NAFEMS *The NAFEMS Benchmark Challenge*

Assessment of a Simply Supported Plate with Uniformly Distributed Load

A building has a floor opening that has been covered by a durbar plate with a yield stress of 275MPa. The owner has been instructed by his insurers that for safety the load carrying capacity of the plate needs to be assessed. The owner has calculated (possibly unrealistically but certainly conservatively) that if 120 people each weighing 100kg squeeze onto the plate then it must be able to cope with 100kN/m^2. He has found, in the *Steel Designers' Manual*, that the plate should be able to withstand 103kN/m^2. This is rather close to the required load and looking in *Roark's Formulas for Stress and Strain* he finds that the collapse load is more like 211kN/m^2 which he feels does provide an adequate factor of safety. However, with the huge difference between the two published values he has asked you to provide him with an independent assessment of the load carrying capacity of the plate.

The Challenge

As an experienced engineer you realise that under increasing load the plate will eventually reach first yield after which the stress will redistribute until the final collapse load is reached. You will appreciate that the steel will have some work hardening capability and that if transverse displacements are considered then some membrane action will occur. However, opting for simplicity and realising that ignoring these two strength enhancing phenomena will lead to a degree of conservatism in your assessment, you decide that this is a limit analysis problem in which the flexural strength of the plate governs collapse.

Unless you have specialist limit analysis software you will decide to tackle this as an incremental non-linear plastic problem with a bi-linear stress/strain curve and a von Mises yield criterion.

Please carry out an assessment of the strength of the plate and provide your best estimate of the actual collapse load together with evidence of the verification you have conducted sufficient to convince the owner and his risk averse insurer.

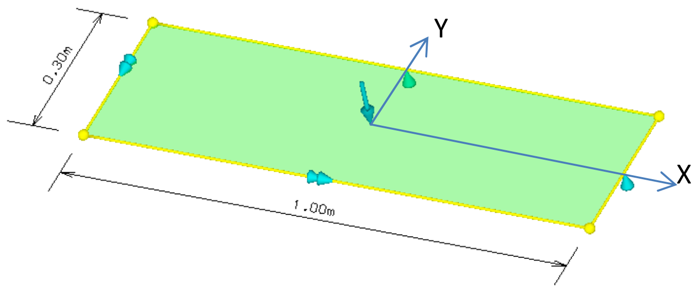


Plate Properties

Length (l) = 2m $E = 210GPa$

Breadth (b) = 0.6m $\nu = 0.3$

Thickness (t) = 0.01m (10mm)

Simply supported with an applied pressure (p)

Figure 1: Symmetric quarter model of challenge plate and boundary conditions

Raison d'être for the Challenge

This challenge derives from an observation that the strength of steel plates, such as the one in this challenge, as quoted in the Steel Designers' Manual is significantly lower than that derived from standard reference texts such as Roark. The strength of a steel plate such as the challenge plate can be assessed using limit analysis techniques. Ramsay Maunder Associates (RMA) has developed a software tool for the lower-bound limit analysis of steel plates. This software tool uses equilibrium finite element (EFE) technology and will be used to assess the strength of the challenge plate. As specialist limit analysis software will not be available to most practising engineers the problem will need to be tackled using a simulated limit analysis with incremental non-linear material finite elements.

Review of Published Strength Values

Steel Designers' Manual
(6th Edition, p1281)

Ultimate load capacity (kN/m²) for floor plates simply supported on all four edges stressed to 275 N/mm² (Values obtained using Pounder's formula allowing corners to lift)

Thickness on plain mm	Breadth B mm	Length (mm)							
		600	800	1000	1200	1400	1600	1800	2000
10.0	600	172*	126*	112*	107*	105*	103*	103*	103*
	800		97.0	74.5	65.9	62.1	60.1	59.1	58.5
	1000			62.1	49.5	43.9	41.0	39.4	38.5
	1200				43.1	35.4	31.5	29.3	28.0
	1400					31.7	26.6	23.8	22.1
	1600						24.3	20.7	18.6
	1800							19.2	16.6

Values *without* an asterisk cause deflection greater than b/100 at serviceability, assuming that the only dead load present is due to self-weight.

Roark's Formulas for Stress and Strain
(7th Edition, pp453-454)

11.13 Ultimate Strength

Plates of brittle material fracture when the actual maximum tensile stress reaches the ultimate tensile strength of the material. A flat-plate modulus of rupture, analogous to the modulus of rupture of a beam, may be determined by calculating the (fictitious) maximum stress corresponding to the breaking load, using for this purpose the appropriate formula for elastic stress. This flat-plate modulus of rupture is usually greater than the modulus of rupture determined by testing a beam of rectangular section.

Plates of ductile material fail by excessive plastic deflection, as do beams of similar material. For a number of cases the load required to produce collapse has been determined analytically, and the results for some of the simple loadings are summarized as follows.

3. *Rectangular plate, length a, width b; uniform load, edges supported*

$$W_u = \beta \sigma_y t^2$$

where β depends on the ratio of b to a and has the following values (Ref. 44):

b/a	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
β	5.48	5.50	5.58	5.64	5.89	6.15	6.70	7.68	9.69

In each of the above cases W_u denotes the total load required to collapse the plate, t the thickness of the plate, and σ_y the yield point of the material. Accurate prediction of W_u is hardly to be expected; the theoretical error in some of the formulas may range up to 30%, and few experimental data seem to be available.

Corrigendum – The Owner Bodged his Calculation!

