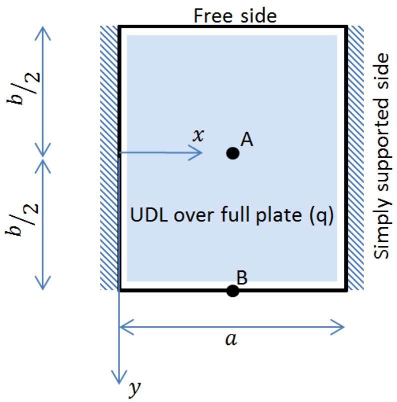
The NAFEMS Benchmark Challenge

Poisson's Ratio for a Steel Plate

As a final year project, a student was asked to conduct an experiment on a thin steel plate, simply supported on two opposite sides and loaded with a uniformly distributed transverse load (UDL). The student was asked to measure the surface strains at the centre of the plate and from these to determine the value of Poisson's ratio for the steel plate.

The student recalled from an earlier lecture that if a plate-membrane is subjected to a uniaxial stress field then the ratio of the direct strain in the lateral direction to that in the longitudinal direction gave Poisson's ratio. Whilst she realised that because of symmetry the point at the centre would have zero shear stress, she was also aware that the state of stress at this point might well not be uniaxial. She recognised that for a force driven problem, such as this one, the stresses would be independent of the Young's modulus but that Poisson's ratio might well change the relative proportion of the two stress components. She also realised that if she took the ratio of the two stresses then this would be independent of the load magnitude. So, if she could find the true ratio of stress (or moments as these are proportional to the stresses) at the centre of the plate then her measured value should agree if her plate had the same value of Poisson's ratio as that used to evaluate the true ratio. For the true ratio she resorted to Timoshenko's text [1] which gives the moments at the centre of the plate for a value of Poisson's ratio of 0.3.

For her experiment she used a plate with an aspect ratio of b/a=0.5 and when she compared the ratio of the stresses she had evaluated (about 10) to that offered in Timoshenko's text (about 12) she realised that the steel of her plate must have a different Poisson's ratio to that of Timoshenko's value (0.3). At this point she was at a loss on how to continue and so she visited her supervisor for advice.



	Point A			Point B	
	$x = a/2,\ y = 0$			$x = a/2,\ y = \pm b/2$	
b/a	$w = \alpha \frac{qa^4}{D}$	$M_x = \beta_1 qa^2$	$M_y = \beta_1' qa^2$	$w = \alpha_2 \frac{qa^4}{D}$	$M_x = \beta_2 qa^2$
	α_1	β_1	β_1'	α_2	β_2
0.5	0.01377	0.1235	0.0102	0.01443	0.1259
1.0	0.01309	0.1225	0.0271	0.01509	0.1318
2.0	0.01289	0.1235	0.0364	0.01521	0.1329
∞	0.01302	0.1250	0.0375	0.01522	0.1330

Figure 1: Plate Configuration and Table 47 from page 219 of Timoshenko's text [1]

The Challenge

As this student's supervisor you were also surprised that the moment or stress ratios were so different particularly as you knew the value of Poisson's ratio for the plate to be very close to 0.3. You decided to try and understand the problem by modelling the plate in finite elements and investigating how the moment ratio changed with Poisson's ratio. The challenge is to conduct this numerical experiment and establish the truth!

[1] S.P. Timoshenko & S. Woinowsky-Krieger, 'Theory of Plates and Shells', 2^{nd} Edition, McGraw-Hill International Series, 28^{th} Printing 1989. ISBN 0-07-Y85820-9.