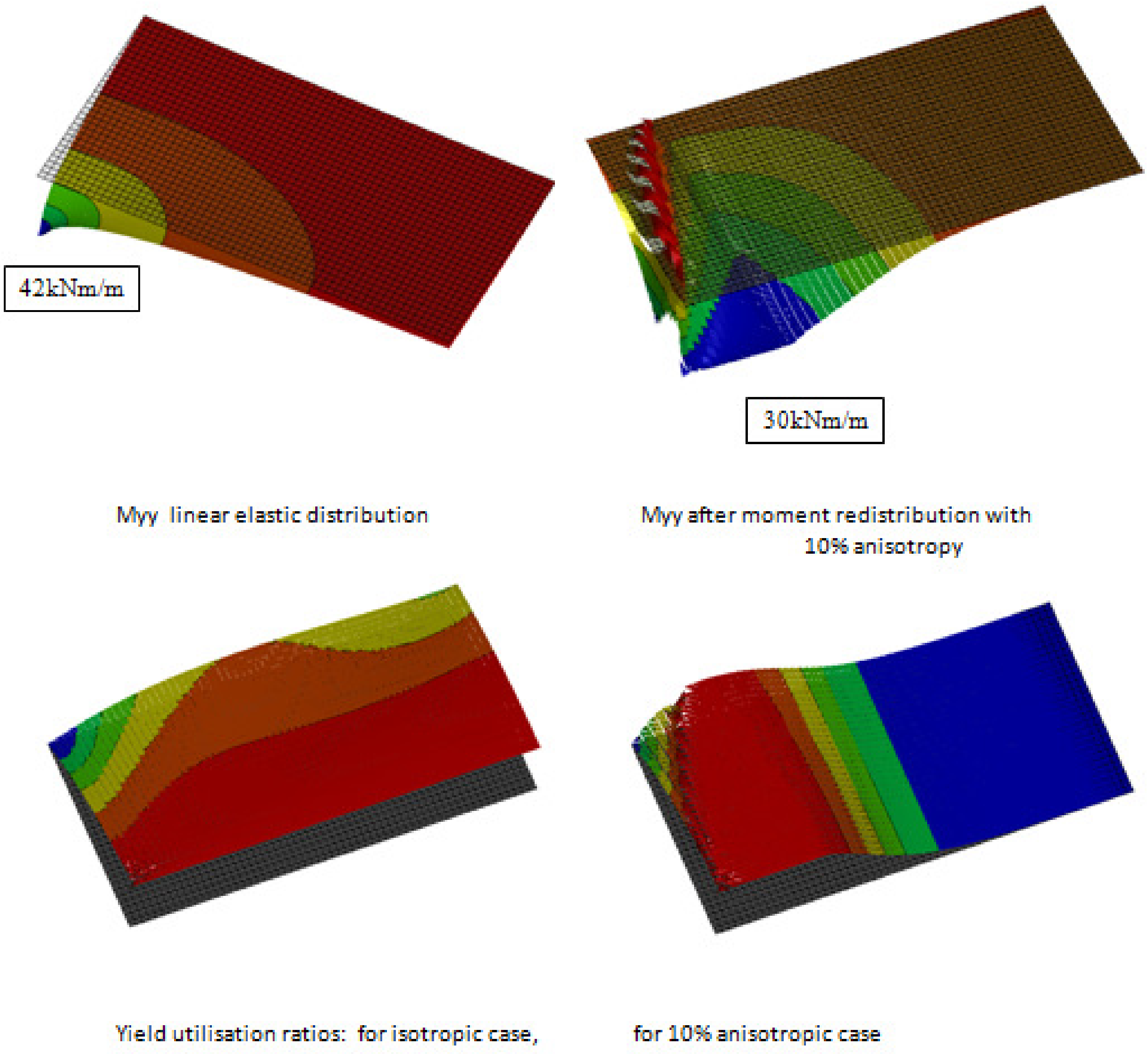


Figure 7: Results for the Verulam slab [B]

Closure

Further information, examples and resources can be found on RMA's website [5].

References

1) www.limitstate.com/slab/verification - Square with three simply supported edges.
2) Search for 'Predicting the Flexural Collapse Load of Concrete Slab Bridges' by A.M. Jackson and C.R. Middleton.
3) The Structural Engineer (5 May 2010).
4) The Structural Engineer (18 January 2011).
5) www.ramsaymaunder.com .