Sharp readers will have noted that the owner of the building made a mistake using the equation from Roark. He calculated W_u correctly but then assumed that this was the applied pressure or UDL. However, this value is the total load and so the pressure is obtained by dividing the total load by the area of the plate:

$$p = \frac{W_u}{Area} = \frac{7.68 \cdot 275E^6 \cdot 0.01^2}{2 \cdot 0.6} = 176kPa$$

It is interesting to notice that whilst the collapse pressure is dependent on the area of the plate, the collapse load is not.

Plate Theory and Boundary Conditions

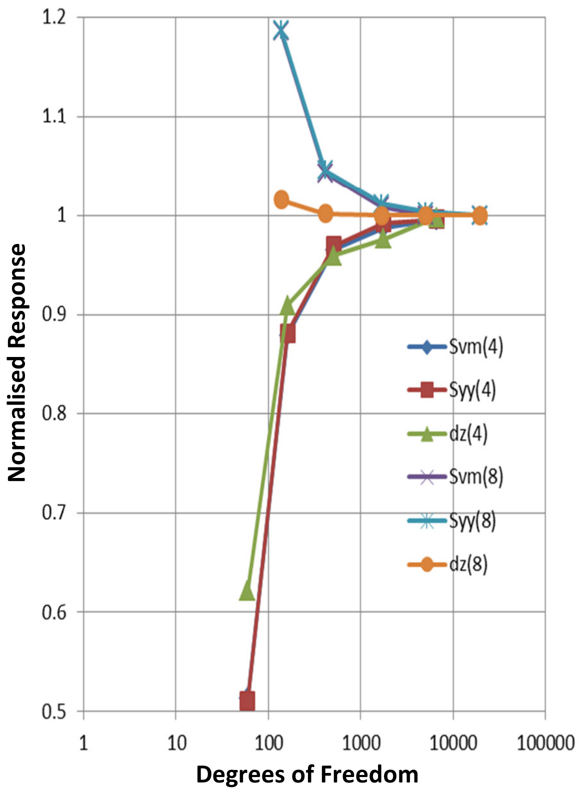
Classical plate theory recognises the need for different formulations for 'thick' and 'thin' plates. For 'thick' plates the appropriate theory is Reisner-Mindlin, which accounts for shear deformation, whereas for 'thin' plates the appropriate theory is Kirchhoff which assumes the section to be rigid in terms of shear deformation. The transition between 'thick' and 'thin' plates occurs at a span to depth ratio of about ten and as the challenge problem has a ratio of 0.6/0.01=60 it is clearly in the realm of 'thin' plate or Kirchhoff theory. In theory, at least, the results produced by 'thick' plate theory should converge to those achieved by 'thin' plate theory as the span to depth ratio increases. However, one needs to be aware that with many finite element formulations of 'thick' plates, a nasty phenomenon known as 'shear-locking' may occur to stymy this principal. Fortunately for the challenge problem the span to depth ratio is not sufficiently large that shear-locking will occur and finite elements of either formulation can be used successfully to solve this problem.

The formulation used for a plate element has implications on the nature of the boundary conditions that might be applied. The thick formulation allows torsional moments and their corresponding twisting rotations to be specified at the boundary whereas the thin formulation does not explicitly control these quantities. As such, for a thick plate, the support conditions may be specified as 'soft' or 'hard' depending on whether the twisting rotations are free or constrained. As the challenge problem is a thin plate problem this distinction is not relevant BUT if the challenge plate is modelled with a thick plate finite element formulation then the simple supports should be interpreted as soft simple supports and the twisting rotation left free.

As with any problem that exhibits symmetry it is worth taking advantage of this property in order to reduce the size of the finite element model and therefore the time to compute a solution. For the challenge problem the upper right hand quarter of the plate will be modelled as shown in figure 1. The figure shows the boundary conditions, (soft) simply supports on the external edges and symmetry conditions (no normal rotation) on the symmetry boundaries. The applied pressure is indicated by a central arrow.

Elastic Solution

A good starting point for understanding the challenge problem is to look at the elastic solution. The finite element software used to generate the elastic solutions was CS1 and the results for a load of 100kPa are shown in figure 2.



The maximum displacement and stresses occur at the centre of the plate and this figure shows the convergence of these quantities with mesh refinement for both four-noded and eight-noded elements. The values have been normalised by the values achieved with the most refined eight-noded mesh:

Svm = 232MPa; Syy = 262MPa; dz = 8.5mm

The normalised peak von Mises stress and peak Syy stress are virtually identical and so the curves lie on top of each other in the figure.

It is interesting to note that the direction in which convergence occurs is dependent on the element type with results converging from *above* the true solution for the eight-noded element and *below* the true solution for the four-noded element.

There is an often presented argument (see [1] for example) that as displacement elements are over-stiff then under applied forces they will tend to deform less and therefore induce less stress than would occur in the actual member. This is of course true in an integral sense and this can be seen in the strain energy convergence for a pure displacement element model under applied forces. In terms of point as opposed to integral values of quantities it is possible however for individual points to behave differently. It is also the case that some elements used in commercial FE systems are not pure displacement elements and so for a number of reasons these arguments are spurious and may not generally hold as seen with this example.

Figure 2: Convergence of stress and displacement for an elastic analysis of the challenge plate

From the elastic analysis we have sufficient results to make the following statements:

1) The peak von Mises stress is 232MPa for a load of 100kPa so that first yield is reached at 275/232=1.185 times the applied pressure which is 118.5kPa.
2) Even if no redistribution occurs across the face of the plate, the applied pressure to cause plastic collapse is 1.5x118.5=178kPa – where the 1.5 factor comes from the increase in load required to develop yield from first yield at the surface to full yield through the section.

It is curious that this value is a little greater than the Roark value of 176kPa. So the Roark value is clearly too conservative especially, as can be seen in the appendix, redistribution across the plate does occur!

The Steel Designers' Manual (SDM) use Pounder's equations which are reproduced from his paper [2] below:

Max Skin Stress - Pounder equation 19

$$\frac{3}{4}\frac{kpb^2}{t^2}\left\{1+\frac{14}{75}(1-k)+\frac{20}{57}(1-k)^2\right\}$$

Maximum Deflection - Pounder equation 19(a)

$$\frac{\nu^2-1}{\nu^2}\cdot\frac{5kpb^4}{32Et^3}\cdot\left\{1+\frac{37}{175}(1-k)+\frac{79}{201}(1-k)^2\right\}$$

where:

$$k=\frac{l^4}{l^4+b^4}$$

Pounder's equations are based on an elastic analysis and for a pressure of 100kPa applied to the challenge plate give a maximum direct stress of 268MPa (which is pleasingly close to the finite element value of 262MPa) and a maximum displacement of 8.72mm (again close to the finite element value of 8.5mm). Comparing the maximum stress with the yield stress gives 275/268=1.03 which corresponds to an applied pressure of 103kPa and is identical to the value reported in the SDM. The approach used by the SDM then is one which limits the maximum principal stress to the yield stress for the material. Thus whilst the SDM quotes these loads as 'Ultimate Load Capacity' the method allows no plastic redistribution and whilst appropriate for brittle materials, is not relevant for the sort of ductile plates being considered – see figure 9 for practical evidence of ductility. In terms of deflection, then the deflection at 'failure' is 1.03x8.72=9mm. It is interesting to note that in the SDM, and based on a breadth/100=6mm deflection limit, the displacement is greater than this value so the asterisk indicating that SLS is not a concern is incorrect!

The SDM suggests that Pounder's equations take into consideration the phenomenon of corner uplift. However, after several readings of Pounder's paper, phrases indicating consideration of corner uplift were not found and so it should be assumed that this phenomenon is not considered. This is not surprising since corner uplift is a non-linear phenomenon that is not simply dealt with in the form of linear analysis considered in Pounder's paper. To consider corner uplift one can perform an incremental finite element analysis progressively releasing any boundary nodes where the reactions are tensile. Such an analysis has been performed with linear elastic material properties and although not reported here, the change in the maximum stress and displacement is observed to be minimal.

Limit Analysis Solution

The idea of limit analysis is to find the collapse load of the structure based on plasticity arguments and a perfectly-plastic material model. For plates such as the challenge plate it is the flexural failure mode that dominates. The collapse solution, in limit analysis, is found directly without recourse to an incremental form of analysis. The yield line technique is perhaps the most widely known form of limit analysis. In this technique the engineer postulates a (kinematically admissible) collapse mechanism comprising sagging and hogging yield lines and then calculates the corresponding collapse load. In general if the collapse mechanism is not the correct one then the technique produces an upper-bound (unsafe) approximation to the true collapse load. To improve yield line solutions some form of geometric optimisation is generally conducted on the collapse mechanism. The collapse mechanism for the quarter model of the challenge slab comprises two sagging yield lines and is simply described with a single geometric variable, g, as shown in figure 3.

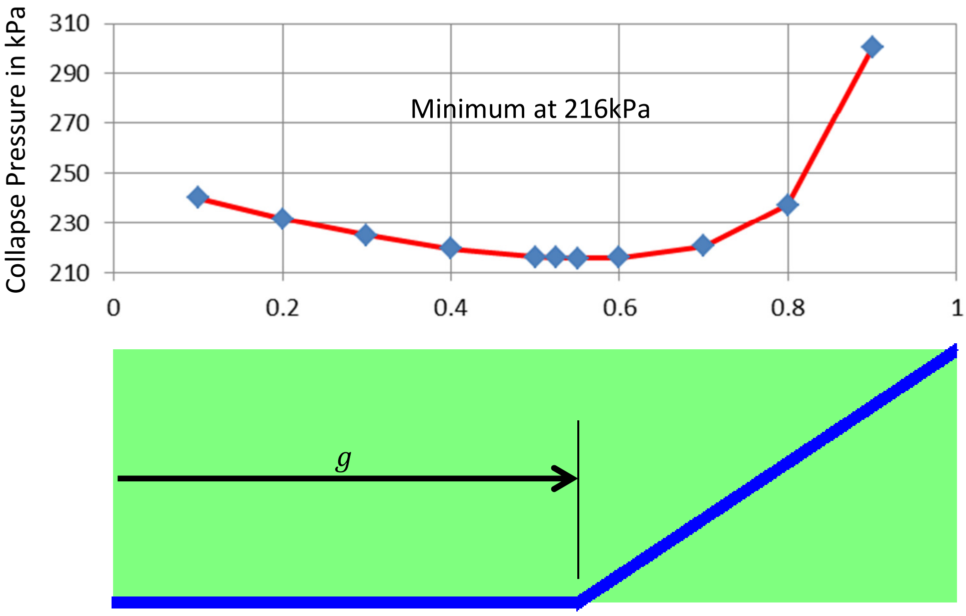


Figure 3: Geometric optimisation of a yield line collapse mechanism

The optimised collapse load by yield line comes out at about 216kPa with a value of $g = 0.55m$. It should be recorded that the collapse load as predicted by the yield line technique is unlikely to be exact since the technique adopts the Nielsen yield criterion rather than the von Mises criterion that should be used for ductile steels – see [3].

RMA develop engineering software based on equilibrium finite elements that produce safe lower-bound limit analysis approximations to the collapse load of structural members such as steel plates using the von Mises yield criterion. This software Equilibrium Finite Elements or EFE has been used to produce a reference solution for the challenge problem. The solution achieved was 231kPa and a contour plot of material utilisation (yield moment divided by the von Mises moment) is shown in figure 4. This result is verified by two of the challenge responders both of whom modelled the plate with solid elements. The results from one of the responders are reproduced in the appendix.

Mesh	Collapse Pressure (kPa)
1x1x4	197
1x3x4	223
3x10x4	231
6x20x4	231

Note: EFE uses triangular elements (see figure 8) and so the plate has been discretised into regular rectangular regions which are then subdivided into four triangular elements. The convergence of collapse pressure with mesh is shown in the table where the safe lower-bound characteristic can be observed with collapse loads converging rapidly from below the true value.

Figure 4: Utilisation contours from EFE for a quarter of the challenge plate

The plot of utilisation shows that, with the exception of a small region around the centre of the long supported edge, the material is fully utilised. The pressure to produce first yield has already been calculated as 118.5kPa which leads (without stress redistribution across the surface of the plate) to a conservative prediction of collapse at 178kPa. The value achieved by EFE, together with the plot of utilisation, shows that significant stress redistribution does occur across the surface of the plate and the evolution of this redistribution can be seen in the appendix.

It is interesting to compare the collapse pressures produced by EFE with those tabulated in Roark. Table 1 reproduces Roark's results and is appended with results achieved by EFE and the percentage difference.

b/a	**1**	**0.9**	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**
β (Roark)	5.48	5.50	5.58	5.64	5.89	6.15	6.70	7.68	9.69
β (EFE)	6.26	6.28	6.40	6.51	6.90	7.42	8.38	10.08	13.67
Difference	14%								41%

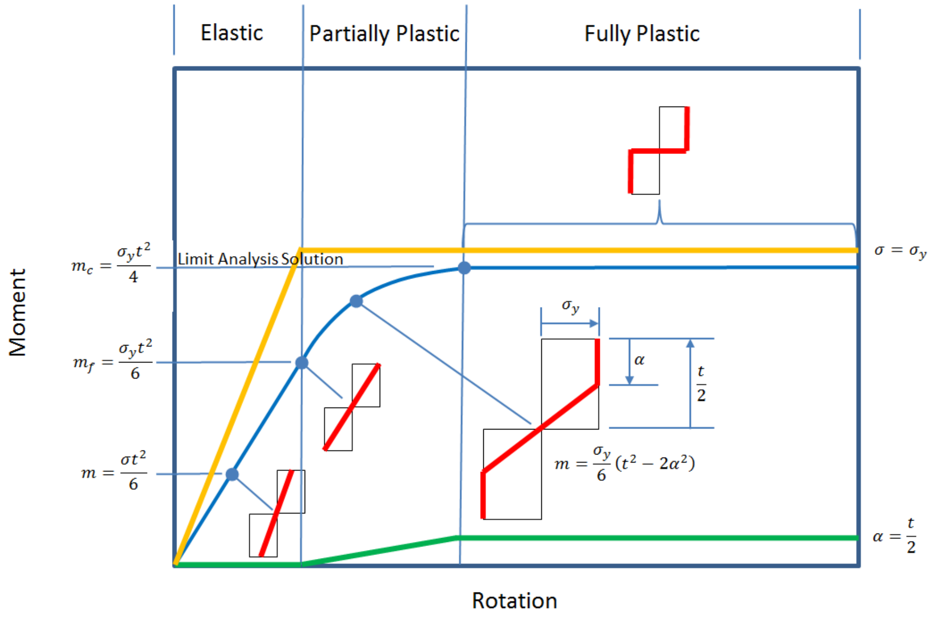
Table 1: Roark's values for β appended with those produced by EFE

It is interesting to see that using dedicated limit analysis software more capacity can be achieved from one's structure – the results from EFE are greater than those produced by Roark. In the text, Roark admits that the results are not expected to be accurate and that they could be up to 30% in error. In practice whilst Roark's values are conservative, for the plate aspect ratios considered, they can be greater than 40% in error!

Simulated Limit Analysis Solutions

Bespoke limit analysis software is only recently becoming available for the practising engineer and in the absence of such tools engineers wishing to undertake this challenge have used simulated limit analysis through incremental non-linear material finite element analysis. Whereas bespoke limit analysis tools require minimal input data and user intervention, conventional finite element systems used to predict the collapse of plates will require more careful consideration if reliable results are to be achieved. To this end a Tips and Hints document on simulating limit analysis results was produced [4].

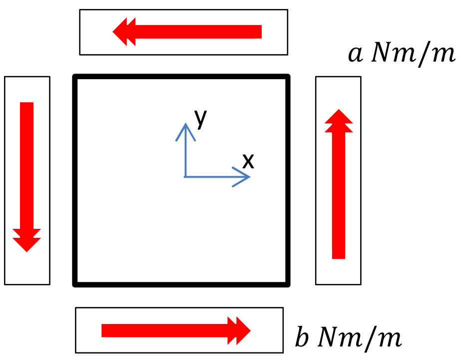
The following figure is reproduced from [4] and shows how the moment/rotation develops with increasing load at a point in a beam or plate.



The yellow line in this figure indicates how the stress at the surface increases linearly until it reaches yield. The green line indicates the degree of penetration of yield through the section.

Figure 5: Moment/Rotation development through the thickness of a plate

If a problem can be concocted where the moment field is constant over the plate then the plot in figure 5 will apply to all points in the plate and the only stress redistribution that can occur will be through the thickness of the plate. Such a constant moment problem was presented in [4] and is reproduced in figure 6.



$$\begin{Bmatrix} m_x \\ m_y \\ m_{xy} \end{Bmatrix} = \lambda \begin{Bmatrix} a \\ b \\ 0 \end{Bmatrix}$$

Geometric Properties: $l = 1m$; $t = 0.01m$

Elastic Properties: $E = 210GPa$; $\upsilon = 0.3$

Plastic Properties: $\sigma_y = 275MPa$

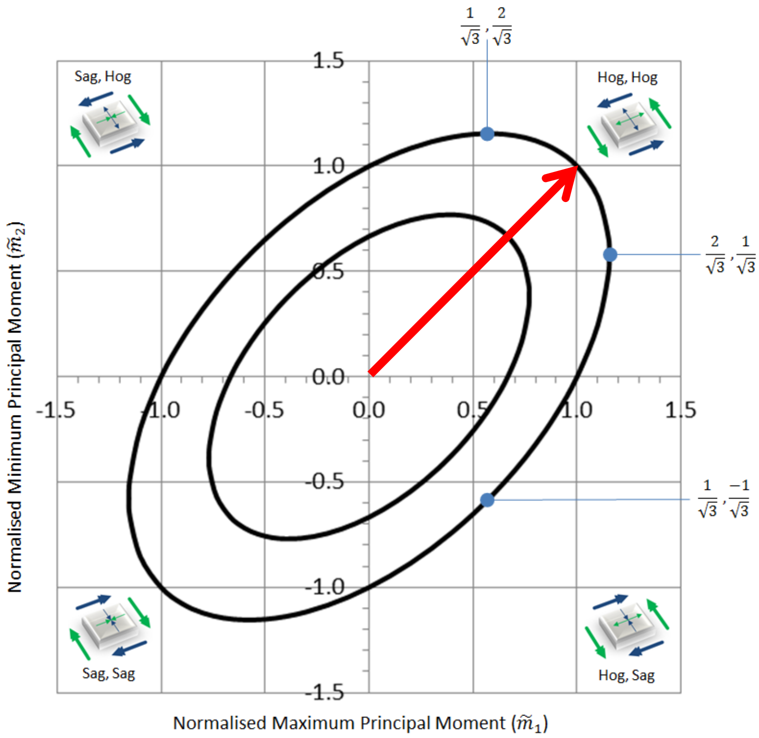
Figure 6: A constant moment plate problem

The manner in which the bending moments redistribute is governed by the chosen yield criterion. The von Mises yield criterion, which is appropriate to steel members and thus for the challenge problem, is shown in figure 7 where it is presented in terms of normalised principal moments which are simply the principal moments ($m_1$and m_2) divided by the collapse moment:

$$m_c = \frac{\sigma_y t^2}{4} = 6875\text{Nm/m}$$

Plastic collapse is governed by the yield criterion and the von Mises yield criterion may be written in terms of the principal moments:

$$m_1^2 + m_2^2 - m_1 m_2 \leq m_c^2$$



The figure shows two von Mises ellipses the smaller one being that of first yield and the large one being that of collapse moment. The inset figures in the corners shows the principal moments acting on an infinitesimal element of the plate (green and blue for the first and second principal moments respectively). They also show the direction of the direct stresses at the top surface of the plate developed by these applied moments.

The red arrow traces the principal moments for the constant moment problem.

Figure 7: The von Mises yield criterion in terms of normalised principal moments

The load factor, λ, is a non-dimensional constant that scales the applied loads and we are interested in determining the value at which the plate will collapse. As the moment field for this problem involves no torsional moments then the principal moments are simply equal to the normal Cartesian components $m_1 = \lambda a$ and $m_2 = \lambda b$. We can thus determine the load factor at collapse, λ_c, in terms of the collapse moment and the amplitudes of the unfactored normal moments:

$$\lambda_c = \frac{m_c}{\sqrt{a^2 + b^2 - ab}}$$

With unit values for the direct moment components, the load factor to cause collapse is equal to the amplitude of the collapse moment:

$$a = b = 1 \rightarrow \lambda_c = 6875$$

The constant moment problem just described is ideal for verifying a finite element system because it has a known exact solution which should be reproduced using a single finite element. This problem will be analysed using the popular commercial finite element tools CS1 and CS2.

As already discussed, a finite element system could get away with a single plate element of the 'thick' type provided the formulation of the element was robust. But, as evidenced by the large number of plate/shell type elements offered in many commercial systems, this ideal seems not to have been realised. This situation is not helpful to the practising engineer who is then faced with selecting an element for his/her plate problem from a wide range of available elements.

A second issue in the simulation of limit analysis with conventional finite element systems is that the solution one is trying to detect is the point at which the structure loses its stiffness when the deflections and strains tend to infinity. This means that one needs to wait for the programme to abort and then have the necessary faith to believe the answer!

The following table presents the finite element collapse moments for the constant moment problem. For both commercial FE systems four noded shell elements with reduced integration were used. In CS1 the element is described as 'a four-node finite strain shell' whilst for CS2 the element is described as 'a four-node doubly curved thin or thick shell, reduced integration, hourglass control, finite membrane strains'. Default solution parameters were used and the number of integration points through the plate thickness was varied. The load applied was 10,000Nm per edge so that the time at collapse for the exact collapse load should be 0.6875 in a 1 second time frame.

Number Integration Points Through Thickness	CS1	CS2
3	/	4583.8 (33.3%)
5 (Default)	6506.3 (5.4%)	6875.4 (0%)
7	6562.5 (4.5%)	6620.4 (3.7%)
9 (Correct Solution)	6875.0 (0%)	6875.1 (0%)
11	/	6783.2 (1.3%)

Table 2: Collapse moments (Nm/m) for two commercial FE systems (constant moment problem)

The default integration scheme for both FE codes is five integration points through the plate thickness. The results indicate, for CS1, that this integration scheme is not sufficient to recover the true solution. It seems, however, that a nine point scheme can provide accurate recovery of the true solution for both CS1 and CS2 although it is interesting that, for CS2, a further increase in the number of integration points leads to a less accurate solution.

Collapse pressures for the challenge plate were generated using fairly coarse meshes in the two commercial finite element systems. The experience gained from the constant moment problems in terms of integration points through the thickness was utilised with nine points being used. The total pressure applied was 1000kPa so that the time at collapse should be around 0.231 in a 1 second time frame. Like the constant moment problem, default solution settings were used for both commercial software tools.

	CS1		CS2	
Mesh	Reduced	Full	Reduced	Full
1x3	387	385	1000 +	386
3x10	235	235	1000 +	241
6x20	225	224	435	234

Table 3: Collapse pressures (kPa) for two commercial FE systems (challenge problem)

The first point to note with regard to the performance of the commercial FE codes on the challenge problem is that convergence is from *above* the true value. This is important since, at least for the challenge problem, coarse meshes produce *unsafe* predictions of the collapse load c.f. the results from EFE which converge from below the true value and are thus always safe irrespective of mesh refinement – see figure 4. It appears also to be the case that reduced integration schemes have a detrimental effect on the prediction of the collapse pressure. This is particularly significant for CS2 which would appear to show that the entire pressure can be taken for the coarser meshes with the reduced integration element. Despite the above concerns, it is pleasing to see, at least for the fully integrated elements, that the collapse load is predicted fairly reasonably (at least in an engineering sense) by both codes. The convergence of the collapse pressure for the challenge plate is shown in figure 8.

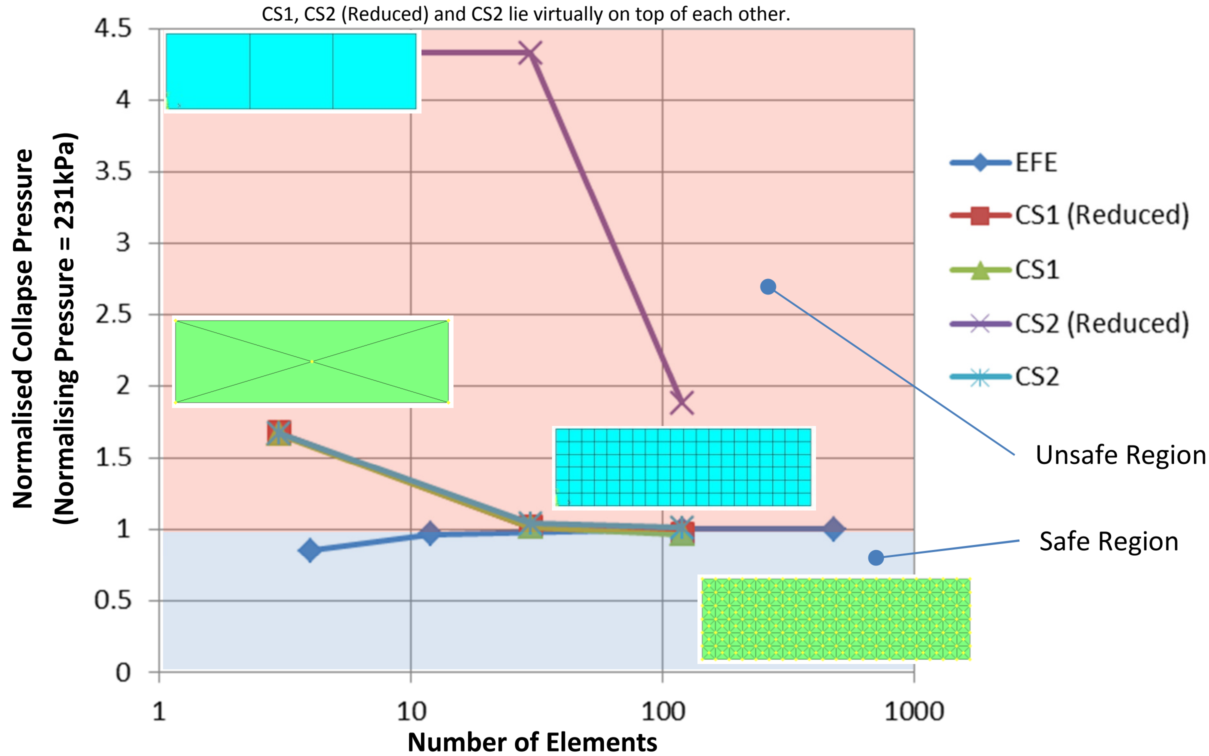


Figure 8: Convergence of the collapse pressure for the challenge plate

Discussion

This challenge has proved interesting at a number of levels. In developing the limit analysis tool EFE and testing it on published results, RMA has been long aware that the results presented in the SDM and in Roark, whilst conservative, are incorrect.

The results published in the SDM are based on the elastic equations of Pounder which, although stated as taking account of corner uplift do not include this non-linear phenomenon – and anyway corner uplift does not significantly influence the maximum deflection and stress for such a plate configuration. The SDM uses Pounder's equations to predict the maximum principal stress in the plate and then factors this with the yield stress to provide what it calls the 'Ultimate Load Capacity'. This phrase is generally understood to mean the load at which *failure* will occur and in the context of steel plates one would generally interpret this as the load at which *failure by collapse* occurs. However, it is clear, from this study, that the meaning implied in the SDM is the load at which first yield occurs. This might of course mean actual failure if the plate were made of a brittle material

(Pounder's work did deal with cast steel plates) but as used by the SDM in the context of Durbar plates, which offer considerable ductility – see figure 9, this is rather misleading.

In contrast to the SDM, Roark makes clear that it is talking about the plastic collapse of plates so we understand the loads it provides to be collapse loads. The results are produced in tabular format for a practical range of plate aspect ratios. The result for the challenge plate aspect ratio is interesting since it is less than 1.5 times the load to produce first yield. It is known, even without allowing for any plastic redistribution across the plate, that the redistribution through the thickness will give a collapse pressure 1.5 times that required to produce first yield and so, as in general redistribution across the plate can and does occur (see appendix), it is clear that this result is going to be rather conservative. The accompanying text in Roark states that the results are likely to be up to 30% in error (hopefully on the safe side - although not specified!) and it is seen using the dedicated and verified limit analysis software EFE that for the largest aspect ratio plate considered in the table, the error is actually greater than 40% - see table 1.

The published results for the collapse load of plates such as the challenge plate are thus rather disappointing and do not serve the practising engineer who might be designing a new plate or assessing an existing plate for a change in duty.

The yield line analysis of the plate gives a collapse load of 216kPa which is some 6% below the correct (verified) value. This serves to remind us that whilst the yield line technique *is* appropriate for the limit analysis of reinforced concrete slabs which adhere to the Nielsen yield criterion, when used for metallic plates that obey the von Mises yield criterion differences may occur. In this case the yield line result is conservative but there are other cases where it can be considerably unsafe.

Whilst RMA have the bespoke limit analysis software EFE to conduct collapse analyses on plates such as the challenge plate, it was realised that most practising engineers would not have access to such software and would need to use conventional finite element codes to simulate the limit analysis solution. To assist in the understanding and verification of simulated limit analysis [4] was prepared. A constant moment problem was presented in this document which possessed a known exact solution both in terms of elasticity and plasticity and for which a finite element system should be able to recover the exact solution with a single four-noded plate/shell element. Analysis of this problem with two commercial FE systems showed that with the default settings one of the systems was *incapable* of capturing the exact result. Further investigation showed that the default integration scheme through the plate thickness was the cause for this issue and that increasing from the default five points to nine points through the thickness cured this problem.

Simulated limit analysis using incremental procedures in a conventional finite element system was recognised as a rather unsatisfactory procedure since one is attempting to predict, with accuracy, a point of structural instability where the displacements and strains tend to infinity. A high degree of reliance/faith on the solution settings is required as the solution only appears when the software has aborted the solution process. Nonetheless, the default settings in the two commercial FE systems tested for this challenge problem did seem appropriate and led to results for the challenge problem that were not unreasonable. It was noted however that for the plate elements used, convergence with mesh refinement was from *above* the exact solution. This form of convergence is potentially <u>unsafe</u> since if the engineer does not perform adequate mesh refinement then the prediction will be *greater* than the exact value. It was also noted for one of the commercial FE systems (CS2) that the reduced integration shell element was incapable of converging on the true collapse load. It is not surprising then that of the responders who used plate/shell elements to model the challenge plate there was a considerable range of results submitted.

Two responders used solid elements to model the challenge plate. Both demonstrated convergence to a solution which was virtually identical to that produced by EFE. The results from one of these analyses are presented in the appendix and served to verify EFE.

Thinking from the perspective of the practising engineer then it has become clear that whilst conventional FE systems can simulate limit analysis problems they are rather unsuited to the problem. Different elements perform in different and sometimes unpredictable manners, the default settings are sometimes not appropriate and the need to rely on a myriad of solution settings for the software to abort at the correct load seems a rather unsatisfactory way of going about things. As usual there is also the essential requirement for good simulation governance by way of verification through mesh convergence studies and when this is not appropriately applied, at least for the plate elements tested, the results might be considerably unsafe. In contrast to this a dedicated or bespoke limit analysis software tool such as EFE requires no the setting of and reliance on solution parameters and achieves a guaranteed safe solution, irrespective of mesh refinement in a fraction of the time taken to simulate the problem in conventional FEA – even for the most refined mesh the solution in EFE took less than a second whereas for the solid models used by some of the respondents the solution took in excess of 5 minutes.

Closure

Beyond simple linear elastic analysis, limit analysis provides further useful information regarding one of the possible failure modes of a structure, i.e. plastic collapse. It provides invaluable information about how much more load your structure can take beyond that predicted from a simple linear-elastic analysis or, alternatively, it enables you to make the most of your available structural capacity. It provides another tool for the engineer to use in the understanding of his/her structure and its response to load.

Published collapse loads for plates aimed at helping the practising engineer are seen to be significantly different (although conservative) from the result produced through limit analysis. This means that engineers using these texts will design structures with more material than is really necessary and/or they will underestimate the residual strength when reassigning the duty of a structural plate element potentially unnecessarily requiring its strengthening or replacement. Neither situation is satisfactory. Commercially available finite element software tools are able to simulate limit analysis solutions with reasonable accuracy BUT the results they produce are strongly dependent on the engineer being skilled in non-linear FEA and having the time and interest in ensuring appropriate simulation governance or verification.

Limit analysis is conservative since it ignores strength enhancing phenomenon such as strain hardening and membrane actions. Whilst large permanent deformations might, in some applications, be considered unacceptable, there are plenty of examples of plates, such as the challenge plate, where large permanent deformations do not affect the serviceability of the plate – see figure 9 for example.

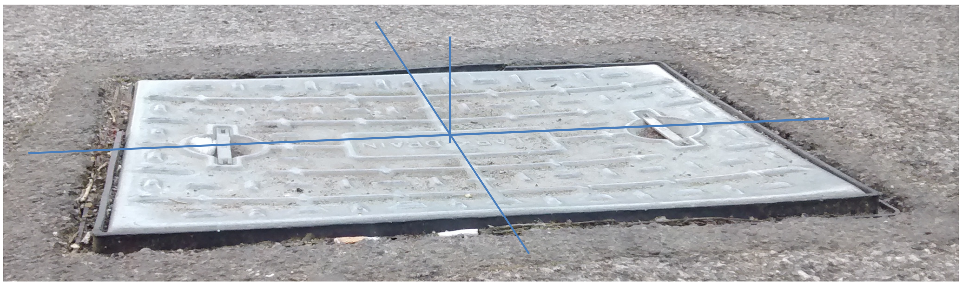


Figure 9: Permanent and large deflections of a plate

The challenge, as presented, was for engineers to 'convince the owner of the building and his risk averse insurer' that the plate, as fitted by the owner, was fit for duty. Given that the collapse pressure is 231kPa then this is some 2.3 times the maximum load the plate is ever likely to see.

The owner of the building (Ferdinand Frugal of Frugal Castle) was rather peeved at the outcome of this study in that he realises he could have got away with using a thinner (8mm instead of 10mm) plate:

$$t_{new} > \sqrt{\frac{100kPa \cdot 10^2}{231kPa}} = 6.58mm$$

The difference in the cost of the plate is some £64 which Ferdinand feels would have been better spent on a case of cheap Whisky for his guests!

Product	Description	UOS	Qty	Price	Value	
35016318	Mild Steel Durbar 2000 x 1000 x 8mm Grade S235	EACH	1	£121.18	£121.18	Delete
350163110	Mild Steel Durbar 2000 x 1000 x 10mm Grade S235	EACH	1	£173.94	£173.94	Delete
			Update			

Standard Delivery charges apply to the UK Mainland only. For delivery outside of the UK Mainland please contact your local branch.

Goods Value	£295.12
VAT Value	£59.02
Invoice Total	£354.14

Continue Shopping | Empty Basket | Checkout

Figure 10: Cost of Durbar plate for the challenge problem

References

[1] 'The fib Model Code for Concrete Structures 2010: A Critique of the Comments on Finite Element Analysis' (http://www.ramsay-maunder.co.uk/r--d/resources/)

[2] C.C. Pounder, 'The Design of Flat Plates', The Association of Engineering and Shipbuilding Draughtsmen, 4th Edition, 1949.

[3] 'Yield Line Analysis of Reinforced Concrete Slabs: is the 10% Rule Safe?' http://www.ramsay-maunder.co.uk/downloads/bm_yield_line_article.pdf

[4] 'Simulated Limit Analysis of Plates using Incremental Plasticity Finite Elements' (http://www.ramsay-maunder.co.uk/r--d/resources/)

Appendix – Development of Plasticity and Verification of EFE

The results shown in this appendix were generated by Matt Watkins of ESRD Inc. using his company's finite element software StressCheck. The software provides objective solution verification information by p-refinement which is an essential component of simulation governance. Like one other responder Matt used solid elements to model the challenge plate and the collapse pressure from both responders who used solid elements produced collapse pressures of about 231kPa agreeing almost exactly with the result produced by EFE.

The following figure shows the development of utilisation with increasing pressure from the elastic solution through to collapse. The development of plasticity across the plate is clearly seen with the utilisation at collapse being rather similar to that produced by EFE – c.f. figure 4.

103kPa

118 (first yield)

142

170

231 (collapse)

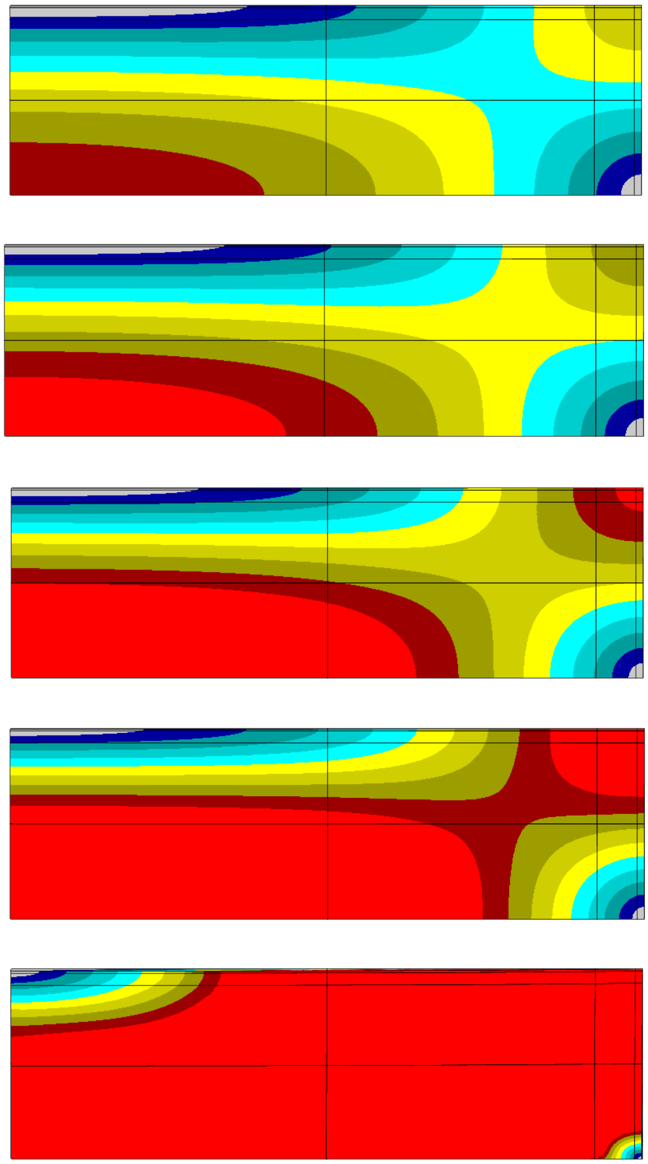


Figure 11: Utilisation contours from the elastic solution to collapse